New A hybrid Hestenes-Stiefel and Dai-Yuan conjugate gradient algorithms for unconstrained optimization

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Abstract
In this Research we developed a New Hybrid method of conjugate gradient type, this Method depends basically on combining Hestenes-Stiefel and Dai-Yuan algorithms by using spectral direction conjugate algorithm, which is developed by Yang Z & Kairong W [19]. The developed method becomes converged by assuming some hypothesis. The numerical results show the efficiency of the developed method for solving test Unconstrained Nonlinear Optimization problems.

1- Introduction

The non-linear conjugate gradient (CG) method is a very useful technique for solving large scale unconstrained minimization problems and has wide applications in many fields [11]. This method is an iterative process which requires at each iteration the current gradient and previous direction, which is characterized by low memory requirements and strong local and global convergence properties [4 and 16].

In this paper, we focus on conjugate gradient methods applied to the non-linear unconstrained minimization problem:

$$\min f(x), \quad x \in R^n \; . \; ... \quad (1)$$

Where $f: R^n \to R$ is continuously differentiable function and bounded below. A conjugate gradient method generates a sequence x_k, $k \ge 1$ starting from an initial guess $x_1 \in R^n$, using the recurrence

$$x_{k+1} = x_k + \alpha_k d_k \quad \cdots \quad (2)$$

Where the positive step size α_k is obtained by a line search, and the directions d_k are generated by the rule:

$$d_1 = -g_1$$
$$d_{k+1} = -g_{k+1} + \beta_k d_k \quad \cdots \quad (3)$$

Where $g_k = \nabla f(x_k)$, and let $y_k = g_{k+1} - g_k$ and $s_k = x_{k+1} - x_k$, here β_k is the CG update parameter. Different CG methods corresponding to different choice for the parameter β_k see [2,5 and 12]. The first CG algorithm for non-convex problems was proposed by Fletcher and Revees(FR) in 1964 [13], which defined as

$$\beta_k^{FR} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k} . \quad \cdots \quad (4)$$

We know that the other equivalents forms for β_k are Polack-Ribeir (PR) and Hestenes- Stiefel (HS) for example

$$\beta_k^{PR} = \frac{g_{k+1}^T y_k}{g_k^T g_k} . \quad \text{and} \quad \beta_k^{HS} = \frac{g_{k+1}^T y_k}{d_k^T y_k} . \cdots \quad (5)$$

Although all the above formulas are equivalent for convex quadratic functions, but they have different performance for non-quadratic functions, the performance of a non-linear CG algorithm strongly depends on coefficient β_k. Dai and Yuan (DY) in [7] proposed a non-linear CG method (2) and (3) with β_k defined as

$$\beta_k^{DY} = \frac{g_{k+1}^T g_{k+1}}{d_k^T y_k} . \cdots \quad (6)$$

Which generates a descent search directions

$$d_k^T g_k < 0. \quad \cdots \quad (7)$$

At every iteration k and convergence globally to the solution if the following standard Wolfe conditions are used to accept the step-size α_k [3]:

$$f(x_k + \alpha_k d_k) \le f(x_k) + c_1 \alpha_k g_k^T d_k \cdots \quad (8)$$
$$g(x_k + \alpha_k d_k)^T d_k \ge c_2 g_k^T d_k \cdots \quad (9)$$

Where $0 < c_1 < c_2 < 1$. Condition (8) stipulates a decrease of f along d_k if (7) satisfied. Condition (9) is called the curvature condition and it's role is to force α_k to be sufficiently far a way from zero [16]. Which could happen if only condition (8) were to be used. Conditions (8) and (9) are called standard Wolfe conditions (SDWC). Notice that if equation (8) satisfied then always there exists $\bar{\alpha} > 0$ such that for any $\alpha_k \in [0, \bar{\alpha}]$ the conditions (8) and (9) will be satisfied according to the theorem (1) given later. If we wish to find a point α_k , which is closer to a solution of the one dimensional problem

$$\underset{\alpha>0}{Min}\, \phi(\alpha) = \min_{\alpha>0} f(x_k + \alpha d_k) \quad \cdots \quad (10)$$

Than a point satisfying (8) and (9) we can impose on α_k the strong Wolfe conditions (STWC):

$$f(x_k + \alpha_k d_k) \le f(x_k) + c_1 \alpha_k g_k^T d_k \cdots \quad (11)$$
$$|g(x_k + \alpha_k d_k)^T d_k| \le c_2 |g_k^T d_k| \cdots \quad (12)$$

Where $0 < c_1 < c_2 < 1$. In contrast to (SDWC) $g_{k+1}^T d_k$ cannot be arbitrarily large [16]. The (STWC) with the sufficient descent property

$$d_k^T g_k < -c\|g_k\|, \; c \in (0,1) \quad \cdots \quad (13)$$

Widely used in the convergence analysis for the CG methods.

Theorem (1)

Assume that f is continuously differentiable and that is bounded below along the line $x = x_k + \alpha d_k$, $\alpha \in (0,\infty)$. Suppose also that d_k is a direction of descent (7) is satisfied if $0 < c_1 < c_2 < 1$ then there exist nonempty intervals of step lengths satisfying the (SDWC) and (STWC) conditions, For proof see [16].

The Fletcher-Revees (FR) and Dai-Yuan (DY) methods have common numerator $g_{k+1}^T g_{k+1}$. One theoretical difference between these methods and other choices for the update parameter β_k is that the global convergence theorems only require the Lipschitz assumption not the bounded ness assumption [11].

The global convergence for the methods with $g_{k+1}^T g_{k+1}$ in the numerator of β_k established with exact and inexact line searches for general functions [3,8 and 20]. Despite the strong convergence theory that has been developed for methods with $g_{k+1}^T g_{k+1}$ in the numerator of β_k, these methods are all susceptible to jamming, that is they begin to take small steps without making significant progress to the minimum [11]. On the other hand the convergence of the methods with $g_{k+1}^T y_k$ in the numerator (PR) and (HS) for general non-linear function are uncertain, in general the performance of these methods is better than the performance of the methods with $g_{k+1}^T g_{k+1}$ in the numerator of β_k see [11], but they have weaker convergence theorems.

This paper is organized as follows in section 2 New A hybrid conjugate gradient algorithm for unconstrained optimization. In section 3 we will show that our algorithm satisfies descent condition for every iteration. Section 4 we will show that our algorithm satisfies Global convergence condition for every iteration. Section 5 presents numerical experiments and comparisons.

2- New A hybrid conjugate gradient algorithm for unconstrained optimization

In this section, we derive New A hybrid conjugate gradient algorithm for unconstrained optimization. Based on combining Hestenes-Stiefel and Dai-Yuan algorithms by using direction conjugate algorithm.

We know the direction formula

$$d_{k+1} = -g_{k+1} + \beta_{k+1} d_k \quad \ldots (14)$$

Hestenes-Stiefel algorithm

$$\beta_k^{HS} = \frac{g_{k+1}^T y_k}{d_k^T y_k} \quad \ldots. \quad (15)$$

Dai-Yuan algorithm

$$\beta_k^{DY} = \frac{\|g_{k+1}\|^2}{y_k^T d_k} \quad \ldots..(16)$$

Suppose that

$$\beta_{k+1}^{*} = \eta \beta_{k+1}^{HS} \quad \ldots.(17)$$

$$\beta_{k+1}^{**} = \eta \beta_{k+1}^{DY} + (1-\eta)\beta_{k+1}^{HS} \quad \ldots (18)$$

$$d_{k+1} = -g_{k+1} + \eta \beta_{k+1}^{HS} d_k \quad \ldots...(19)$$

$$d_{k+1} = -g_{k+1} + (\eta \beta_{k+1}^{DY} + (1-\eta)\beta_{k+1}^{HS}) d_k \quad \ldots (20)$$

Equality of equation (19) with (20) and note of the d_k equal in equations (19) and (20) we get

$$-g_{k+1} + \eta \beta_{k+1}^{HS} d_k = -g_{k+1} + (\eta \beta_{k+1}^{DY} + (1-\eta)\beta_{k+1}^{HS}) d_k \quad \ldots(21)$$

Subtracting the g_{k+1} of two said from above equation we have

$$\eta \beta_{k+1}^{HS} d_k = (\eta \beta_{k+1}^{DY} + (1-\eta)\beta_{k+1}^{HS}) d_k \quad \ldots(22)$$

After some algebra, we get

$$\eta = \frac{\beta_{k+1}^{HS}}{2\beta_{k+1}^{HS} - \beta_{k+1}^{DY}} \quad \ldots..(23)$$

Substituting η in the equation (20)

$$\beta_{k+1}^{KH1} = \frac{\beta_{k+1}^{HS}\beta_{k+1}^{DY}}{2\beta_{k+1}^{HS} - \beta_{k+1}^{DY}} + \left(1 - \frac{\beta_{k+1}^{HS}}{2\beta_{k+1}^{HS} - \beta_{k+1}^{DY}}\right)\beta_{k+1}^{HS} \quad \ldots. (24)$$

After some algebra of above equation we get a new formula denote by β_{k+1}^{KH1} is defined by

$$\beta_{k+1}^{KH1} = \frac{(g_{k+1}^T y_k)^2}{d_k^T y_k (2 g_{k+1}^T y_k - \|g_{k+1}\|^2)} \quad \ldots. (25)$$

Substituting above equation in spectral direction conjugate algorithm, which is developed by Yang Z & Kairong W [19]. There for we have

$$d_{k+1} = -(\xi + \beta_{k+1}^{KH1} \frac{d_k^T y_k}{\|g_{k+1}\|^2}) g_{k+1} + \beta_{k+1}^{KH1} d_k \quad \ldots (26)$$

New Algorithm $KH1$

Step 1. Initialization. Select $x_1 \in R^n$ and the parameters $0 \le \xi \le 1$.

Compute $f(x_1)$ and g_1. Consider $d_1 = -g_1$ and set the initial guess $\alpha_1 = 1/\|g_1\|$.

Step 2. Test for continuation of iterations. If $\|g_{k+1}\| \le 10^{-6}$, then stop.

Step 3. Line search. Compute $\alpha_{k+1} > 0$ satisfying the Wolfe line search condition (11) and (12) and update the variables $x_{k+1} = x_k + \alpha_k d_k$.

Step 4. β_{k+1}^{KH1} conjugate gradient parameter which defined in (25) and (26).

Step 5. Direction new computation, Compute $d_{k+1} = -(\xi + \beta_{k+1}^{KH1} \frac{d_k^T y_k}{\|g_{k+1}\|^2}) g_{k+1} + \beta_{k+1}^{KH1} d_k$. If the restart criterion of Powell $|g_{k+1}^T g_k| \ge 0.2 \|g_{k+1}\|^2$, is satisfied, then set $d_{k+1} = -g_{k+1}$

Otherwise define $d_{k+1} = d$. Compute the initial guess $\alpha_k = \alpha_{k-1} \|d_{k-1}\| / \|d_k\|$, set $k = k+1$ and continue with step2 .

3- The Descent Property of the New Method

Below we have to show the descent property for our proposed a hybrid conjugate gradient algorithm, denoted by β_{k+1}^{KH1}. In the following Theorem.

Theorem (2)

suppose that $2g_{k+1}^T y_k - \|g_{k+1}\|^2 > 0$ then the search direction d_{k+1} and β_{k+1}^{KH1} given in equation

$$d_{k+1} = -(\xi + \beta_{k+1}^{KH1} \frac{d_k^T y_k}{\|g_{k+1}\|^2}) g_{k+1} + \beta_{k+1}^{KH1} d_k$$

(**)

Will hold for all $k \geq 1$

Proof:-

The proof is by induction.

1- If k=1 then $g_1^T d_1 < 0$ $d_1 = -g_1 \rightarrow < 0$.

Since it by assumption $2g_{k+1}^T y_k - \|g_{k+1}\|^2 > 0$ then $\beta_{k+1}^{KH1} > 0$ there for $d_k^T y_k > 0$ by standard Wolfe conditions.

2- Let the relation $g_k^T d_k < 0$ for all k .

3- We prove that the relation is true when $k = k+1$ by multiplying the equation (**) in g_{k+1} we obtain

$$g_{k+1}^T d_{k+1} = -(\xi + \beta_{k+1}^{KH1} \frac{d_k^T y_k}{\|g_{k+1}\|^2}) g_{k+1}^T g_{k+1} + \beta_{k+1}^{KH1} g_{k+1}^T d_k$$

…(27)

$$g_{k+1}^T d_{k+1} = -\xi g_{k+1}^T g_{k+1} - (\beta_{k+1}^{KH1} \frac{d_k^T y_k}{\|g_{k+1}\|^2}) g_{k+1}^T g_{k+1} + \beta_{k+1}^{KH1} g_{k+1}^T d_k$$

… (28)

$$g_{k+1}^T d_{k+1} = -\xi g_{k+1}^T g_{k+1} - \beta_{k+1}^{KH1} d_k^T y_k + \beta_{k+1}^{KH1} g_{k+1}^T d_k$$

…(29)

After some algebra, we get

$$g_{k+1}^T d_{k+1} = -\xi g_{k+1}^T g_{k+1} - \beta_{k+1}^{KH1} d_k^T g_{k+1} + \beta_{k+1}^{KH1} d_k^T g_k + \beta_{k+1}^{KH1} g_{k+1}^T d_k$$

…(30)

$$g_{k+1}^T d_{k+1} = -\xi g_{k+1}^T g_{k+1} - \beta_{k+1}^{KH1} d_k^T g_{k+1} + \beta_{k+1}^{KH1} d_k^T g_k + \beta_{k+1}^{KH1} g_{k+1}^T d_k$$

…(31)

$$g_{k+1}^T d_{k+1} = -\xi g_{k+1}^T g_{k+1} + \beta_{k+1}^{KH1} d_k^T g_k < 0 \quad \text{…. (32)}$$

$$g_{k+1}^T d_{k+1} < 0 \quad \text{…………….(33)}$$

4- Global convergence analysis

Next we will show that CG method with β_{k+1}^{KH1} converges globally. We need the following assumption for the convergence of the proposed new algorithm.

Assumption (1)

1-Assume f is bound below in the level set $S = \{x \in R^n : f(x) \leq f(x_\circ)\}$; In some Initial point.

2-f is continuously differentiable and its gradient is Lipshitz continuous, there exist $L > 0$ such that:

$$\|g(x) - g(y)\| \leq L\|x - y\| \quad \forall x,y \in N \quad \text{…(34)}$$

3- f is uniformly convex function, then there exists a constant $\mu > 0$ such that

$$(\nabla f(x) - \nabla f(y))^T (x - y) \geq \mu \|x - y\|^2 \text{ , for any } x, y \in S$$

…(35)

or equivalently

$$y_k^T s_k \geq \mu \|s_k\|^2 \quad \text{and} \quad \mu \|s_k\|^2 \leq y_k^T s_k \leq L \|s_k\|^2 \quad \cdots$$

(36)

On the other hand, under Assumption(1), It is clear that there exist positive constants B such

$$\|x\| \leq B \quad , \forall x \in S \cdots (37)$$

$$\|\nabla f(x)\| \leq \bar{\gamma} \quad , \forall x \in S \quad \cdots (38)$$

Lemma(1)

Suppose that Assumption (1) and equation (37) hold. Consider any conjugate gradient method in from (2) and (3), where d_k is a descent direction and α_k is obtained by the strong Wolfe line search. If

$$\sum_{k>1} \frac{1}{\|d_{k+1}\|^2} = \infty \cdots (39)$$

then we have

$$\liminf_{k \to \infty} \|g_k\| = 0$$

More details can be found in [1,9 and 14].

Theorem (3)

Suppose that Assumption (1) and equation (37) and the descent condition hold. Consider a conjugate gradient method in the form

$$d_{k+1} = -(\xi + \beta_{k+1}^{KH1} \frac{d_k^T y_k}{\|g_{k+1}\|^2}) g_{k+1} + \beta_{k+1}^{KH1} d_k$$

where α_k is computed from Wolfe line search condition (11) and (12) with $2g_{k+1}^T y_k - \|g_{k+1}\|^2 > 0$,

If the objective function is uniformly on set S, then $\liminf_{n \to \infty} \|g_k\| = 0$.

Proof:-

Firstly, we need substituting our new β_{k+1}^{KH1}, in the direction d_{k+1} there for we obtain

$$d_{k+1} = -(\xi + \beta_{k+1}^{KH1} \frac{d_k^T y_k}{\|g_{k+1}\|^2}) g_{k+1} + \beta_{k+1}^{KH1} d_k \quad \cdots (40)$$

After simplify above equation we get

$$d_{k+1} = -\xi g_{k+1} - \beta_{k+1}^{KH1} \frac{d_k^T y_k}{\|g_{k+1}\|^2} g_{k+1} + \beta_{k+1}^{KH1} d_k \quad \text{….}$$

(41)

$$d_{k+1} = -\xi g_{k+1} - \beta_{k+1}^{KH1} (\frac{d_k^T y_k}{\|g_{k+1}\|^2} g_{k+1} - d_k) \quad \cdots$$

(42)

$$d_{k+1} = -\xi g_{k+1} - \beta_{k+1}^{KH1} (\frac{d_k^T y_k g_{k+1} - \|g_{k+1}\|^2 d_k}{\|g_{k+1}\|^2})$$

…(43)

$$\|d_{k+1}\|^2 = \left\| -\xi g_{k+1} - \beta_{k+1}^{KH1} (\frac{d_k^T y_k g_{k+1} - \|g_{k+1}\|^2 d_k}{\|g_{k+1}\|^2}) \right\|^2$$

….(44)

$$\|d_{k+1}\|^2 \le \xi \|g_{k+1}\|^2 + \beta_{k+1}^{KH1} \left\| \left(\frac{d_k^T y_k g_{k+1} - \|g_{k+1}\|^2 d_k}{\|g_{k+1}\|^2} \right) \right\|^2 \quad \ldots(45)$$

Using Chouchy Schwartez inequality together with equation (37) we get

$$\|d_{k+1}\|^2 \le \xi \|g_{k+1}\|^2 + \beta_{k+1}^{KH1} \left(\frac{L\|d_k\|^2 \|g_{k+1}\|^2 + \|g_{k+1}\|^2 \|d_k\|^2}{\|g_{k+1}\|^2} \right) \quad \ldots(46)$$

$$\|d_{k+1}\|^2 \le \xi \|g_{k+1}\|^2 + \beta_{k+1}^{KH1} (cL + c) \quad \ldots(47)$$

By using Assumption(1) we get

$$\|d_{k+1}\|^2 \le \xi \overline{\gamma}^2 + c\beta_{k+1}^{KH1}(L+1) \quad \ldots(48)$$

$$\|d_{k+1}\|^2 \le u \frac{1}{\overline{\gamma}^2} \quad \ldots(49)$$

$$\sum_{k=1}^{\infty} \frac{1}{\|d_k\|^2} \ge \frac{1}{u} \overline{\gamma}^2 \sum_{k \ge 1} 1 = \infty \quad \ldots(50)$$

By using Lemma (1) then we get

$$\lim_{k \to \infty} \|g_k\| = 0 \quad \ldots(51)$$

5- Numerical results and comparisons

In this section, we compare the performance of new formal β_{k+1}^{KH1} developed a New Hybrid method of conjugate gradient method to other classical conjugate gradient method (Hestenes-Stiefel and Dai-Yuan algorithms). we have selected (20) large scale unconstrained optimization problem, for each test problems taken from (Andrie, 2008) [6]. For each test function we have considered numerical experiments with the number of variables $n = 100, \ldots, 1000$. These two new versions are compared with well-known conjugate gradient algorithm, the Hestenes-Stiefel and Dai-Yuan algorithms. All these algorithms are implemented with standard Wolfe line search conditions (11) and (12) with. In all these cases, the stopping criteria is the $\|g_k\| = 10^{-6}$. All codes are written in doble precision FORTRAN Language with F77 default compiler settings. The test functions usually start point standard initially summary numerical results recorded in the figures (1),(2),(4),(3). The performance profile by Dolan and More´ [10] is used to display the performance of the developed a New Hybrid method of conjugate gradient algorithm with Hestenes-Stiefel and Dai-Yuan algorithms. Define $p = 200$ as the whole set of n_p test problems and $S = 3$ the set of the interested solvers. Let $l_{p,s}$ be the number of objective function evaluations required by solver S for problem p. Define the performance ratio as

$$r_{p,s} = \frac{l_{p,s}}{l_p^*} \quad \ldots(52)$$

Where $l_p^* = \min\{l_{p,s} : s \in S\}$. It is obvious that $r_{p,s} \ge 1$ for all p, s. If a solver fails to solve a problem, the ratio $r_{p,s}$ is assigned to be a large number M. The performance profile for each solver S is defined as the following cumulative distribution function for performance ratio $r_{p,s}$,

$$\rho_s(\tau) = \frac{size\{p \in P : r_{p,s} \le \tau\}}{n_p} \quad \ldots(53)$$

Obviously, $p_s(1)$ represents the percentage of problems for which solver S is the best. See [10] for more details about the performance profile. The performance profile can also be used to analyze the number of iterations, the number of gradient evaluations and the cpu time. Besides, to get a clear observation, we give the horizontal coordinate a log-scale in the following figures.

Note:-

1- By using wolfe conditions (11) and (12) to choose α_k [18].

2- $\xi = 0$.

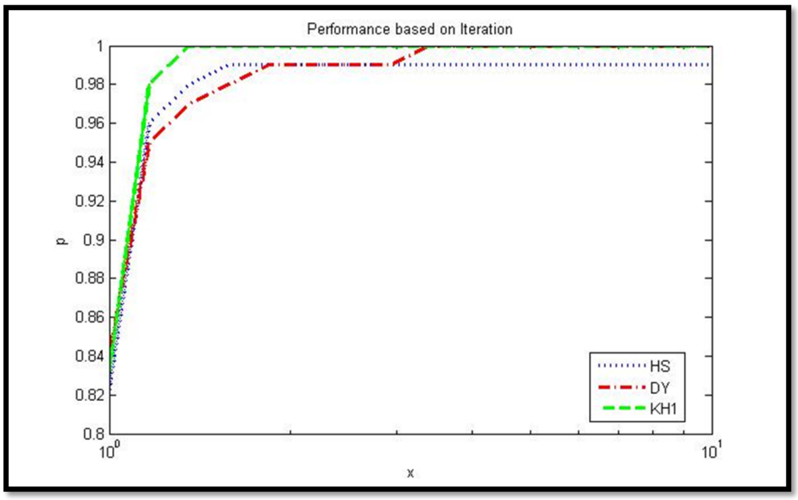

Figure (1). Comparison based on number of iteration for the algorithms HS, DY and KH1.

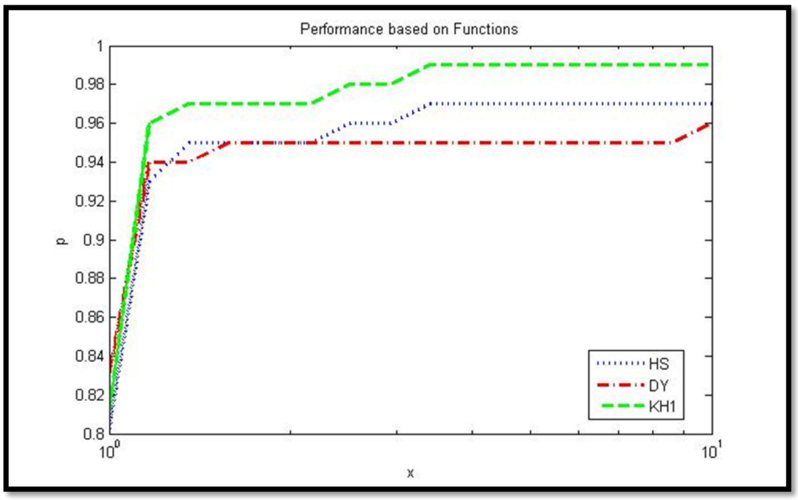

Figure (2). Comparison based on number of function evaluations for the algorithms HS, DY and KH1.

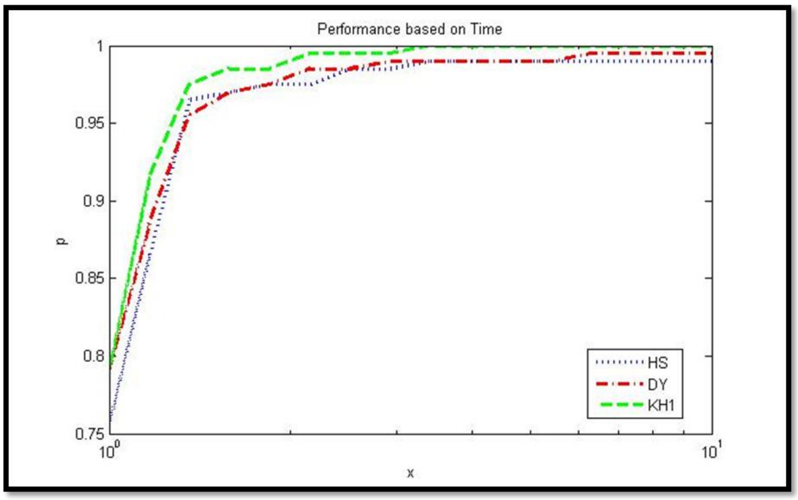

Figure (3). Comparison based on time for the algorithms HS, DY and KH1.

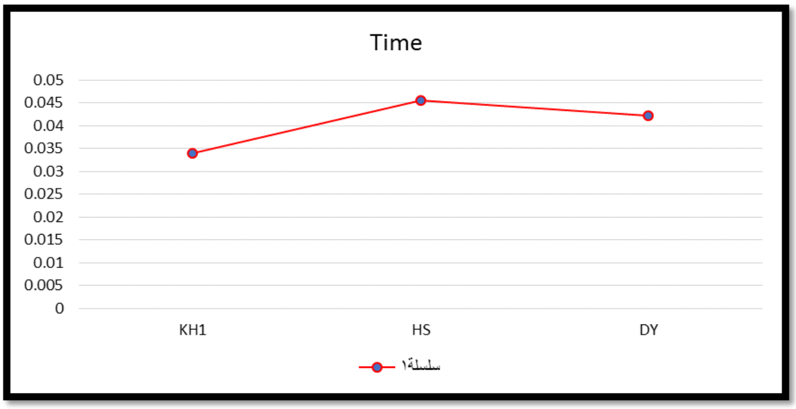

Figure (4). Comparison based on total time needed for solving 200 test problem .

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خوارزمية جديدة مهجنة بين هستينس–استفيل وداي–يوان في التدرج المترافق في الامثلية غير المقيدة

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الملخص

في هذا البحث تم تطوير طريقة جديدة من طرائق التدرج المترافق الهجينة. وتعتمد هذه الطريقة في الأساس على تهجين خوارزميات هستينس–استفيل وداي–يوان. وذلك بأستخدام خوارزمية متجهات مترافقة طيفية المطورة من قبل Yang Z & Kairong W [19]. والطريقة المطورة ذات تقارب شامل تحت فرضيات معينة. وأشارت النتائج العديدة الى كفاءة هذه الطريقة في حل دوال الاختبار اللاخطية في الامثلية غير المقيدة المعطى.