Stopping Power for Tungsten $^{183}_{74}W$

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Abstract
In this paper we report the values of radiative, collisional and total stopping power of electrons (β⁻) for Tungsten element by using Bethe-Bloch relativistic formula. All related equations were inputted into MathCad2012 using the ionization potential value for Tungsten in the energy range (0.1 - 10) MeV. The results show that the collisional stopping power dominates radiative stopping power. This is in excellent agreement with the values of Estar in the energy range (0.01 – 3) MeV, but at energy levels greater than 3 MeV the Bethe-Bloch relativistic formula requires some corrections to minimize errors in results.

Keywords: Bethe-Bloch, stopping power, radiative, collision, mean excitation energy.

1. Introduction
The Stopping Power (SP) of a medium toward a penetrating charged particle is the energy loss suffered by the particle per unit path length [1]. The study of SP of electron and positron through matter is an effective tool for exploring the structure of matter. SP is important in a variety of applications such as Radiation Physics, Chemistry, Biology and Medicine; especially for β^- and β^+ in matter which is widely used in medical applications [2].

The name Tungsten is derived from the Swedish term meaning "heavy stone". Tungsten has been assigned the chemical symbol (W) after its German name *Wolfram*, while sometimes regarded as a scarce or exotic metal. Tungsten abundance in nature is actually about the same as that of copper [3]. The density of W is 19.3 g/cm^3 and its melting point 3380 ^{0}C [4] .The SP calculations for β^- are studied in two different ways; the first considers the interactions of incoming electrons and positrons with electron targets, this is called collisional stopping power (S_{Coll}). The second considers the fact that accelerated charged particles are radiated, which is called radiative stopping power (S_{Rad}) or Bremsstrahlung loss which we will discuss in detail in the next section.

The total stopping power (S_{Tot}) is calculated using the following equation [5]:

$$S^{\pm}_{Tot}(E) = S^{\pm}_{Coll}(E) + S^{\pm}_{Rad}(E) \quad \text{-----(1)}$$

Where the signs (+) and (-) refer to the positron and the electron respectively.

Extensive studies have been undertaken by many researchers [5-14]. Berger and Seltzer [6] modified the Bethe – Heitler theory by introducing empirical corrections to calculate the mean energy loss by Bremsstrahlung. Btra [12] also calculated the SP for positron using two-parameter approximation valid for positron energy (1 – 5000) keV in which it was also found that there are multiple scattering distributions which exhibit differences between β^- and β^+. Jablonski et al [13] reported an improved predictive formula for electron SP and based on their analysis it fit the SP calculated from the optical data for 37 elements in the energy range (30 – 200) keV. Zhenya et al [14] systemically studied the SP and the mean free path in amino acids.

The aim of this work is to use Bethe-Bloch theory to obtain the electronic SP of Tungsten which is of a huge importance in various applications.

2. Methodology
In this section collisional and radiative interactions in tungsten will be discussed as well as mean excitation energies.

2.1 The Collisional Stopping Power
The S_{Coll} is defined as the inelastic collision of β^- and β^+ with atomic electrons, this results in excitation or ionization which ultimately end with heating of the absorber (through atomic and molecular vibrations) unless the ions and electrons can be separated using an electric field as in radiation detectors. The S_{Coll} of Beta particles is different from heavy charged particles due to two physical reasons. Firstly, an electron can lose a large fraction of its energy in a single collision with an atomic electron which has an equal mass. Secondly, a β^- particle is identical to the atomic electron with which it collides. In quantum mechanics β^+ is an electron antiparticle. The identity of particles implies that one cannot distinguish experimentally between the incident and struck electron after collision. Energy loss is when an electron of lower energy is treated as a struck particle after collision unlike a heavy charged particle. The identity of β^- particle and the relation of β^+ to atomic electrons impose certain symmetry requirements that are described by their collisions with atom. The S_{Coll} of Beta particles can be illustrated using the following equation [15].

$$-\left(\frac{dE}{dx}\right)^{\pm}_{coll} = \frac{4\pi r_e^2 mc^2 nZ}{\beta^2 A}\left[\ln\frac{mc^2\tau\sqrt{\tau+2}}{I\sqrt{2}} + F^-(\beta)\right] \quad \text{---- (2)}$$

Where

$$F^-(\beta^-) = \frac{1-\beta^2}{2}\left[1 + \frac{\tau^2}{8} - (2\pi+1)\ln 2\right] \quad \text{---- (3)}$$

Equation (3) is a dimensionless function depending on the kinetic energy (E) of the incident electron in MeV. Where, β = v/c and $\tau = E/mc^2$. β^- is expressed in multiples of electron rest mass energy mc^2. The other symbols in equation (2) represent:

r_e - classical radius of electron .
A - mass number of the target material.
Z - atomic number of the target material.
x - distance travelled by the particle in material.

I - the mean excitation potential of target material.
n - the material electron density which can be calculated using the following equation [15]:

$$n = N_A Z \rho / A M_u \text{ ------------(4)}$$

Where N_A is Avogadro's number, ρ is the density of target material and M_u is the molar mass constant. By substituting equation (4) into equation (2), we obtain a more simplified formula in units of MeV/cm [10].

$$\left(-\frac{dE}{dx}\right)^{\pm}_{Coll} = \frac{5.08\times10^{-31} n}{\beta^2}\left[\ln\frac{3.61\times10^{5}\tau\sqrt{\tau+2}}{I} + F^{-}(\beta^{-})\right] \text{ --- (5)}$$

Where (–*dE*) is the energy increment lost in the infinitesimal material thickness (*dx*). Hence a higher SP means a shorter range the particle can penetrate in the material. The SP is proportional inversely with the incident particle velocity and ionization energy. On the other hand, the mass stopping power (–*dE/ρdx*) of a material is obtained by dividing the SP by its density. Common units for mass stopping power are MeV.g^{-1}.cm^2. The mass stopping power is a useful quantity because it expresses the rate of energy loss of charged particles per g.cm^{-2} of the medium traversed.

2.2 The Radiative Stopping Power

The radiative term, $(dE/dx)_{rad}$, accounts for the energy loss due to bremsstrahlung, Cerenkov radiation or nuclear interactions. S_{rad} represents the inelastic collision with a nucleus, which produces quanta of electromagnetic radiation (photon of energy (h*v*)), as shown in fig. (1). Bethe and Heitler obtained an approximate relation between the S_{Coll} and S_{Rad} by the following relation [7].

$$S^{\pm}_{Rad} = S^{\pm}_{Coll}\left(\frac{EZ}{800}\right) \text{ ------ (6)}$$

By combining equation (1) and (6) we get [5]:

$$S_{tot} = S^{\pm}_{Coll}\left(1+\frac{EZ}{800}\right) \text{ ------- (7)}$$

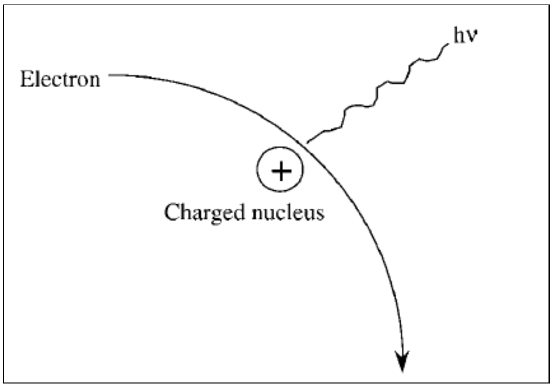

Fig.1 Illustrates the radiative stopping power mechanism which produces Bremsstrahlung radiation.

2.3 Mean excitation energies

Mean Ionization parameter (*I*) of an atom or molecule describes the average minimum energy required to remove an electron from a certain electron shell to infinity which is measured in units of electron volt (eV). *I* can be given by semi-empirical formula [16]:

$$I = 52.8 + 8.71\,Z \quad \text{for} \quad Z > 13 \text{ ------ (8)}$$

I represents a function of the medium's atomic number; this has been provided in literature.

3. Results and Discussion

The results of S_{Coll}, S_{Rad} and S_{Tot} are given in tables 1, 2 and 3, and figures 2, 3 and 4, respectively. These results are obtained by applying equations 2 - 8 using Mathcad2012 for tungsten in energy range (0.1 - 10) MeV. When the electron enters a medium it will lose its kinetic energy and also change its direction continuously, therefore the electron will suffer many deviations at large angles along its path length after it approaches the nuclear field of the target atom. The collisional interaction between incident and orbital electrons are due to the interaction of the electrical fields of both electrons. As the incident electron approaches the orbital electrons no actual contact occurs between them, but its interaction is like the interaction of similar magnetic poles. We assume that *I* is the mean ionization of target atoms and the energy lost in the interaction is given by following equation [17]:

$$E_{loss} = E - I \text{ ------------ (9)}$$

The S_{Rad} value is proportional to the incident electron energy and this situation can be explained as the slow electrons (low energy electrons) spend most of their time interacting with orbital electrons and this indicates that these electrons have a high probability of interacting with atomic electrons, while the fast electrons have a low probability of interacting with atomic electrons and pass over the columbic field without being influenced by the electrons, thus this induces the electrons to open up more channels of radiative energy loss. By comparing the results of S_{Coll} and S_{Rad}, we found that the S_{Coll} dominates the S_{Rad} due to the low energy range therefore S_{Tot} value is largely influenced by S_{Coll}. In all figures, at energies greater than 3 MeV we observe a divergence between the present results and Estar results. This could be due to Bethe-Bloch relativistic formula used in the calculations of this work requiring some corrections to density and shells to minimize the errors.

Table 1. Comparison between S_{Coll} for this work and Estar at different energies for Tungsten.

E (MeV)	S_{Coll} present work	S_{Coll} Estar	*E* (MeV)	S_{Coll} present work	S_{Coll} Estar
0.1	2.0128	2.05	1	1.058	1.02
0.125	1.7854	1.81	1.25	1.0661	1.02
0.15	1.6302	1.65	1.5	1.0788	1.02
0.175	1.5181	1.53	1.75	1.0932	1.03
0.2	1.4337	1.44	2	1.1080	1.04
0.25	1.3166	1.32	2.5	1.1365	1.06
0.3	1.2406	1.23	3	1.1627	1.07
0.35	1.1886	1.18	3.5	1.1865	1.09
0.4	1.1518	1.14	4	1.2081	1.10
0.45	1.1251	1.11	4.5	1.2277	1.11
0.5	1.1055	1.09	5	1.2457	1.13
0.55	1.0910	1.07	5.5	1.2623	1.14
0.6	1.0802	1.06	6	1.2776	1.15
0.7	1.0666	1.04	7	1.3051	1.16
0.8	1.0599	1.03	8	1.3293	1.18
0.9	1.0575	1.02	9	1.3509	1.19
			10	1.3703	1.20

Table 2. Comparison between S_{Rad} for this work and Estar at different energies for Tungsten.

E (MeV)	S_{Rad} present work	S_{Rad} Estar	E (MeV)	S_{Rad} present work	S_{Rad} Estar
0.1	0.0346	0.0408	1	0.0963	0.116
0.125	0.0360	0.0436	1.25	0.1179	0.139
0.15	0.0374	0.0460	1.5	0.1430	0.162
0.175	0.0389	0.0481	1.75	0.1730	0.187
0.2	0.0403	0.0502	2	0.2000	0.212
0.25	0.0431	0.0541	2.5	0.2590	0.263
0.3	0.0459	0.0580	3	0.3208	0.316
0.35	0.0488	0.0618	3.5	0.3921	0.370
0.4	0.0516	0.0657	4	0.4602	0.425
0.45	0.0544	0.0696	4.5	0.5325	0.481
0.5	0.0580	0.0735	5	0.5973	0.537
0.55	0.0609	0.0776	5.5	0.6707	0.595
0.6	0.0649	0.0816	6	0.7403	0.652
0.7	0.0733	0.0899	7	0.8939	0.770
0.8	0.0808	0.0984	8	1.040	0.889
0.9	0.0885	0.107	9	1.1824	1.010
			10	1.3403	1.130

Table 3. Comparison between S_{Tot} for this work and Estar at different energies for Tungsten.

E (MeV)	S_{Tot} present work	S_{Tot} Estar	*E* (MeV)	S_{Tot} present work	S_{Tot} Estar
0.1	2.0474	2.0908	1	1.1543	1.136
0.125	1.8214	1.8536	1.25	1.184	1.159
0.15	1.6676	1.696	1.5	1.222	1.182
0.175	1.557	1.5781	1.75	1.2662	1.217
0.2	1.474	1.4902	2	1.3081	1.252
0.25	1.3597	1.3741	2.5	1.3957	1.323
0.3	1.2865	1.288	3	1.4835	1.386
0.35	1.2374	1.2418	3.5	1.5786	1.460
0.4	1.2034	1.2057	4	1.6683	1.525
0.45	1.1795	1.1796	4.5	1.7602	1.591
0.5	1.1635	1.1635	5	1.843	1.667
0.55	1.1519	1.1476	5.5	1.933	1.735
0.6	1.1451	1.1416	6	2.0179	1.802
0.7	1.1399	1.1299	7	2.199	1.930
0.8	1.1407	1.1284	8	2.3693	2.069
0.9	1.146	1.127	9	2.5333	2.20
			10	2.7106	2.33

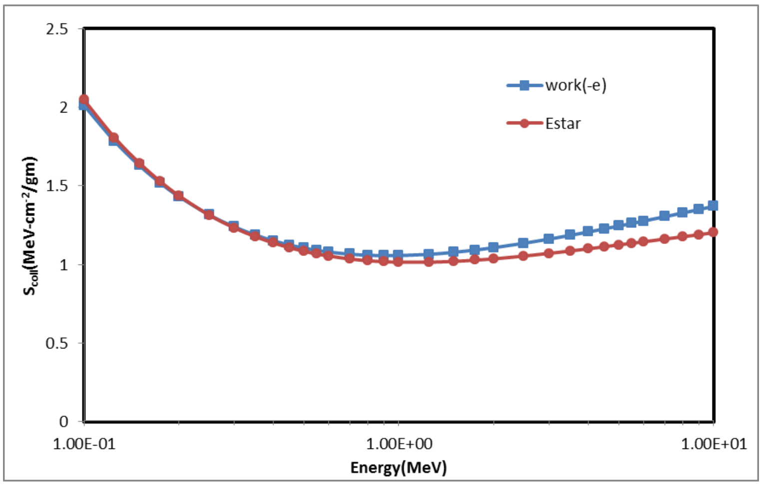

Fig. 2 Compares the S_{Coll} of this work with Estar for Tungsten.

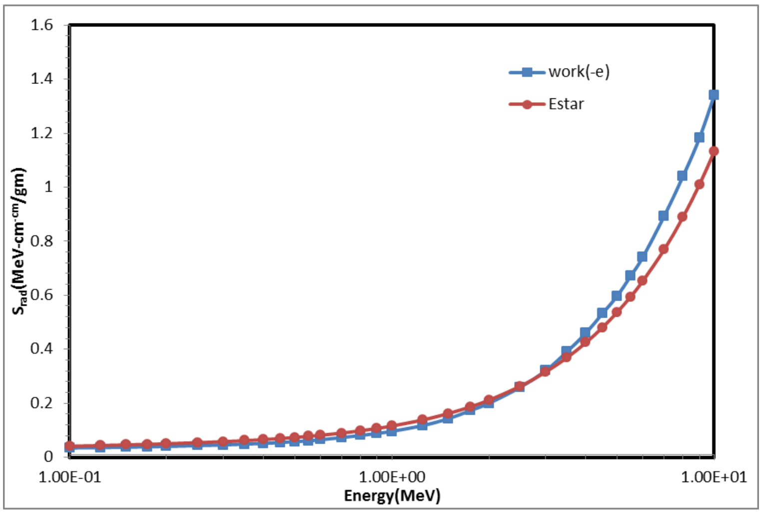

Fig. 3 Compares the S_{Rad} of this work with Estar for Tungsten.

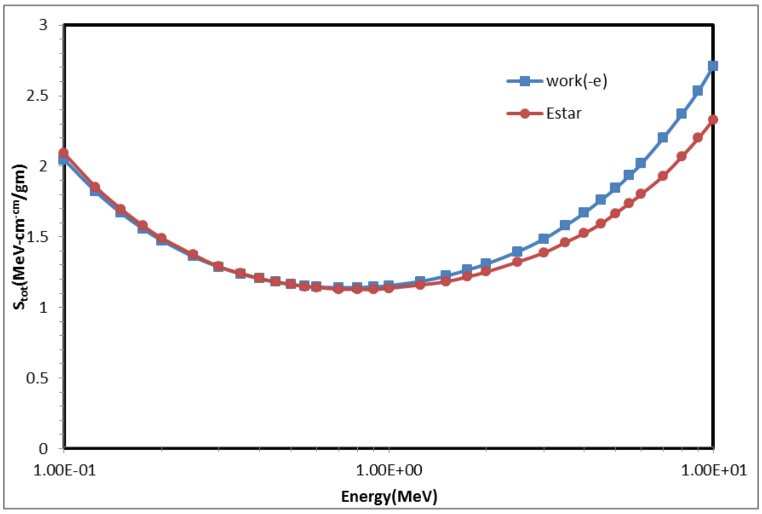

Fig. 4 Compares the S_{Tot} of this work with Estar code for Tungsten.

4. Conclusion

Through the findings of this research work, we can conclude the following:

1. The calculations indicate that S_{Tot} decreases with increasing particle incident energy, and this energy depends upon the particle velocity which limits the type of interactions with the target.

2. With increasing particle incident energy, S_{Rad} values increase due to the fact that the electrons reach the nuclear field producing Bremsstrahlung radiation.

3. Interactions between low energy incident electrons and valence electrons in tungsten are largely radiative while at higher energies they are collisional.

4. Although S_{rad} is significantly lower than S_{Coll} it still influences the interaction hence affecting $S_{Tot.}$

5. The S_{Tot} value depends on particle incident energy and the atomic number of the target.

5. References

[1] A. Mozumder, "Fundamentals of Radiation Chemistry" Ch. 2, pp.12-24 AP Academic Press, (1999) USA.

[2] Mustafa Cagatay Tufan and Hasan Gumus, "A study on the calculation of topping power and CSDA range for incident positrons", journal of nuclear material 412, pp.308-314 (2011).

[3] Lexicon Universal Encyclopedia W Lexicon publication Inc New york, (1988), USA.

[4] Halliday, Resnick and Walker, "Fundamentals of Physics 6th edition" appendix G p.1152 John Wiley & sons Inc USA (2001).

[5] P. B. Pal, V. P. Varshney and D. K. Gupta, "Semiempircal stopping power equation for positrons" Journal of applied physics 60 (1986).

[6] M. J. Berger, S. M. Seltzer, "Stopping powers and ranges of electrons and positrons" (National Bureau of Standards Report, NBSIR 82-2550 A, (1982).

[7] H. A. Bethe, Z., Phys. 76, p.293 (1932).

[8] C. Moler, Ann. Phys., 14, p.568 (1932).

[9] H. J. Bhabha, Proc. R. Soc. London Ser A. pp.154 -194 (1936).

[10] F. Fohrlch and B. C. Carlson, Phys. Rev. 93, p. 38 (1953).

[11] H. Bethe and W. Heitler, Proc. R. Soc London A., 146, p.83 (1934).

[12] R. K. Btra and M. L. Sehgal, Nucl. Phys. A. 156, p. 314 (1970).

[13] A. Jablonski, S. Tanuma and C. J. Powell, "modified predicative formula for electron stopping power "J. Applied physics, p.103 (2006).

[14] Zhenyu Tan, Yueyuan Xia, Mingwmino Zhao and Xian gdong Liu, "Electron stopping power and inelastic mean free path in a acids and protein over energy range 20-20KeV" Rad. Biophysics, p. 450 Vol. 45 (2008).

[15] ICRU "Stopping powers for electrons and positrons (International Commission on Radiation Units and Measurements" Bethesda, MD Report, No. 37, 1984.

[16] Mustafa Cagatay Tufan, Onder Kabaday and Hasan Gumus, "Calculation of the stopping power for intermediate energy positrons" Chinese Journal of physics Vol. 44, No. 4, pp. 290-296 China (2006).

[17] Kenth Krane, " Modern Physics 2nd edition" ch. 5, p.145, John Wiley & sons Inc., USA (1996).

قدرة الإيقاف لعنصر التنكستن $^{183}_{74}W$

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الملخص

قدمنا في هذا العمل البحثي قيم قدرة الإيقاف الاشعاعية (S_{Rad}) والتصادمية (S_{Coll}) والقدرة الكلية (S_{tot}) لجسيمات بيتا السالبة (β^-) الساقطة على مادة التنكستن باستخدام معادلة بيث – بلوخ النسبية ، حيث كتب جميع المعادلات خلال البرنامج الحسابي (MathCad2012) ولمدى طاقة من (0.1 – 10) ميكا إلكترون فولت ، وقد بينت النتائج التي حصلنا عليها الهيمنة الكبيرة لقدرة الإيقاف التصادمية اكثر من قدرة الايقاف الاشعاعية في قيم قدرة الايقاف الكلية. وبمقارنة النتائج مع (Estar code) وجدنا انها تطابق بشكل جيد لمدى الطاقة من (0.1 – 3) ميكا إلكترون فولت، اما بالنسبة للطاقة الاكثر من 3 ميكا إلكترون فولت فان معادلة بيث – بلوخ النسبية تتطلب ادخال بعض التصحيحات لتقليل نسبة الخطأ.

الكلمات المفتاحية: معادلة بيث – بلوخ ، قدرة الايقاف ، الاشعاعية ، التصادمية ، معدل طاقة التأيين.