K- constant type Kahler and Nearly Kahler manifolds for conharmonic curvature tensor

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Abstract
The constant of permanence conharmonic type kahler and nearly kahler manifold conditions are obtained when the Nearly Kahler manifold is a manifold conharmonic constant type (K). and also proved that M nearly kahler manifold of pointwise constant holomorphic sectional conharmonic (PHKm(X)) – curvature) curvature tensor if the components of holomorphic sectional (HS- curvature) curvature tensor in the adjoint G-structure space that satisfies condition .
Keywords. Conharmonic constant type kahler and nerly kahler manifolds, manifold pointwise constant holomorphic sectional conharmonic .

Definition(1) [5]
Let (M, J, g) is NK- manifold of dimension 2n , K - tensor conharmonic curvature. that components tensor Riemann- Christoffel on space of the adjoint, G-structure will be Reminded [13] look like:

$$1)\ R^{a}_{bcd}=R^{a}_{\hat b\hat c\hat d}=0;\ 2)\ R^{a}_{bc\hat d}=-R^{a}_{bd\hat c}=A^{ad}_{bc}-B^{adh}B_{hbc};$$
$$3)\ R^{\hat a}_{\hat bcd}=R^{\hat a}_{b\hat c\hat d}=0;\ 4)\ R^{\hat a}_{\hat bc\hat d}=-R^{\hat a}_{\hat bd\hat c}=-A^{bd}_{ac}+B^{bdh}B_{hac};$$
$$5)\ R^{a}_{\hat bc\hat d}=R^{a}_{\hat b\hat cd}=0;\ 6)\ R^{a}_{\hat bcd}=2B^{abh}B_{hcd};\ 7)\ R^{\hat a}_{bcd}=0;$$
$$8)\ R^{\hat a}_{bc\hat d}=R^{\hat a}_{b\hat cd}=0;\ 9)\ R^{\hat a}_{b\hat c\hat d}=2B^{cdh}B_{hab};\ 10)\ R^{\hat a}_{\hat b\hat c\hat d}=0.$$
...(1)

and the components of Ricci tensor S on space of the adjoint G-structure look like:

$$1)\ S_{ab}=0;\ 2)\ S_{\hat a\hat b}=0;\ 3)\ S_{a\hat b}=S_{b\hat a}=A^{ac}_{bc}+3B^{ach}B_{bch}.$$
….(2)

At last scalar curvature χ nearly Kahler manifolds in the space of the adjoint G-structure is calculated under the formula

$$\chi = 2A^{ab}_{ab}+6B^{abc}B_{abc}.\ ....(3)$$

Proposition(2) [5]
Let (M, J, g) - NK- manifold. The curvature tensor conharmonic was introduced will be Reminded Ishii [11] as a tensor of type (4, 0) on n-dimensional Riemannian manifold, determined by the formula

$K(X,Y,Z,W) = R(X,Y,Z,W)$ ⍰ $\frac{1}{2(n-1)}$ $[g(X,W)S(Y,Z)-$ $g(X,Z)S(Y,W)$ + $g(Y,Z)S(X,W)$ ⍰ $g(Y,W)S(X,Z)]$ …(4)

Where R – the Riemann curvature tensor, S - Ricci tensor. This tensore is invariant under conharmonic transformations, i.e. with conformal transformations of space keeping a harmony of functions. Let's consider properties tensor conharmonic curvature: [6]

$$1)\ K(X,Y,Z,W)=R(X,Y,Z,W)-\frac{1}{2(n-1)}[g(X,W)S(Y,Z)-g(X,Z)S(Y,W)]-$$
$$-\frac{1}{2(n-1)}[g(Y,Z)S(X,W)-g(Y,W)S(X,Z)]=-R(Y,X,Z,W)+$$
$$+\frac{1}{2(n-1)}[-g(X,W)S(Y,Z)+g(X,Z)S(Y,W)]+$$
$$+\frac{1}{2(n-1)}[-g(Y,Z)S(X,W)+g(Y,W)S(X,Z)]=-K(Y,X,Z,W).$$

Properties are similarly proved:
1) $K(X,Y,Z,W)$ = ⍰ $K(X,Y, W, Z)$.
2) $K(X,Y,Z,W) = K(Z,W, X,Y)$.
3) $K(X,Y,Z,W) + K(Y,Z, X,W) + K(Z, X,Y,W) = 0$.

Thus conharmonic curvature tensor satisfies all the properties of algebraic curvature tensor:
1) $K(X,Y,Z,W)$ = ⍰ $K(Y, X , Z,W)$;
2) $K(X,Y,Z,W)$ = ⍰ $K(X, Y, W , Z)$;
3) $K(X,Y,Z,W) + K(Y, Z, X, W) + K(Z, X, Y, W) = 0$;
4) $K(X,Y,Z,W) = K(Z, W, X, Y)$; $X ,Y, Z, W \in X(M)$
,where $X(M)$ is module of vector fields on a manifold M (5)

Lets Calculate Components of the Components tensor curvature on space of the adjonit G-structure for Nearly Kahler manifold, In terms of the covariant components of the form[5]
We shall write down as :

$$K_{ijkl} = R_{ijkl} - \frac{1}{2(n-1)}(g_{ik}\ \delta_{jl} + g_{jl}\ \delta_{jk} - g_{jl}\ \delta_{jk} - g_{jk}\ \delta_{il})\ ….(6)$$

we will extract compounds the tensor conharmonic which may extracted from the source in another way, as in the following theory.

Theorem(3) :
The components of The conharmonic constant concircular tenser of Nearly Kahler -manifold in the adjoint G-structure space are given by the following forms:

1- $K_{\hat a bc\hat d} = A^{ad}_{bc} - B^{adh}B_{hbc} - \frac{1}{2(n-1)}(\delta^a_c\ \delta^d_b + \delta^d_b\ \delta^a_c)$ ….(7)

2- $K_{ab\hat c\hat d} = 2\ B^{cdh}\ B_{abh} - \frac{1}{2(n-1)}(\delta^c_a\ \delta^d_b + \delta^d_b\ \delta^c_a) + (\delta^d_a\ \delta^c_b + \delta^c_b\ \delta^d_a)$ …(8)

and the other are conjugate to the above components or equal to zero

proof:

$$K_{ijkl} = R_{ijkl} - \frac{1}{2(n-1)}(g_{ik}\ \delta_{jl} + g_{jl}\ \delta_{jk} - g_{jl}\ \delta_{jk} - g_{jk}\ \delta_{il})$$

1- Put i = a j = b k = c and l = d

$$K_{abcd} = R_{abcd} - \frac{1}{2(n-1)}(g_{ac}\ \delta_{bd} + g_{bd}\ \delta_{ac} - g_{ad}\ \delta_{bc} - g_{bc}\ \delta_{ad})$$
$$= 0 - \frac{1}{2(n-1)}(0)$$

$= 0$

2- Put $I = \hat{a}$ $j = b$ $k = c$ and $l = d$

$K_{\hat{a}bcd} = R_{\hat{a}bcd} - \frac{1}{2(n-1)} (g_{\hat{a}c} \; \delta_{bd} + g_{bd}\, \delta_{\hat{a}c} - g_{\hat{a}d}\; \delta_{bc} - g_{bc}\, \delta_{\hat{a}d})$

$= 0 - \frac{1}{2(n-1)} (0)$

$= 0$

3- put $i = a$, $j = \hat{b}$, $k = c$ and $l = d$

$K_{a\hat{b}cd} = R_{a\hat{b}cd} - \frac{1}{2(n-1)} (g_{ac} \; \delta_{\hat{b}d} + g_{\hat{b}d}\, \delta_{ac} - g_{ad}\; \delta_{\hat{b}c} - g_{\hat{b}c}\, \delta_{ad})$

$= 0 - \frac{1}{2(n-1)} (0)$

$= 0$

4- Put $i = a$, $j = b$, $k = \hat{c}$ and $l = d$

$K_{ab\hat{c}d} = R_{ab\hat{c}d} - \frac{1}{2(n-1)} (g_{a\hat{c}} \; \delta_{bd} + g_{bd} \; \delta_{a\hat{c}} - g_{ad}\; \delta_{b\hat{c}} - g_{b\hat{c}} \; \delta_{ad})$

$= 0 - \frac{1}{2(n-1)} (0)$

$= 0$

5- Put $i = a$, $j = b$, $k = c$ and $l = \hat{d}$

$K_{abc\hat{d}} = R_{abc\hat{d}} - \frac{1}{2(n-1)} (g_{ac} \; \delta_{b\hat{d}} + g_{b\hat{d}} \; \delta_{ac} - g_{a\hat{d}}\; \delta_{bc} - g_{bc} \; \delta_{a\hat{d}})$

$= 0 - \frac{1}{2(n-1)} (0)$

$= 0$

6- Put $i = \hat{a}$ $j = \hat{b}$ $k = c$ and $l = d$

$K_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} - \frac{1}{2(n-1)} (g_{\hat{a}c} \; \delta_{\hat{b}d} + g_{\hat{b}d} \; \delta_{\hat{a}c} - g_{\hat{a}d}\; \delta_{\hat{b}c} - g_{\hat{b}c} \; \delta_{\hat{a}d})$

$= 0 - \frac{1}{2(n-1)} (0)$

$= 0$

7- Put $i = a$ $j = b$ $k = \hat{c}$ and $l = \hat{d}$

$K_{ab\hat{c}\hat{d}} = R_{ab\hat{c}\hat{d}} - \frac{1}{2(n-1)} (g_{a\hat{c}} \; \delta_{b\hat{d}} + g_{b\hat{d}} \; \delta_{a\hat{c}} - g_{a\hat{d}}\; \delta_{b\hat{c}} - g_{b\hat{c}} \; \delta_{a\hat{d}})$

$K_{ab\hat{c}\hat{d}} = 2 B^{cdh} B_{abh} - \frac{1}{2(n-1)} (\delta^c_a \; \delta^d_b + \delta^d_b \; \delta^c_a) + (\delta^d_a \; \delta^c_b + \delta^c_b \; \delta^d_a)$

8- Put $i = \hat{a}$ $j = b$ $k = c$ and $l = \hat{d}$

$K_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} - \frac{1}{2(n-1)} (g_{\hat{a}c} \; \delta_{b\hat{d}} + g_{b\hat{d}} \; \delta_{\hat{a}c} - g_{\hat{a}d}\; \delta_{bc} - g_{bc} \; \delta_{\hat{a}\hat{d}})$

$K_{\hat{a}bc\hat{d}} = A^{ad}_{bc} - B^{adh} B_{hbc} - \frac{1}{2(n-1)} (\delta^a_c \; \delta^d_b + \delta^d_b \; \delta^a_c)$

i.e for Nearly Kahler, only two conharmonic invariant don't equal to zero ;

$K_1 = K_{(1)} = -K_2$ with component $\{K^a{}_{bc\hat{d}}, K^{\hat{a}}{}_{\hat{b}\hat{c}d}\}$ *and* $K_4 = K_{(4)}$ with component $\{K^a{}_{\hat{b}cd}, K^{\hat{a}}{}_{b\hat{c}\hat{d}}\}$, other components conharmonic curvature Tensor equal to zero .

Definition (4) : [4]

Suppose that $\lambda(X,Y,Z,W) = K(X,Y,Z,W) - K(X,Y,JZ,JW)$

Consider the following tensor $\lambda(X,Y) = \lambda(X,Y,Y,X)$

We say that an almost hermition manifold M is of constant type at $p \in M$

Provided that for all $X \in T_P(M)$, where $T_P(M)$ is tangent space of M at the point p

$\lambda(X,Y) = \lambda(X,Z)$ ....(9)

Remark(5):

1- If (9) holds for all $p \in M$ then the manifold M has pointwise constant type.

2- If (9)is constant function ,then (M,J,g) has a global constant type .

Definition (6) :

An almost hermition manifold M^{2n} conharmonic type (K- constant type)

If $\forall$ X,Y,Z,W $\in$X (M^{2n}) that $\lambda(X,Y,Z,W) = K(X,Y,Z,W) - K(X,Y,JZ,JW)$

Consider the following tensor $\lambda(X,Y) = \lambda(X,Y,Y,X)$

We say that an almost hermition manifold M is of constant type at $p \in M$

Provided that for all $X \in T_P(M)$, where $T_P(M)$ is tangent space of M at the point p

$\lambda(X,Y) = \lambda(X,Z)$

Theorem (7):

If M is Nearly Kahler manifold of The conharmonic tenser then M manifold conharmonic constant concircular if and only if

$\lambda(X,Y) = \lambda(X,Z) = -8B^{cdh} B_{abh} - \frac{2}{(n-1)} (\delta^c_a \; \delta^d_b + \delta^d_b \; \delta^c_a) + (\delta^d_a \; \delta^c_b + \delta^c_b \; \delta^d_a)$

Proof :

suppose that M is Nearly Kahler manifold of conharmonic tenser

At first we find the following result

By using definition(6) it follows :-

1- $\lambda(X,Y) = K(X,Y,Y,X) - K(X,Y,JY,JX)$

Let M^{2n} Nearly Kahler manifold To compute the $K(X,Y,Y,X)$and $K(X,Y,JY,JX)$ on the space of the adjont G-structure

(i) $K(X,Y,Y,X) = K_{ijkl} X^i Y^j Y^k X^l = K_{abcd} X^a Y^b Y^c X^d + K_{ab\hat{c}d} X^a Y^b Y^{\hat{c}} X^d + K_{abc\hat{d}} X^a Y^b Y^c X^{\hat{d}} + K_{ab\hat{c}\hat{d}} X^a Y^b Y^{\hat{c}} X^{\hat{d}} + K_{\hat{a}bcd} X^{\hat{a}} Y^b Y^c X^d$
$+ K_{\hat{a}b\hat{c}d} X^{\hat{a}} Y^b Y^{\hat{c}} X^d + K_{\hat{a}\hat{b}cd} X^{\hat{a}} Y^{\hat{b}} Y^c X^d + K_{\hat{a}\hat{b}\hat{c}d} X^{\hat{a}} Y^{\hat{b}} Y^{\hat{c}} X^d$
$+ K_{\hat{a}\hat{b}c\hat{d}} X^{\hat{a}} Y^{\hat{b}} Y^c X^{\hat{d}} + K_{\hat{a}\hat{b}\hat{c}\hat{d}} X^{\hat{a}} Y^{\hat{b}} Y^{\hat{c}} X^{\hat{d}} + K_{a\hat{b}\hat{c}d} X^a Y^{\hat{b}} Y^{\hat{c}} X^d$
$+ K_{a\hat{b}cd} X^a Y^{\hat{b}} Y^c X^d + K_{\hat{a}b\hat{c}\hat{d}} X^{\hat{a}} Y^b Y^{\hat{c}} X^{\hat{d}} + K_{\hat{a}bc\hat{d}} X^{\hat{a}} Y^b Y^c X^{\hat{d}}$
$+ K_{a\hat{b}c\hat{d}} X^a Y^{\hat{b}} Y^c X^{\hat{d}} + K_{a\hat{b}\hat{c}\hat{d}} X^a Y^{\hat{b}} Y^{\hat{c}} X^{\hat{d}}$

By using the properties of conharmonic tenser equation (5) We get :

$K(X,Y,Y,X) = K_{\hat{a}bc\hat{d}} X^{\hat{a}} Y^b Y^c X^{\hat{d}} + K_{\hat{a}b\hat{c}d} X^{\hat{a}} Y^b Y^{\hat{c}} X^d$
$+ K_{\hat{a}\hat{b}cd} X^{\hat{a}} Y^{\hat{b}} Y^c X^d + K_{a\hat{b}c\hat{d}} X^a Y^{\hat{b}} Y^c X^{\hat{d}} + K_{a\hat{b}\hat{c}d} X^a Y^{\hat{b}} Y^{\hat{c}} X^d +$
$K_{ab\hat{c}\hat{d}} X^a Y^b Y^{\hat{c}} X^{\hat{d}}$ ....(10)

(II) $K(X,Y,JY,JX)$

In the adjoint G-structure space the components

$K(X,Y,JY,JX) = K_{ijkl} X^i Y^j (JY)^k (JX)^l = K_{abcd} X^a Y^b (JY)^c (JX)^d$
$+ K_{ab\hat{c}d} X^a Y^b (JY)^{\hat{c}} (JX)^d + K_{abc\hat{d}} X^a Y^b (JY)^c (JX)^{\hat{d}}$
$+ K_{ab\hat{c}\hat{d}} X^a Y^b (JY)^{\hat{c}} (JX)^{\hat{d}} + K_{a\hat{b}cd} X^a Y^{\hat{b}} (JY)^c (JX)^d$
$+ K_{a\hat{b}\hat{c}d} X^a Y^{\hat{b}} (JY)^{\hat{c}} (JX)^{\hat{d}} + K_{\hat{a}b\hat{c}d} X^{\hat{a}} Y^b (JY)^{\hat{c}} (JX)^d$
$+ K_{\hat{a}bcd} X^{\hat{a}} Y^b (JY)^c (JX)^d + K_{a\hat{b}c\hat{d}} X^a Y^{\hat{b}} (JY)^c (JX)^{\hat{d}}$
$+ K_{\hat{a}b\hat{c}d} X^{\hat{a}} Y^b (JY)^{\hat{c}} (JX)^d + K_{\hat{a}bc\hat{d}} X^{\hat{a}} Y^b (JY)^c (JX)^{\hat{d}}$
$+ K_{\hat{a}\hat{b}cd} X^{\hat{a}} Y^{\hat{b}} (JY)^c (JX)^d + K_{\hat{a}\hat{b}\hat{c}d} X^{\hat{a}} Y^{\hat{b}} (JY)^{\hat{c}} (JX)^d$
$+ K_{\hat{a}\hat{b}\hat{c}\hat{d}} X^{\hat{a}} Y^{\hat{b}} (JY)^{\hat{c}} (JX)^{\hat{d}}$

By using the properties of conharmonic tenser equation (5) We get :

$K(X,Y,JY,JX) = K_{\hat{a}bc\hat{d}} X^{\hat{a}} Y^b (Y)^c (X)^{\hat{d}} + K_{\hat{a}b\hat{c}d} X^{\hat{a}} Y^b (JY)^{\hat{c}} (JX)^d$
$- K_{\hat{a}\hat{b}cd} X^{\hat{a}} Y^{\hat{b}} (JY)^c (JX)^d - K_{a\hat{b}c\hat{d}} X^a Y^{\hat{b}} (JY)^c (JX)^{\hat{d}}$
$+ K_{a\hat{b}\hat{c}d} X^a Y^{\hat{b}} (JY)^{\hat{c}} (JX)^d - K_{ab\hat{c}\hat{d}} X^a Y^b (JY)^{\hat{c}} (JX)^{\hat{d}}$ ...(11)

Making use of the equation (10) and (11) we get :

$K(X,Y,Y,X) - K(X,Y,JY,JX) = K_{\hat{a}bc\hat{d}} X^{\hat{a}} Y^b Y^c X^{\hat{d}} + K_{\hat{a}b\hat{c}d} X^{\hat{a}} Y^b Y^{\hat{c}} X^d$
$+ K_{\hat{a}\hat{b}cd} X^{\hat{a}} Y^{\hat{b}} Y^c X^d + K_{a\hat{b}c\hat{d}} X^a Y^{\hat{b}} Y^c X^{\hat{d}} + K_{a\hat{b}\hat{c}d} X^a Y^{\hat{b}} Y^{\hat{c}} X^d$
$+ K_{ab\hat{c}\hat{d}} X^a Y^b (Y)^{\hat{c}} (X)^{\hat{d}} - K_{\hat{a}bc\hat{d}} X^{\hat{a}} Y^b (JY)^c (JX)^{\hat{d}}$
$+ K_{\hat{a}b\hat{c}d} X^{\hat{a}} Y^b (JY)^{\hat{c}} (JX)^d - K_{\hat{a}\hat{b}cd} X^{\hat{a}} Y^{\hat{b}} (JY)^c (JX)^d$
$- K_{a\hat{b}c\hat{d}} X^a Y^{\hat{b}} (JY)^c (JX)^{\hat{d}} + K_{a\hat{b}\hat{c}d} X^a Y^{\hat{b}} (JY)^{\hat{c}} (JX)^d$
$- K_{ab\hat{c}\hat{d}} X^a Y^b (JY)^{\hat{c}} (JX)^{\hat{d}}$
$= -4 K_{\hat{a}\hat{b}cd} X^{\hat{a}} Y^{\hat{b}} Y^c X^d$

This is the K_4 of theory (3) equation (8) and the compensation we get :

$= -4[2B^{cdh} B_{abh} - \frac{1}{2(n-1)} (\delta_a^c \; \delta_b^d + \delta_b^d \; \delta_a^c) + (\delta_a^d \; \delta_b^c + \delta_b^c \; \delta_a^d)$
$= -8B^{cdh} B_{abh} - \frac{2}{(n-1)} (\delta_a^c \; \delta_b^d + \delta_b^d \; \delta_a^c) + (\delta_a^d \; \delta_b^c + \delta_b^c \; \delta_a^d)$ ....(12)

2- $\lambda(X,Z) = K(X,Z,Z,X) - K(X,Z,JZ,JX)$

To compute the $K(X,Z,Z,X) and \; K(X,Z,JZ,JX)$ on the space of the adjont G-structure

(I) $K(X,Z,Z,X) = K_{ijkl} X^i Z^j Z^k X^l = K_{abcd} X^a Z^b Z^c X^d + K_{ab\hat{c}d} X^a Z^b Z^{\hat{c}} X^d +$
$K_{abc\hat{d}} X^a Z^b Z^c X^{\hat{d}} + K_{ab\hat{c}\hat{d}} X^a Z^b Z^{\hat{c}} X^{\hat{d}} + K_{\hat{a}bcd} X^{\hat{a}} Z^b Z^c X^d$
$+ K_{\hat{a}b\hat{c}d} X^{\hat{a}} Z^b Z^{\hat{c}} X^d + K_{\hat{a}\hat{b}cd} X^{\hat{a}} Z^{\hat{b}} Z^c X^d + K_{\hat{a}\hat{b}\hat{c}d} X^{\hat{a}} Z^{\hat{b}} Z^{\hat{c}} X^d$
$+ K_{\hat{a}\hat{b}c\hat{d}} X^{\hat{a}} Z^{\hat{b}} Z^c X^{\hat{d}} + K_{\hat{a}\hat{b}\hat{c}\hat{d}} X^{\hat{a}} Z^{\hat{b}} Z^{\hat{c}} X^{\hat{d}} + K_{a\hat{b}\hat{c}d} X^a Z^{\hat{b}} Z^{\hat{c}} X^d$
$+ K_{a\hat{b}cd} X^a Z^{\hat{b}} Z^c X^d + K_{\hat{a}b\hat{c}\hat{d}} X^a Z^b Z^{\hat{c}} X^{\hat{d}} + K_{\hat{a}bc\hat{d}} X^{\hat{a}} Z^b Z^c X^{\hat{d}}$
$+ K_{a\hat{b}c\hat{d}} X^a Y^{\hat{b}} Y^c X^{\hat{d}} + K_{a\hat{b}\hat{c}\hat{d}} X^a Y^{\hat{b}} Y^{\hat{c}} X^{\hat{d}}$

By using the properties of conharmonic tenser equation (5) We get :

$K(X,Z,Z,X) = K_{\hat{a}bc\hat{d}} X^{\hat{a}} Z^b Z^c X^{\hat{d}} + K_{\hat{a}b\hat{c}d} X^{\hat{a}} Z^b Z^{\hat{c}} X^d$
$+ K_{\hat{a}\hat{b}cd} X^{\hat{a}} Z^{\hat{b}} Z^c X^d + K_{a\hat{b}c\hat{d}} X^a Z^{\hat{b}} Z^c X^{\hat{d}} + K_{a\hat{b}\hat{c}d} X^a Z^{\hat{b}} Z^{\hat{c}} X^d +$
$K_{ab\hat{c}\hat{d}} X^a Z^b Z^{\hat{c}} X^{\hat{d}}$ .....(13)

(II) $K(X,Z,JZ,JX)$

In the adjoint G-structure space the components

$K(X,Z,JZ,X) = K_{ijkl} X^i Z^j (JZ)^k (JX)^l = K_{abcd} X^a Z^b (JZ)^c (JX)^d$
$+ K_{ab\hat{c}d} X^a Z^b (JZ)^{\hat{c}} (JX)^d + K_{abc\hat{d}} X^a Z^b (JZ)^c (JX)^{\hat{d}}$
$+ K_{ab\hat{c}\hat{d}} X^a Z^b (JZ)^{\hat{c}} (JX)^{\hat{d}} + K_{a\hat{b}cd} X^a Z^{\hat{b}} (JZ)^c (JX)^d$
$+ K_{a\hat{b}\hat{c}\hat{d}} X^{\hat{a}} Z^{\hat{b}} (JZ)^{\hat{c}} (JX)^{\hat{d}} + K_{\hat{a}b\hat{c}d} X^{\hat{a}} Z^b (JZ)^{\hat{c}} (JX)^d$
$+ K_{\hat{a}bcd} X^{\hat{a}} Z^b (JZ)^c (JX)^d + K_{a\hat{b}c\hat{d}} X^a Z^{\hat{b}} (JZ)^c (JX)^{\hat{d}}$
$+ K_{\hat{a}b\hat{c}d} X^{\hat{a}} Z^b (JZ)^{\hat{c}} (JX)^d + K_{\hat{a}bc\hat{d}} X^{\hat{a}} Z^b (JZ)^c (JX)^{\hat{d}}$
$+ K_{\hat{a}\hat{b}cd} X^{\hat{a}} Z^{\hat{b}} (JZ)^c (JX)^d + K_{\hat{a}\hat{b}\hat{c}d} X^{\hat{a}} Z^{\hat{b}} (JZ)^{\hat{c}} (JX)^d$
$+ K_{\hat{a}\hat{b}\hat{c}\hat{d}} X^{\hat{a}} Z^{\hat{b}} (JZ)^{\hat{c}} (JX)^{\hat{d}}$

By using the properties of conharmonic tenser equation (5) We get :

$K(X,Z,JZ,JX) = K_{\hat{a}bc\hat{d}} X^{\hat{a}} Z^b Z^c X^{\hat{d}} + K_{\hat{a}b\hat{c}d} X^{\hat{a}} Z^b (JZ)^{\hat{c}} (JX)^d$
$- K_{\hat{a}\hat{b}cd} X^{\hat{a}} Z^{\hat{b}} (JZ)^c (JX)^d - K_{a\hat{b}c\hat{d}} X^a Z^{\hat{b}} (JZ)^c (JX)^{\hat{d}}$
$+ K_{a\hat{b}\hat{c}d} X^a Z^{\hat{b}} (JZ)^{\hat{c}} (JX)^d - K_{ab\hat{c}\hat{d}} X^a Z^b (JZ)^{\hat{c}} (JX)^{\hat{d}}$ ....(14)

Making use of the equation (13) and (14) we get :

$K(X,Y,Y,X) - K(X,Y,JY,JX) = K_{\hat{a}bc\hat{d}} X^{\hat{a}} Z^b Z^c X^{\hat{d}} + K_{\hat{a}b\hat{c}d} X^{\hat{a}} Z^b Z^{\hat{c}} X^d$
$+ K_{\hat{a}\hat{b}cd} X^{\hat{a}} Z^{\hat{b}} Z^c X^d + K_{a\hat{b}c\hat{d}} X^a Z^{\hat{b}} Z^c X^{\hat{d}} + K_{a\hat{b}\hat{c}d} X^a Z^{\hat{b}} Z^{\hat{c}} X^d$
$+ K_{ab\hat{c}\hat{d}} X^a Z^b (Z)^{\hat{c}} (X)^{\hat{d}} - K_{\hat{a}bc\hat{d}} X^{\hat{a}} Z^b (JZ)^c (JX)^{\hat{d}}$
$+ K_{\hat{a}b\hat{c}d} X^{\hat{a}} Z^b (JZ)^{\hat{c}} (JX)^d - K_{\hat{a}\hat{b}cd} X^{\hat{a}} Z^{\hat{b}} (JZ)^c (JX)^d$
$- K_{a\hat{b}c\hat{d}} X^a Z^{\hat{b}} (JZ)^c (JX)^{\hat{d}} + K_{a\hat{b}\hat{c}d} X^a Z^{\hat{b}} (JZ)^{\hat{c}} (JX)^d$
$- K_{ab\hat{c}\hat{d}} X^a Z^b (JZ)^{\hat{c}} (JX)^{\hat{d}}$
$= -4 K_{\hat{a}\hat{b}cd} X^{\hat{a}} Z^{\hat{b}} JZ^c X^d$

This is the K_4 of theory (3) equation (8) and the compensation we get :-

$= -4[2B^{cdh}B_{abh} - \frac{1}{2(n-1)}(\delta^c_a\ \delta^d_b + \delta^d_b\ \delta^c_a) + (\delta^d_a\ \delta^c_b + \delta^c_b\ \delta^d_a)]$

$= -8B^{cdh}B_{abh} - \frac{2}{(n-1)}(\delta^c_a\ \delta^d_b + \delta^d_b\ \delta^c_a) + (\delta^d_a\ \delta^c_b + \delta^c_b\ \delta^d_a)$ ….(15)

From equation (12) and (15) it follows

$\lambda(X,Y) = \lambda(X,Z)$

$= -8B^{cdh}B_{abh} - \frac{2}{(n-1)}(\delta^c_a\ \delta^d_b + \delta^d_b\ \delta^c_a) + (\delta^d_a\ \delta^c_b + \delta^c_b\ \delta^d_a)$

Thus by definition (6) we get :

M is constant type if and only if

$\lambda(X,Y) = -8B^{cdh}B_{abh} - \frac{2}{(n-1)}(\delta^c_a\ \delta^d_b + \delta^d_b\ \delta^c_a) + (\delta^d_a\ \delta^c_b + \delta^c_b\ \delta^d_a)$

Now we studding Nearly Kahler manifold of apointwise holomorphic sectional conhormonic $(PMK_M(X))$ curvature tensor.

Let $\{M^{2n},g,J\}$ Almost Hermitian manifold X(M) module smooth vector field on manifold M^{2n}

Definition (8): [2]

Two –dimensions Level two-dimensional $\sigma \in T_m(M)$,m ∈ M called holomorphic, if $J_m(\sigma) = \sigma$

Theorem (9) :[2]

Two –dimensions Level two-dimensional $\sigma \in T_m(M)$,m ∈ M holomorphic if and only if when $\sigma = L(X, JX)$where X∈ X(M)- some vector, L denoted taking linear hull

Defintion (10):[3]

Sectional curvature almost Hermitian manifold {M,g,J} on the direction two –dimensions Level two-dimensional $\sigma \in T_m(M)$,m ∈ M called holomorphic sectional curvature (don't equal zero) vector $X \in \sigma$ and denoted $H_m(X)$,With form

$H_m(X) = \frac{<Rm\ (X,JX)JX,X>}{\|X\|4}$; m ∈ M, X $\in T_m(M)$

- If $H_m(X)$ Do not depend on the choice of the point $\sigma \in T_m(M)$ then manifold involving M pointwise constant holomorphic sectional curvature.

- If Hm(X)Do not depend on the choice of the point m then manifold involving M globol constant holomorphic sectional curvature.

Definition (11) :

Let M be an almost Hermitian manifold , aholomorphic sectional curvature conhormonic $(HK_m(X)$- curature) tensor of amanifold M in the direction X∈ X(M) , X≠0 is afunction H(X) which is defind as :

$K(X, JX, X,JX) = MK_m(X)\ \|X\|^4$, ∀ X∈ X(M)

$(HK_m(X) = \frac{<Km\ (X,JX)JX,X>}{\|X\|4}$; m ∈ M, X $\in T_m(M)$

where$\|X\|^2 = <X,X>$ …(16)

Definition (12):

A manifold M is called a manifold apointwise constant holomorphic sectional conhormoic curvature $(PHK_m(X))$ – curvature tensor , if $HK_m(X)$ does not depend on X

i.e $K(X, JX, X,JX) = C\ \|X\|^4$; X∈ X(M), C∈ $C^\infty(M)$ ….(17)

Definition (13):

A manifold M apointwise constant holomorphic sectional conhormoic curvature is called global constant holomorphic sectional conhormoic curvature if $HK_m(X)$ has constant

i.e $(HK_m(X)$ does not depend on m ∈ M)

Theorem(14): **[5]**

Almost Hermitian manifold (M,g,J) is manifold constant HK- curvature if and only if , ,when in the adjoint G-structure space

$K^{ad}_{bc} = \frac{c}{2}\tilde{\delta}^{ad}_{bc}$

Lemma(15): **[4]**

If M almost Hermitian manifold of pointwise holomorphic sectional curvature tensor then we have:

$\|X\|^4 = 2\tilde{\delta}^{bc}_{ad}\ X^a X^d X_b X_c$

Theorem (16) :

Nearly Kahler manifold of pointwise constant holomorphic sectional conharmonic $(PHK_m(X))$ – curvature) curvature tensor if the components of holomorphic sectional (HS-curvature) curvature tensor in the adjoint G-structure space that satisfies the following condition:

$A^{bc}_{ad} = \frac{c}{2}\tilde{\delta}^{bc}_{ad}$

Proof :

Suppose that M is Nearly Kahler manifold of PHK-curvatue tensor

According to the the definition (12) we have :

$K(X, JX, X,JX) = C\ \|X\|^4$ ….(18)

Where C∈ $C^\infty(M)$

By using Lemma(15) ,the equation (18) becomes:-

$K(X, JX, X,JX) = 2\tilde{\delta}^{bc}_{ad}\ X^a X^d X_b X_c$ ….(19)

In the adjoint G-structure space the eqution (19) can be written as the following form:

$K_{ijkl}X^i(JX)^j(X)^k(JX)^l = K_{abcd}X^a(JX)^bX^c(JX)^d + K_{ab\hat{c}d}X^a(JX)^bX^{\hat{c}}(JX)^d$
$+ K_{abc\hat{d}}X^a(JX)^bX^c(JX)^{\hat{d}} + K_{ab\hat{c}\hat{d}}X^a(JX)^bX^{\hat{c}}(JX)^{\hat{d}}$
$+ K_{\hat{a}\hat{b}\hat{c}\hat{d}}X^{\hat{a}}(JX)^{\hat{b}}X^{\hat{c}}(JX)^{\hat{d}} + K_{a\hat{b}\hat{c}\hat{d}}X^a(JX)^{\hat{b}}X^{\hat{c}}(JX)^{\hat{d}}$
$+ K_{a\hat{b}\hat{c}d}X^a(JX)^{\hat{b}}X^{\hat{c}}(JX)^d + K_{a\hat{b}cd}X^a(JX)^{\hat{b}}X^c(JX)^d$
$+ K_{a\hat{b}\hat{c}\hat{d}}X^a(JX)^{\hat{b}}X^{\hat{c}}(JX)^{\hat{d}} + K_{\hat{a}b\hat{c}\hat{d}}X^{\hat{a}}(JX)^bX^{\hat{c}}(JX)^{\hat{d}}$
$+ K_{\hat{a}\hat{b}cd}X^{\hat{a}}(JX)^{\hat{b}}X^c(JX)^d + K_{\hat{a}bcd}X^{\hat{a}}(JX)^bX^c(JX)^d$
$+ K_{\hat{a}b\hat{c}d}X^{\hat{a}}(JX)^bX^{\hat{c}}(JX)^d + K_{\hat{a}\hat{b}\hat{c}d}X^{\hat{a}}(JX)^{\hat{b}}X^{\hat{c}}(JX)^d$
$+ K_{\hat{a}bc\hat{d}}X^{\hat{a}}(JX)^bX^c(JX)^{\hat{d}} + K_{\hat{a}\hat{b}c\hat{d}}X^{\hat{a}}(JX)^{\hat{b}}X^c(JX)^{\hat{d}}$
$= 2C\tilde{\delta}^{bc}_{ad}\ X^aX^dX_bX_c$

By using the properties $(JX)^a = \sqrt{-1}\ X^a$, $(JX)^{\hat{a}} = -\sqrt{-1}\ X^{\hat{a}}$ we get :-

$< K(X, JX), JX, X > = -2 K_{\hat{a}\,bcd}\ X^{\hat{a}}\ X^{b}\ X^{c}\ X^{d} + 4 K_{\hat{a}\,bc\hat{d}}\ X^{\hat{a}}\ X^{b}\ X^{c}\ X^{\hat{d}}$

$+ 2 K_{\hat{a}\,\hat{b}\,\hat{c}d}\ X^{\hat{a}}\ X^{\hat{b}}\ X^{\hat{c}}\ X^{d} + K_{\hat{a}\,\hat{b}cd}\ X^{\hat{a}}\ X^{\hat{b}}\ X^{c}\ X^{d}$

$- K_{\hat{a}\,\hat{b}\,\hat{c}\hat{d}}\ X^{\hat{a}}\ X^{\hat{b}}\ X^{\hat{c}}\ X^{\hat{d}} - K_{a\,bcd}\ X^{a}\ X^{b}\ X^{c}\ X^{d}$

$= 2\, C\delta^{bc}_{ad}\ X^{a} X^{d} X_{b} X_{c}$

By using the properties of conharmonic tenser equation (5) We get :

$K_{\hat{a}\,bcd}\ X^{\hat{a}}\ X^{b}\ X^{c}\ X^{d} = - K_{\hat{a}\,bcd}\ X^{\hat{a}}\ X^{b}\ X^{c}\ X^{d} = K_{\hat{a}\,bcd}\ X^{\hat{a}}\ X^{b}\ X^{c}\ X^{d}$

T.e $K_{\hat{a}\,bcd} = 0$

Similarly:

$K_{a\,bcd} = 0$, $K_{\hat{a}\,\hat{b}cd} = 0$, $K_{\hat{a}\,\hat{b}\,\hat{c}d}$, $K_{\hat{a}\,\hat{b}\,\hat{c}\hat{d}} = 0$

we get :

$< K(X, JX), JX, X > = 4 K_{\hat{a}\,bc\hat{d}}\ X^{\hat{a}}\ X^{b}\ X^{c}\ X^{\hat{d}} = 2 C\delta^{bc}_{ad}\ X^{a} X^{d} X_{b} X_{c}$

By theorm (3) the eqution (8) we show :

$4 \left\{ A^{ad}_{bc} - B^{adh} B_{hbc} - \frac{1}{2(n-1)} \delta^{c}_{a}\delta^{d}_{b} + \delta^{d}_{b}\delta^{a}_{c} \right\}$

$= 2\,\delta^{bc}_{ad}\ X^{a} X^{d} X_{b} X_{c}$

If we compute anti symmetrizing structure tensor we have :

$A^{ad}_{bc} - \frac{1}{4(n-1)} (\delta^{a}_{b}\delta^{d}_{c} + \delta^{a}_{c}\delta^{d}_{b} + \delta^{d}_{b}\delta^{a}_{c} + \delta^{d}_{c}\delta^{a}_{b}) = \frac{c}{2}\delta^{ad}_{cb}$

Explain that $\tilde{A}$ tensor type $\binom{2\ \ 2}{0\ \ 0}$ with component

$\tilde{A} = A^{ad}_{bc} - \frac{1}{4(n-1)} (\delta^{a}_{b}\delta^{d}_{c} + \delta^{a}_{c}\delta^{d}_{b} + \delta^{d}_{b}\delta^{a}_{c} + \delta^{d}_{c}\delta^{a}_{b})$

Symmetric for each Identical for each pair supernal or subjacent and we get :

$\tilde{A}^{ad}_{bc} = \frac{c}{2}\delta^{ad}_{cb}$.

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النوع الثابت (K) لمنطويات كوهلر ومنطوي كوهلر التقريبي لتنزر الانحناء الكونهورموني

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الملخص

ان الشروط الثابتة للأنواع الكونهورمونية لمنطوي كوهلر ومنطوي كوهلر التقريبي نحصل عليها عندما يكون منطوي كوهلر التقريبي هو منطوي كونهورموني من النوع الثابت **(K)**. وكذلك اثبات ان **(M)** هو منطوي كوهلر تقريبي ذات ثابت نقطي هلومورفك في القاطع الكونهورموني لتنزر الانحناء **(PHKm(X)) – curvature)** اذا كانت محتويات القاطع الهولومورفك **(HS- curvature)** لتنزر الانحناء في بنية الفضاء المركب – **G** يحقق الشروط محدد.