A New Parameterized Conjugate Gradient Method based on Generalized Perry Conjugate Gradient Method

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Abstract
A New Parameterized Conjugate Gradient Method based on Generalized Perry Conjugate Gradient Method is proposed to be based on Perry's idea, the descent condition and the global convergent is proven under Wolfe condition. The new algorithm is very effective for solve the large-scale unconstrained optimization problem.

Keywords: Perry Conjugate Gradient, Unconstrained optimization,

1. Introduction
Consider the problem

$$\min_{x\in R^n} f(x) \quad \dots (1)$$

is a large-scale unconstrained optimization problem, where f a continuously differentiable function, and the popular method for such problems is the nonlinear conjugate gradient method. The line search of steepest descent (the simplest type of conjugate gradient) is

$$x_{k+1} = x_k + \alpha_k d_k \quad \dots\dots(2)$$

Where the d_k is the direction of algorithm given by

$$\begin{cases} d_1 = -g_1 \\ d_{k+1} = -g_{k+1} + \beta_k d_k, \quad \forall k > 1 \end{cases} \quad \dots\dots(3)$$

where $g_k = g(x_k) = \nabla f(x_k)$ and $\beta_k \in R$ is parameter which characterized the method. The numerical experiment of the nonlinear conjugate gradient method is affected by β_k; Therefore there are several types of conjugate gradient according to the choice of β_k, such as HS method[1], FR method [2], PRP method [3], DY method [4] and LS method [5], and so on.

The generalized of the method of Perry conjugate gradient was proposed in [6],[7] with search direction

$$\begin{cases} d_1 = -g_1 \\ d_{k+1} = -P_{k+1}g_{k+1} = g_{k+1} + \frac{y_k^T g_{k+1}}{s_k^T y_k} s_k + \sigma \frac{s_k^T g_{k+1}}{y_k^T u_k} u_k, \quad \forall k > 1 \end{cases}$$

$\dots(4)$

where

where

$$P_{k+1} = (I - \frac{s_k y_k^T}{s_k^T y_k} + \sigma \frac{u_k s_k^T}{y_k^T u_k}) \quad \dots(5)$$

where P_{k+1} is called the iteration matrix of generalized Perry conjugate gradient method and $y_k = g_{k+1} - g_k$, $s_k = x_{k+1} - x_k$, $s_k = \alpha_k d_k$, α_k is the line search and σ is a parameter defined as follows

$$\sigma = c\frac{\|y_k\|^2}{s_k^T y_k}, \; c > 0$$

The choice of σ and u_k such that $y_k^T u_k \neq 0$ correspond to different of generalized Perry conjugate gradient method. The case of $u_k = s_k$ and $\sigma = 2\|y_k\|^2 / (s_k^T y_k)$ was discussed in [8] and [9]. When $u_k = y_k$ was studied in [6].

2-The New algorithm Parameterized Conjugate Gradient Method based on Generalized Perry Conjugate Gradient Method (PGPCG)

In this paper, we suggest u_k as a linear combination of s_k and y_k as follows

$$u_k = \gamma y_k + (1-\gamma)s_k \quad \dots(6)$$

and

$$\sigma = c\frac{\|y_k\|^2}{s_k^T y_k}, c > 0 \quad \dots(7)$$

then by substitute equ. (6) and equ. (7) in equ. (4) we get

$$d_{k+1} = -P_{k+1}g_{k+1} = g_{k+1} + \frac{y_k^T g_{k+1}}{s_k^T y_k} s_k - \sigma \frac{s_k^T g_{k+1}}{y_k^T(\gamma y_k + (1-\gamma)s_k)}(\gamma y_k + (1-\gamma)s_k)$$

$$\Rightarrow d_{k+1} = -g_{k+1} + \frac{y_k^T g_{k+1}}{s_k^T y_k} s_k - \frac{\sigma\gamma s_k^T g_{k+1}}{\gamma\|y_k\|^2 + (1-\gamma)y_k^T s_k} y_k - \frac{\sigma(1-\gamma)s_k^T g_{k+1}}{\gamma\|y_k\|^2 + (1-\gamma)y_k^T s_k} s_k$$

$$\begin{cases} d_1 = -g_1 \\ d_{k+1} = -P_{k+1}g_{k+1}, \quad \forall k > 1 \end{cases} \quad \dots(8)$$

where

$$P_{k+1} = (I - \frac{y_k s_k^T}{s_k^T y_k} + \frac{\sigma\gamma y_k s_k^T}{\gamma\|y_k\|^2 + (1-\gamma)y_k^T s_k} + \frac{\sigma(1-\gamma)s_k^T s_k}{\gamma\|y_k\|^2 + (1-\gamma)y_k^T s_k})$$

$\dots(9)$

Algorithm (2.1)

Step(1):We select the initial point $x_0 \in R^n$, and the accurate solution $\varepsilon > 0$ and we find $d_0 = -g_0$, and we set $k = 0$.

Step(2): If $\|g_k\| \le \varepsilon$, then stop and set the optimal solution is x_k,

Else, go to step(3).

Step(3): Compute the stepsize α_k by Wolfe line search conditions

$$f_{k+1} \le f_k + c_1\alpha_k d_k^T g_k \quad \dots(10)$$

$$d_k^T g_{k+1} \ge c_2 d_k^T g_k \quad \dots(11)$$

and go to step(4).

Step(4): Calculate $x_{k+1} = x_k + \alpha_k d_k$ and compute $f(x_{k+1}), g_{k+1}$ and

$$s_k = x_{k+1} - x_k,\ y_k = g_{k+1} - g_k.$$

Step(5): We compute the search direction by the equation (9) and set

$d_{k+1} = -P_{k+1} g_{k+1}$, if Powell restart [10] is satisfied then set $d_{k+1} = -g_{k+1}$

Else $d_{k+1} = d_k$; $k = k+1$ go to step(2)

3- The Descent Property

We shall prove the descent property for the new formula which was introduce in equation (8)

Theorem(3.1)

The search direction d_k that is suggested by the proposed method in equation (7) satisfies the descent property for all k, when the step size α_k determined by the Wolfe conditions (10),(11).

Proof: we want to prove that $d_k^T g_k < 0, \quad \forall k \ge 1$

We shall proof by using the mathematical induction

First, $d_1 = -g_1 \Rightarrow d_1^T g_1 < 0$

And we suppose that $d_k^T g_k < 0$, is true for all k, and we will prove that the relation is true when $k = k+1$,

Multiply the equation (8) from left by g_{k+1}

$$d_{k+1}^T g_{k+1} = -g_{k+1}^T g_{k+1} + \frac{y_k^T g_{k+1}}{s_k^T y_k} s_k^T g_{k+1} - \frac{\sigma\gamma s_k^T g_{k+1}}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} y_k^T g_{k+1} - \frac{\sigma(1-\gamma) s_k^T g_{k+1}}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} s_k^T g_{k+1}$$

By lipschits condition $\|y_k\| \le L s_k^T y_k$, where $L > 0$,then $\frac{1}{\|y_k\|} \ge \frac{1}{L s_k^T y_k}$ and $-\frac{1}{\|y_k\|} \le -\frac{1}{L s_k^T y_k}$

$$\Rightarrow d_{k+1}^T g_{k+1} \le -\|g_{k+1}\|^2 + \frac{y_k^T g_{k+1}}{s_k^T y_k} s_k^T g_{k+1} - \frac{\sigma\gamma s_k^T g_{k+1}}{\gamma s_k^T y_k + (1-\gamma) s_k^T y_k} y_k^T g_{k+1} - \frac{\sigma(1-\gamma) s_k^T g_{k+1}}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} s_k^T g_{k+1}$$

$$\Rightarrow d_{k+1}^T g_{k+1} \le -\|g_{k+1}\|^2 + \left(1 - \frac{\sigma\gamma}{\gamma L + (1-\gamma)}\right) \frac{s_k^T g_{k+1}}{s_k^T y_k} y_k^T g_{k+1} - \frac{\sigma(1-\gamma) s_k^T g_{k+1}}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} s_k^T g_{k+1}$$

Let $w = \frac{\sigma\gamma}{\gamma L + (1-\gamma)} > 0$,

$$\Rightarrow d_{k+1}^T g_{k+1} \le -\|g_{k+1}\|^2 + (1-w) \frac{s_k^T g_{k+1}}{s_k^T y_k} y_k^T g_{k+1} - \frac{\sigma(1-\gamma)(s_k^T g_{k+1})^2}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k}$$

and by using the inequality $u^T v \le \frac{1}{2}\left(a\|u\|^2 + \frac{\|v\|^2}{a}\right), \quad \forall a > 0$ and $u^T v \le \|u\|\,\|v\|$ With the choice $u_k = (g_{k+1}^T s_k) y_k^T$ and $v_k = g_{k+1}(s_k^T y_k)$

We have

$$(1-w)\frac{s_k^T g_{k+1}(y_k^T g_{k+1})}{s_k^T y_k} = (1-w)\frac{s_k^T g_{k+1}(s_k^T y_k)(y_k^T g_{k+1})}{(s_k^T y_k)^2}$$

$$= (1-w)\frac{((g_{k+1}^T s_k) y_k^T)^T (g_{k+1}(s_k^T y_k))}{(s_k^T y_k)^2}$$

$$\le \frac{(1-w)}{2(s_k^T y_k)^2}\left(a (g_{k+1}^T s_k)^2 \|y_k\|^2 + \frac{1}{a}\|g_{k+1}\|^2 (s_k^T y_k)^2\right)$$

$$\le \frac{a(1-w)\|g_{k+1}\|^2 \|s_k\|^2 \|y_k\|^2}{2(s_k^T y_k)^2} + \frac{(1-w)\|g_{k+1}\|^2}{2a}$$

Combining this with the above equation gives

$$\Rightarrow d_{k+1}^T g_{k+1} \le -\|g_{k+1}\|^2 + \frac{a(1-w)\|g_{k+1}\|^2 \|s_k\|^2 \|y_k\|^2}{2(s_k^T y_k)^2} + \frac{(1-w)\|g_{k+1}\|^2}{2a} - \frac{\sigma(1-\gamma)\|g_{k+1}\|^2 \|s_k\|^2}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k}$$

$$\Rightarrow d_{k+1}^T g_{k+1} \le \left(\frac{a(1-w)\|s_k\|^2 \|y_k\|^2}{2(s_k^T y_k)^2} + \frac{(1-w)}{2a} - 1 - \frac{\sigma(1-\gamma)\|s_k\|^2}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k}\right)\|g_{k+1}\|^2$$

Now, let

$$m_k = \frac{\|s_k\|^2 \|y_k\|^2}{(s_k^T y_k)^2}$$

Since $(u^T v)^2 \le \|u\|^2 \|v\|^2$ then $m_k \ge 1$

$$\Rightarrow d_{k+1}^T g_{k+1} \le \left(\frac{a(1-w)m_k}{2} + \frac{(1-w)}{2a} - 1 - \frac{\sigma(1-\gamma)\|s_k\|^2}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k}\right)\|g_{k+1}\|^2$$

Now, we have two cases, when $w \ge 1$ then

$$\left(\frac{a(1-w)m_k}{2} + \frac{(1-w)}{2a} - 1 - \frac{\sigma(1-\gamma)\|s_k\|^2}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k}\right) < 0$$

So the descent property satisfy.

The other case, when $w < 1$, we let $\tau = a(1-w)m_k$, we select a positive number a such that $0 < \tau < 1$

Then

$$\Rightarrow d_{k+1}^T g_{k+1} \le \left(\frac{\tau}{2} + \frac{(1-w)^2 m_k}{2\tau} - 1 - \frac{\sigma(1-\gamma)\|s_k\|^2}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k}\right)\|g_{k+1}\|^2$$

Now, when $(1-w)^2 m_k \le \tau^2$ then $m_k \le \frac{\tau^2}{(1-w)^2}$ thus

$$\Rightarrow d_{k+1}^T g_{k+1} \le \left(\frac{\tau}{2} + \frac{(1-w)^2 \tau^2}{2\tau(1-w)^2} - 1 - \frac{\sigma(1-\gamma)\|s_k\|^2}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k}\right)\|g_{k+1}\|^2$$

$$\Rightarrow d_{k+1}^T g_{k+1} \le \left(\frac{\tau}{2} + \frac{\tau}{2} - 1 - \frac{\sigma(1-\gamma)\|s_k\|^2}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k}\right)\|g_{k+1}\|^2$$

$$\Rightarrow d_{k+1}^T g_{k+1} \le \left(\tau - 1 - \frac{\sigma(1-\gamma)\|s_k\|^2}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k}\right)\|g_{k+1}\|^2$$

Clearly $\left(\tau - 1 - \frac{\sigma(1-\gamma)\|s_k\|^2}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k}\right) < 0$

So the descent property satisfy for $w < 1$.

4- The global Convergent

Assumption(4.1):

(i) The level set $S = \{x \in R^n : f(x) \le f(x_0)\}$ is closed and bounded on the initial point.

(ii) The objective function is continuously differentiable In a neighborhood N of S, and its gradient is Lipchitz continuous, i.e. there exists a constant L>0 such that

$\|g(x)-g(y)\| \le L\|x-y\|, \quad for \quad all \quad x,y \in N. \quad \cdots$ (12)

(iii) The objective function is Uniformly Convex Function and there exist constant $\lambda > 0$ satisfy the following

$(g(x)-g(y))^T (x-y) \le \lambda \|x-y\|^2, \quad for \quad any \quad x,y \in S.$

Or $y_k^T s_k \ge \lambda \|s_k\|^2$

under these assumptions on f, there exists a constant $\ell \ge 0$ such that $\|g(x)\| \le \ell$, and by (i) then there exists $\|x\| \le M, \quad \forall x \in S$

$\underline{\eta} \le \|g(x)\| \le \bar{\eta} \quad ,\forall x \in S$ [11]

Lemma (4.2):

Let assumptions (4.1.i) and (4.1.iii) holds and consider any conjugate gradient method, where d_{k+1} is a descent direction and the step size α_k is obtained by the strong Wolfe line search. If

$$\sum_{k \ge 1} \frac{1}{\|d_{k+1}\|^2} = \infty \quad \ldots..(13)$$

Then

$$\lim_{k\to\infty}(\inf \|g_k\|) = 0 \quad \ldots (14)$$

Theorem (4.3):

We let that the assumptions (4.1.i) and (4.1.iii) with decsent property holds, the method of conjugate gradient and the equation

$$\Rightarrow d_{k+1} = -g_{k+1} + \frac{y_k^T g_{k+1}}{s_k^T y_k} s_k - \frac{\sigma\gamma s_k^T g_{k+1}}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} y_k - \frac{\sigma(1-\gamma) s_k^T g_{k+1}}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} s_k$$

and $x_{k+1} = x_k + \alpha_k d_k$

Where the step size α_k is obtained by the strong Wolfe line search and the objective function is a uniformly convex function, then

$$\lim_{k\to\infty}(\inf \|g_k\|) = 0$$

Proof:

$$d_{k+1} = -g_{k+1} + \frac{y_k^T g_{k+1}}{s_k^T y_k} s_k - \frac{\sigma\gamma s_k^T g_{k+1}}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} y_k - \frac{\sigma(1-\gamma) s_k^T g_{k+1}}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} s_k$$

$$\Rightarrow \|d_{k+1}\|^2 = \left\| -g_{k+1} + \frac{y_k^T g_{k+1}}{s_k^T y_k} s_k - \frac{\sigma\gamma s_k^T g_{k+1}}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} y_k - \frac{\sigma(1-\gamma) s_k^T g_{k+1}}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} s_k \right\|^2$$

$$\Rightarrow \|d_{k+1}\|^2 \le \|g_{k+1}\|^2 + \left\| \frac{y_k^T g_{k+1}}{s_k^T y_k} s_k \right\|^2 + \left| \frac{\sigma\gamma}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} \right| \|s_k^T g_{k+1} y_k\|^2 + \left| \frac{\sigma(1-\gamma)}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} \right| \|s_k^T g_{k+1} s_k\|^2$$

By Couchy Schwartz inequality

$$\Rightarrow \|d_{k+1}\|^2 \le \|g_{k+1}\|^2 + \left| \frac{1}{s_k^T y_k} \right| \|y_k\|^2 \|g_{k+1}\|^2 \|s_k\|^2 + \left| \frac{\sigma\gamma}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} \right| \|y_k\|^2 \|g_{k+1}\|^2 \|s_k\|^2 + \left| \frac{\sigma(1-\gamma)}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} \right| \|g_{k+1}\|^2 \|s_k\|^4$$

$$\Rightarrow \|d_{k+1}\|^2 \le \|g_{k+1}\|^2 + \left(\left| \frac{1}{s_k^T y_k} \right| + \left| \frac{\sigma\gamma}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} \right| \right) L \|g_{k+1}\|^2 \|s_k\|^4 + \left| \frac{\sigma(1-\gamma)}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} \right| \|g_{k+1}\|^2 \|s_k\|^4$$

$$\Rightarrow \|d_{k+1}\|^2 \le \|g_{k+1}\|^2 + \left(\left| \frac{1}{s_k^T y_k} \right| + \left| \frac{\sigma\gamma}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} \right| \right) L \|g_{k+1}\|^2 \|s_k\|^4 + \left| \frac{\sigma(1-\gamma)}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} \right| \|g_{k+1}\|^2 \|s_k\|^4$$

Let $c = \|d_{k+1}\|^2$ then

$$\Rightarrow \|d_{k+1}\|^2 \le \|g_{k+1}\|^2 + \left(\left| \frac{1}{s_k^T y_k} \right| + L \left| \frac{\sigma\gamma}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} \right| + \left| \frac{\sigma(1-\gamma)}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} \right| \right) \|g_{k+1}\|^2 c^2$$

Now let

$$\varphi = \left| \frac{1}{s_k^T y_k} \right| + L \left| \frac{\sigma\gamma}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} \right| + \left| \frac{\sigma(1-\gamma)}{\gamma\|y_k\|^2 + (1-\gamma) y_k^T s_k} \right|$$

$$\Rightarrow \|d_{k+1}\|^2 \le \|g_{k+1}\|^2 + c^2 \phi \|g_{k+1}\|^2$$

Then by use assumption (i)

$$\Rightarrow \|d_{k+1}\|^2 \le \bar{\eta}^2 + c^2 \phi \bar{\eta}^2$$

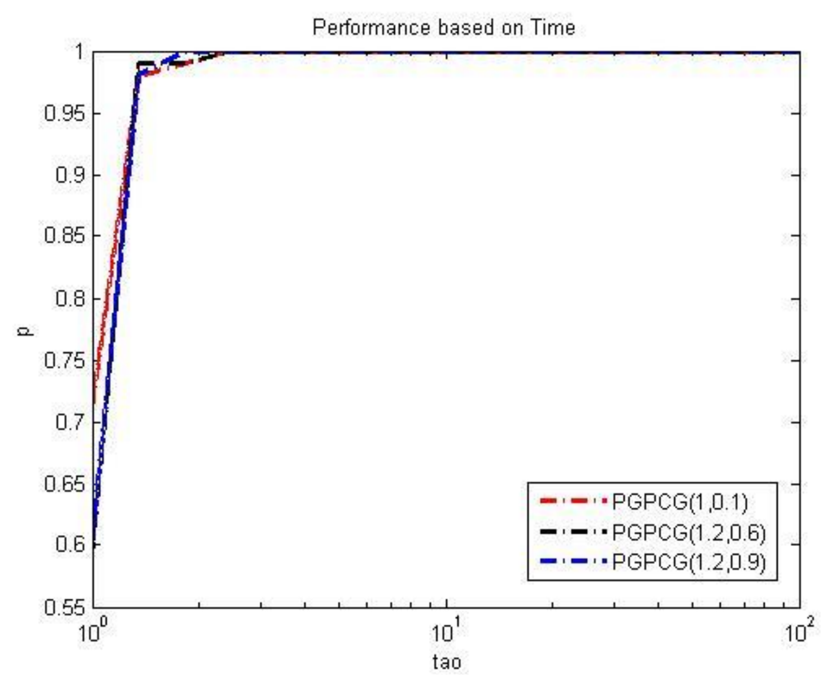
$$\Rightarrow \frac{1}{\|d_{k+1}\|^2} \ge \frac{1}{\bar{\eta}^2 + c^2 \phi \bar{\eta}^2}$$

$$\Rightarrow \sum_{k=0}^{\infty} \frac{1}{\|d_{k+1}\|^2} \ge \frac{1}{\bar{\eta}^2 + c^2 \phi \bar{\eta}^2} \sum_{k=0}^{\infty} 1 = \infty$$

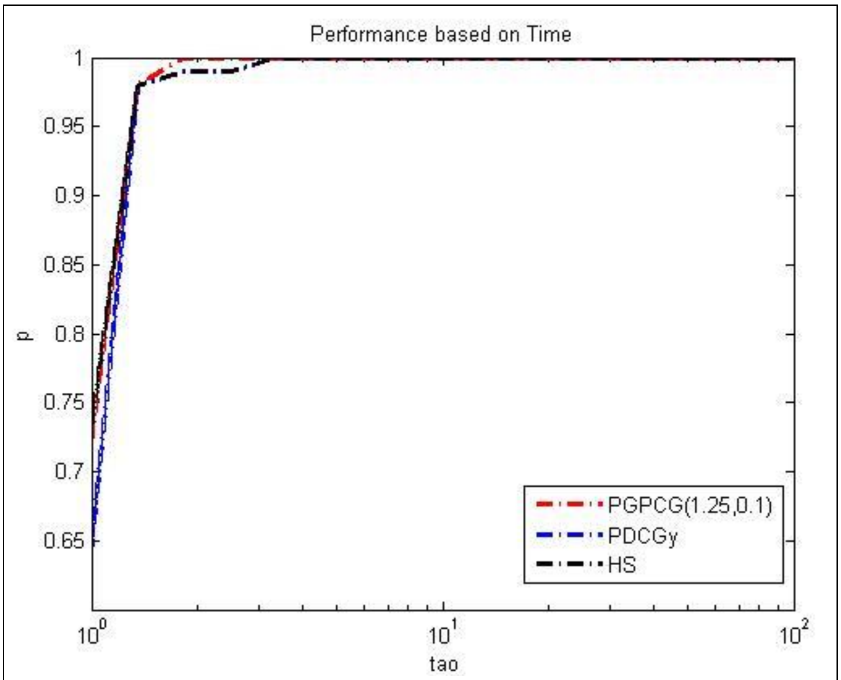
By using the lemma(4.2) then $\lim_{k\to\infty}(\inf \|g_k\|) = 0$

5. Numerical experiment

The numerical results of the new algorithm (**PGPCG**) are displayed in this section. The performance of the proposed algorithm **PGPCG** was made with different values of c and γ in (8), so we denote it by PGPCG (c, γ). We used 51 unconstrained test problems, the number of variable of each test function is 100, 200, ...,1000 respectively with the stopping condition was $\|g_k\| \le 10^{-6}$. All codes are written in Fortran 77 on PC, Intel(R) Celeron(R) CPU N2840 @ 2.16GH 2.16GH, RAM 4.00 GB. The Fig(1) presents the performance of the algorithm **PGPCG** with different values of c and γ, the performance dependent on (Dolan and More)[12] criteria where the result show that the performance of PGPCG(1,0.1) . The Fig(2) present to compare the effective of the new algorithm with HS [1] and PDCGy [6].



Fig(1)



Fig(2)

6. Conclusion

The new algorithm presented a new form of three term conjugate gradient with descent property and global convergence with the Wolfe line search. The new algorithm gives a good result compares with the other algorithms. The optimal value of γ in the search direction (8) may be a good further works of this paper.

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طريقة تدرج مترافق بمعلمة جديدة معتمدة على تعميم طريقة التدرج المترافق لبيري العامة

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الملخص

طريقة تدرج مترافق بمعلمة جديدة معتمدة على تعميم طريقة التدرج المترافق لبيري العامة قدمت خوارزمية معتمدة على فكري بيري، مع تحقيق خاصية الانحدار والتقارب الشمولي تحت شروط وولف. النتائج العددية للخوارزمية الجديدة اثبتت فعاليتها في حل مسائل الامثلية غير المقيدة ذات القياس العالي.