Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php/j

Some Result on Supra Separation Axioms via Graph Theory

Fatima Ibrahim Fathi[1], **Nabeel Ezzulddin Arif**[1], **Luma Saad Abdalbaqi**[2]

[1] *Department of Mathematic, College of pure sciences, University of Tikrit, Tikrit, Iraq*

[2] *Department of Mathematic, College of pure sciences, University of Tikrit, Tikrit , Iraq*

https://doi.org/10.25130/tjps.v27i1.88

ARTICLE INFO.

Article history:

-Received: **20 / 10 / 2021**

-Accepted: **25 / 11 / 2021**

-Available online: **/ / 2022**

Keywords: Supra Topology , Topological Graph , supra gT_0, gT_1, gT_2, gT_3, gT_4.

Corresponding Author:

Name: Fatima Ibrahim Fathi

E-mail:

fatimah.i.fathi35516@st.tu.edu.iq

nabarif@tu.edu.iq

lumahhany1977@tu.edu.iq

Tel:

ABSTRACT

The Concepts of supra topological graph and introduces of class supra separation axioms on supra topological graph (supra gT_0, supra gT_1,supra gT_2,supra gT_3,supra gT_4), many relations among them were studied. Gave results for them.

1. Introduction

A graph G (V', E), where $V \neq \varphi$ set is supposed to be "vertices or nodes" and E $\subseteq G(V)$is spoken be "edges or links" [1]. lis supposed to be "loop" [1-2].Assuming G = (V'(G), E(G)) be a graph , H a "sub graph " of G if V'(H) ⊆ V'(G) and E(H) ⊆ E(G)), (H ⊆ G) [3]. If graph have no loops and no multi edge "simple graph" [4]. A circuit in a graph G that contain each vertex in G precisely once, aside from the beginning and finishing vertex that shows up twice is known as Hamiltonian circuit. in the event that it contains a Hamiltonian.circuit. A Hamiltonian.path is simple that contains all vertices of G where the end focuses many be unmistakable [7]. A connected graph G is Eulerian if there exists a closed trail containing every edge of G, it is also called Euler.path. A Euler path that is called Euler.circuit. A graph which has a Eulerian.circuit is called an Eulerian graph [7]. The supra topological spaces had been introduced by A.S. Mashhour at [8]. In 1983. If (X) is a non-unfilled set, a collection (μ) is subfamily of (X) if (i) X, ∅ ∈ (μ) (ii) if $Á_i \in (\mu)$ for i ∈ y than∪ $Á_i \in (\mu)$ [5]. "The separation axioms" at supra topological space [5]. Consequently, the (X, μ) is called supra T_0-space if for any pair from different points from X, ∃ at least one point set which incorporate one from them anyway not the other[6]. (X, μ) is called supra T_1-space if for any pair of different points from X, there exist two open seats Á, É ∈ μ, such that x ∈ Á, y ∉ Á and x∉ É ,y∈ É [6]. (X, μ) is classified supra T_2-space, if for any every pair from different points can be separated by disjoint open set [6]. (X, μ) is classified " supra regular.space" if ∀ nonempty closed set F and a point x which does not belong to F, ∃open set Á,É, such that x ∈ Á,F⊆ É and Á ∩É =∅

Also this space is "regular" and T_1-space thin, at that point, it is designated "T3-space"[6]. A supra topological space (x,μ) is called "supra normal" space if and provided that for each pair R,W of disjoint closed subsets of Y, there is a couple (Á, É) of disjoint open subsets of X such that R ⊆ Á,W ⊆ É and A∩É =∅ [5]. A normal & supra T_1-space is supposed to be supra T4-space [5]. In this work, we show some new definitions from supra separation-axioms count at the supra topological graph. Utilizing the definition from supra topological graph which established at the neighboring from the decision from vertices and edges between vertices at the graph. We introduce new supra separation-axioms at graphs is said to be a graph supra separation axioms say (supra

gT_0, supra gT_1, supra gT_2, supra gT_3 , supra gT_4), Finally, we gave an examination for certain instances of these graphs supra division aphorisms. Examination is giving a definition for graphs supra-separation-axioms.

2. Graph Separation Axioms

Definition 2.1 Let G =(V',E) be a graph with supra topological graph (V'(G), μ_G). The post class 0f all vertices (v'_iR) for all $v'_i \in$V'(G). The complement 0f these p0st class is called supra closed post class and denoted by supra C (v'_iR)

Definition2.2A supra topological graph (V'(G), μ_G), is said to be supra gT_0 if for any pair of vertices of V'(G), there exist no less than one post class of any vertex which contains one of them however not the e other. All in all, a supra topological. graph (V'(G), μ_G) is supposed to be a supra gT_0 if for any v'_1,v'_2 $\in$V'(G). $v'_1 \neq v'_2$, there exist a post class of any vertex R is supposed to be $v'_1\in$ R however $v'_2\notin$ R.

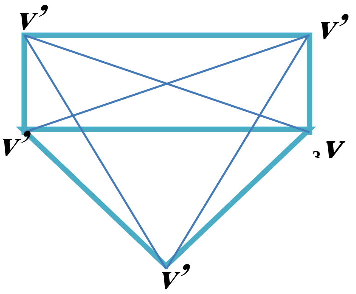

Fig. 1: A Complete graph

Example 2.3. Let G = (V', E) be a graph as shown in Figure 1. We construct its supra topology. The post classes of the vertices are the following :V'(G) = $\{v_i$/i=1,2,3,4,5$\}$E(G)=$\{v_1v_i$:/i=2,3,4,5$\}$، ∪$\{v_2v_i$:/i=1 ,3,4,5$\}$∪$\{v_3v_i$:/i=1,2,4,5$\}$∪$\{v_4v_i$: / i=1,2,3,5$\}$∪$\{v_5v_i$: / i=1,2,3,4$\}$

The post class of the vertices v'_1R ={v'_2, v'_3, v'_4, v'_5 }v'_2R ={v'_2, v'_3, v'_4, v'_5,v'_5R ={v'_1, v'_2, v'_3, v'_4 }v'_3R={v'_1, v'_2 ,v'_4,v'_5 }v'_4R ={v'_1, v'_2,v'_3,v'_5 }،

Thin, a subbase of a supra topology is . S_G = {{v'_2, v'_3, v'_4, v'_5}, { v'_2, v'_3, v'_4, v'_5 }, {v'_1, v'_2 ,v'_4,v'_5},{v'_1,v'_2,v'_3,v'_5},{v'_1, v'_2,v'_3,v'_4}}. The base B_G = {V'(G),ϕ,{v'_3,v'_4,v'_5},{v'_2,v'_4,v'_5},{v'_2, v'_3, v'_5 },{ v'_2, v'_3,v'_4},{v'_1,v'_4, v'_5},{v'_1,v'_3,v'_5},{v'_1,v'_2,v'_4},{v'_1,v'_2,v'_5},{v'_1,v'_2},{v'_1,v'_2,v'_3},{v'_2,v'_3,v'_4,v'_5},{v'_2,v'_3,v'_4,v'_5},{v'_1,v'_2,v'_4, v'_5},{v'_1,v'_2,v'_3,v'},{v'_1,v'_2,v'_3,v'_4},{v'_1},{v'_2},{v'_3},{v'_4},{v'_5},{v'_1,v'_2,v'_4}}.The supra topology on the post class is of:μ_G={V'(G),ϕ,{v'_2,v'_3, v'_4, v'_5 },{v'_1, v'_3, v'_4, v'_5}, {v'_1,v'_2 ,v'_4, v'_5 },{v'_2, v'_4, v'_5},{v'_1,v'_2,v'_3,v'_5},{v'_2,v'_3,v'_5},{v'_1,v'_2,v'_3,v'_4},{v'_2, v'_3,v'_4},{v'_1,v'_3,v'_4,v'_5},{v'_1,v'_2,v'_4},{v'_1,v'_4,v'_5},{v'_1, v'_3,v'_5},{v'_1,v'_3,v'_4},{v'_1,v'_2,v'_5},{v'_1,v'_2,v'_3},{v'_1,v'_2} ,{v'_1,v'_2,v'_4},{v'_1,v'_3},{v'_1,v'_4},{v'_1,v'_5},{v'_2,v'_3},{v'_2 ,v'_4},{v'_2,v'_5},{v'_3,v'_4},{v'_3,v'_5},{v'_4,v'_5},{v'_1},{v'_2}, {v'_3},{v'_4},{v'_5}}This is called a discrete supra topology and the graph G is called a complete graph.

Remark 2.4.: In a indiscrete supra topological graph V'(G), G isn't supra gT_0, and in a discrete topological graph (G) is supra gT_0.

Definition 2.5A supra topological graph (V'(G), μ_G) is said to be a supra gT_1 if for any pair of distinct vertices of V'(G), there exist two post class of any vertex which contains one of them but not the other. In other words, a supra topological graph (V'(G), μ_G) is said to be a supra gT_1 if for any v'_1,$v'_2 \in$ V'(G), $v'_1 \neq v'_2$, there exist post classes of any vertices R and W such that $v'_1\in$ R, $v'_2\notin$ R and $v'_2\in$ W, $v'_1\notin$ W.

Example.2.6. Let v'_1 R = {v'_2, v'_3, v'_4 , v'_5 },v'_2 R ={v'_1, v'_3, v'_4 , v'_5 },v'_3 R = { v'^{*}_1,v'_2, v'_4 , v'_5 },v'_4 R = {v'_1, v'_2, v'_3, v'_5 },v'_5 R = { v'_1, v'_2, v'_3 ,v'_4 }.Then G is supra gT_0.

Definition 2.7. A supra topological graph (V'(G), μ_G) is said to be a supra gT_2 if ∀ pair of distinct V', which can be separated by disjoint post class. In other words, a supra topological graph (V'*(G), μ_G) is said t0 bi a supra gT_2 if f0r any v'_1,$v'_2 \in$ V'(G), $v'_1 \neq v'_2$, there exist post classes of any vertices R and W s.t $v'_1\in$ R , $v'_2\in$ W, R ∩ W = ϕ**,** The converse isn't accurate over all.

Example 2.8. From example 2.3.v''_1R = {v'_2, v'_3, v'_4, v'_5 },v'_2R = {v'_2, v'_3, v'_4,v'_5},v'_3R = {v'_1,v'_2,v'_4, v'_5 },v'_4R = {v'_1, v'_2, v'_3,v'_5 },v'_5R = {v'_1, v'_2, v'_3,v'_4 }. Thin G is not supra gT_2

Definition 2.9:A supra topological graph (V'(G), μ_G) is said to be a supra gT_3 if it is supra gT_1 and for every non empty closed post class F (a complement of a post class of each vertices) and a vertex v_1 which does not belong to F, there exist post classes of any vertices R and W, such that $v_1 \in$ R, F ⊆ W, R ∩ W = ϕ.

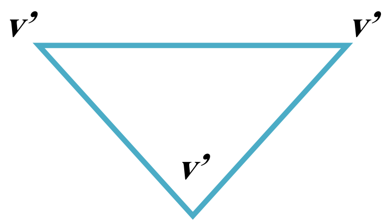

Fig. 2: A supra gT_3

Example 2. 10 Let G be a graph with a supra topological space(V'(G),μ_G) as shown in Figure 2. The post classes are v'_1R ={v'_2},and v'_2R={v'_1},v'_1R={v'_3},v'_3R={v'_1},v'_2R={v'_3},v'_3R={ v'_2}. It's easy to show that G is supra gT_3.

Remark 2. 11 If G graph with a supra topological graph (V'(G), μ_G). Then supra gT_3 is supra gT_2.

Proof: Let G be supra gT_3. G is supra gT_1, byDefinition2.5, ∀ $v_1 \in$ V'(G), and any closed post class F of V'(G), there exist post classis of any R and W such that $v_1 \in$ R, F ⊆ W, R∩W = ϕ . Now, For all x, $y\in$ V'(G) and $x\neq y$. Thin $x\in$ V' (G), {y} is closed post class, {y} ⊆ supraC ({x}). Since G is supra gT_3, then there exist two post classis U and V' s.t $x\in$U, {y} ⊆ V' , U ∩V'= ϕ .So G is supra gT_2.The converse of Remark 3.1.17, The opposite isn't accurate over all as shown example.

Example 2.12 Let G be a graph with a topological graph (V'(G), μ_G),as shown in Figure 3,where : V'(G)={v_i/1≤i≤5},E(G)={v_1v_i :/i=2,5,

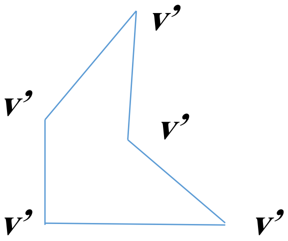

Fig. 3

{v_2v_i :/i=1,3},{v_3v_i :/i=2,4,{v_4v_i :/i=3,5}, {v_5v_i :/ i=1,4}The post classis are: v'_1R = {v'_2, v'_5}, v'_2R = {v'_1 , v'_3}, v'_3R = {v'_2 , v'_4},v'_4R = {v'_3 , v'_5},v'_5R = {v'_1, v'_4}.

The closed post classis are: supra C (v'_1R) = {v'_1,v'_3,v'_4}, supra C (v'_2R) = {v'_2, v'_4 , v'_5}, supra C (v'_3R) = {v'_1, v'_3 , v'_5}, supra C (v'_4R) = {v'_1, v'_2 , v'_4}, supra C (v'_5R) = {v'_2, v'_3 , v'_5}. It's easy to show that G is supra gT$_2$, but it is not supra gT$_3$.

Difiniti0n 2.13A supra t0p0l0gical graph (V'(G), μ_G), is said t0 bi a supra gT$_4$ if it is supra gT$_1$ and for each pair A, B of disjoint closed subsets of V'(G), there is a pair U,V' of disjoint post classes of V'(G) so that A ⊆ U, B ⊆ V' and U ∩ V' = ϕ.

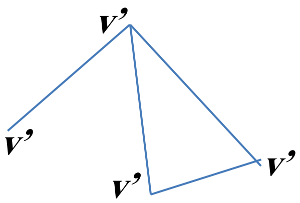

Fig. 4

Example 2.14. The post classes are: v'_1R = {v'_2, v'_3, v'_4},v'_2R = {v'_1, v'_3}, v'_3R = {v'_1, v'_2}, v'_4R = {v'_1}.

The supra closed post classes are: supra C(v'_1R)= {v'_1}, supra C(v'_2R) = {v'_2, v'_4}, supra C (v'_3R) = {v'_3, v'_4}, supra C (v'_4R) = {v'_2, v'_3, v'_4},. It's easy to show that G is supra gT$_4$.

Remark 2.15. one can deduce that each supra gT$_4$ is supra gT$_3$. The opposite isn't accurate overall as in the blew example.

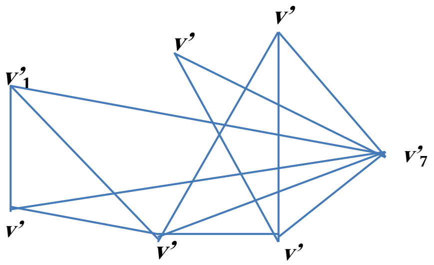

Fig. 5

Example2.16. Let G be a graph with a supra topological space (V'(G), μ_G) as shown figure.5.Where:V'(G) = {v_i :/1 ≤ i ≤7}, E(G) = {v_1v_i :/i= 5,6,7 },{v_2v_i :/i=6,7},{v_3v_i :/i=4,5,7},{v_4v_i :/i=3,5,6,7}, {v_5v_i : / i=1,3,4,6,7},{v_6v_i : / i=1,2,4,5},{v_7v_i : / i=1,2,3,4,,5,6}.

The post class of all vertices are: v'_1R = {v'_5, v'_6, v'_7}, v'_2R = {v'_6, v'_7},v'_3R = {v'_4, v'_5 ,v'_7},v'_4R = {v'_3, v'_5, v'_6, v'_7}, v'_5R = {v'_1, v'_3, v'_4, v'_6 , v'_7}, v'_6R = {v'_1, v'_2, v'_4, v'_5}, v'_7R = {v'_1, v'_2, v'_3, v'_4, v'_5, v'_6}, where : v_1, v_2 ∈V'(G) and v_1 ∈v'_5R, but v_2 ∉ v'_5R. v_1, v_3 ∈V'(G) and v'_3∈v'_4R, but v_1 ∉ v'_4R .v_1, v_4 ∈V'(G) and v_4 ∈v'_3R, but v_1 ∉ v'_3R. v_1, v_5 ∈V'(G) and v_5 ∈v'_1R, but v_1 ∈ v'_1R .v_1, v_6 ∈V'(G) and v_6 ∈v'_1R, but v_1 ∉ v'_1R. v_1, v_7 ∈V'(G) and v_7 ∈v'_1R, but v_1 ∉ v'_1R .v_2, v_3 ∈V'(G) and v_3 ∈v'_4R, but v_2 ∉ v'_4R.v_2, v_4 ∈V'(G) and v_4 ∈v'_3R, but v_2 ∉ v'_3R .v_2, v_5 ∈V'(G) and v_5 ∈v'_1R, but v_2 ∉ v'_1R.v_2, v_6 ∈V'(G) and v_6 ∈v'_1R, but v_2 ∉ v'_1R .v_2, v_7 ∈V'(G) and v_7 ∈v'_1R, but v_2 ∉ v'_1R. v_3, v_4 ∈V'(G) and v_4 ∈v'_3R, but v_3 ∉ v'_3R, v_3, v_5 ∈V'(G) and v_5 ∈v'_1R, but v_3 ∉ v'_1R.v_3, v_6 ∈V'(G) andv_6 ∈v'_1R, but v_3 ∉ v'_1R .v_3, v_7 ∈V'(G) and v_7 ∈v'_1R, but v_3 ∉ v'_1R.v_4, v_5 ∈V'(G) and v_5 ∈v'_1R, but v_4 ∉ v'_1R. v_4, v_6 ∈V'(G) and v_6 ∈v'_1R, but v_4 ∉ v'_1R.v_4, v_7 ∈V'(G) and v_7 ∈v'_1R, but v_4 ∉ v'_1R .v_5, v_6 ∈V'(G) and v_6 ∈v'_2R, but v_5 ∉ v'_2R. v_5, v_7 ∈V'(G) and v_7 ∈v'_2R, but v_5 ∉ v'_2R.v_6, v_7 ∈V'(G) and v_7 ∈v'_3R, but v_6 ∉ v'_3R. Thin G is supra gT$_0$.v_1, v_2 ∈V'(G) and v_1 ∈v'_5R, but ∄ any p0st class contain a vertex b but not contain a , b. Thin G is not supra gT$_1$, and is not supra gT$_2$. The Complement of the post classes are: supra C (v'_1R) = {v'_1,v'_2, v'_3, v'_4}, supraC(v'_2R)={v'_1,v'_2,v'_3,v'_4,v'_5},supra C(v'_3R)={v'_1,v'_2,v'_5,v'_6},supraC(v'_4R)={v'_1,v'_2,v'_4}, supra C (v'_5R) = {v'_2,v'_5}, supra C(v'_6R) ={v'_3,v'_6,v'_7}, supra C (v'_7R) = {v'_7}. The first vertex v'_1 and F= {v'_2}, there exist post classes Rv'_5R,and W= v'_6R, such that v'_1∈ R, F ⊆ W, but R∩W ≠ ∅. Then G is not supra gT$_3$.

Example2.17. Let (V'(G),μ_G) be supra topological. Space as shown in figure 6 Where V'(G)={v_i :/i=1,2,3,4,5},I(G)={v_1v_i :/i=2,5},∪ {v_2v_i :/i=1,3},∪ {v_3v_i :/i=2,4}, ∪{v_4v_i :/i=3,5}, ∪ {v_5v_i :/ i=1,4},

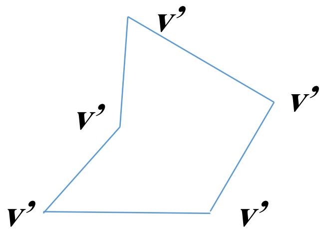

Fig. 6

The post class of all vertices are:$v'_1R = \{v'_2, v'_5\}$, $v'_2R = \{v'_1, v'_3\}$, $v'_3R = \{v'_2, v'_4\}$, $v'_4R = \{v'_3, v'_5\}$, $v'_5R = \{v'_1, v'_4\}$,$v_1, v_2 \in V'(G)$ and $v_2 \in v'_1R$, but $v_1 \notin v'_1R$. $v_1, v_3 \in V'(G)$ and $v_3 \in v'_4R$, but $v_1 \notin v'_4R$. $v_1, v_4 \in V'(G)$and $v_1 \in v'_2R$, but $v_4 \notin v'_2R$.$v_1, v_5 \in V'(G)$ and $v_5 \in v'_1R$, but $v_1 \in v'_1R$. $v_2, v_3 \in V'(G)$ and $v_2 \in v'_1R$, but $v_3 \notin v'_1R$.$v_2, v_4 \in V'(G)$ and $v_2 \in v'_1R$, but $v_4 \notin v'_1R$.$v_2, v_5 \in V'(G)$ and $v_2 \in v'_3R$, but $v_5 \notin v'_3R$.$v_3, v_4 \in V'(G)$ and $v_3 \in v'_2R$, but $v_4 \notin v'_2R$.$v_3, v_5 \in V'(G)$ and $v_3 \in v'_2R$, but $v_5 \notin v'_2R$.$v_4, v_5 \in V'(G)$ and $v_5 \in v'_1R$, but $v_4 \notin v'_1R$. Then G is supra gT_0. $v_1, v_2 \in V'(G)$ and $v_1 \in v'_5R$, but $v_2 \in v'_3R$.$v_1, v_3 \in V'(G)$ and $v_1 \in v'_5R$, but $v_3 \in v'_4R$.$v_1, v_4 \in V'(G)$ and $v_1 \in v'_2R$, but $v_4 \in v'_3R$. $v_1, v_5 \in V'(G)$ and $v_1 \in v'_5R$, but $v_5 \in v'_4R$. $v_2, v_3 \in V'(G)$ and $v_2 \in v'_1R$, but $v_3 \in v'_4R$. $v_2, v_4 \in V'(G)$ and $v_2 \in v'_1R$, but $v_4 \in v'_5R$. $v_2, v_5 \in V'(G)$ and $v_2 \in v'_3R$, but $v_5 \in v'_4R$. $v_3, v_4 \in V'(G)$ and $v_3 \in v'_2R$, but $v_4 \in v'_3R$.$v_3, v_5 \in V'(G)$ and $v_3 \in v'_2R$, but $v_5 \in v'_1R$. $v_4, v_5 \in V'(G)$ and $v_4 \in v'_3R$, but $v_5 \in v'_1R$. Then G is supra gT_1.$v_1, v_2 \in V'(G)$ and$v_1 \in v'_2R$, $v_2 \in v'_1R$and$v'_2R \cap v'_1R=\emptyset$$v_1, v_3 \in V'(G)$ and $v_1 \in v'_5R$, $v_3 \in v'_4R$ and $v'_5R \cap v'_4R = \emptyset$,$v_1, v_4 \in V'(G)$ and $v_1 \in v'_2R$, $v_4 \in v'_3R$ and $v'_2R \cap v'_3R = \emptyset$, $v_1, v_5 \in V'(G)$ and $v_1 \in v'_2R$, $v_5 \in v'_1R$ and $v'_2R \cap v'_1R = \emptyset$, $v_2, v_3 \in V'(G)$ and $v_2 \in v'_1R$, $v_3 \in v'_2R$ and $v'_1R \cap v'_2R = \emptyset$, $v_2, v_4 \in V'(G)$ and $v_2 \in v'_1R$, $v_4 \in v'_5R$ and $v'_1R \cap v'_5R = \emptyset$,$v_2, v_5 \in V'(G)andv_2 \in v'_3R$,$v_5 \in v'_4R$ and$v'_3R \cap v'_4R=\emptyset$,$v_3, v_4 \in V'(G)$and $v_3 \in v'_2R$, $v_4 \in v'_3R$ and $v'_2R \cap v'_3R = \emptyset$, $v_3, v_5 \in V'(G)$ and $v_3 \in v'_2R$, $v_5 \in v'_1R$ and $v'_2R \cap v'_1R =\emptyset$, $v_4, v_5 \in V'(G)$ and $v_4 \in v'_5R$, $v_5 \in v'_4R$ and $v'_5R \cap v'_4R = \emptyset$. Thin G is supra gT_2.

Example 2.18: Let G be a Hamiltonian

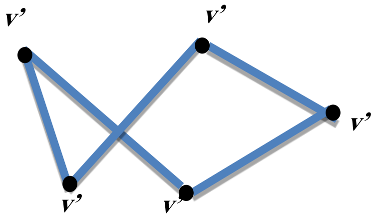

Fig. 7: A Hamiltonian-circuit

circuit graph as shown in Figure 7, The post classes of the vertices are the following:
$v'_1R=\{v'_4, v'_5\}$,$v'_2R=\{v'_3, v'_5\}$,$v'_3R=\{v'_2, v'_4\}$,$v'_4R = \{v'_1, v'_3\}$, $v'_5R = \{v'_1, v'_2\}$. Then, a subbase of a supra topology is $S_G = \{\{v'_4, v'_5\}, \{v'_3, v'_5\}, \{v'_2, v'_4\}, \{v'_1, v'_3\}, \{v'_1, v'_2\}\}$. The base is $B_G = \{\{V'(G), \phi, \{v'_5\}, \{v'_4\}, \{v'_3\}, \{v'_4, v'_5\}, \{v'_3, v'_5\}, \{v'_2, v'_4\}, \{v'_1, v'_3\}, \{v'_1, v'_2\}\}$.$\mu_G = \{V'(G), \phi, \{v'_1\}, \{v'_2\}, \{v'_3\}, \{v'_4\}, \{v'_4\}, \{v'_4, v'_5\}, \{v'_3, v'_5\}, \{v'_2, v'_4\}, \{v'_1, v'_3\}, \{v'_1, v'_2\}, \{v'_1, v'_4\}, \{v'_1, v'_5\}, \{v'_2, v'_3\}, \{v'_2, v'_5\}, \{v'_3, v'_4\}, \{v'_2, v'_3, v'_5\}, \{v'_2, v'_4, v'_5\}, \{v'_3, v'_4, v'_5\}, \{v'_2, v'_3, v'_4\}, \{v'_1, v'_3, v'_4, v'_5\}, \{v'_1, v'_2, v'_4, v'_5\}, \{v'_2, v'_3, v'_4, v'_5\}, \{v'_1, v'_2, v'_3, v'_4\}\}$.**Example 2.19**:

The post classes of the vertices are the following: $v'_1R = \{v'_2, v'_3\}$, $v'_2R = \{v'_1, v'_4\}$, $v'_3R = \{v'_1, v'_4\}$, $v'_4R = \{v'_2, v'_3, v'_5\}$, $v'_5R = \{v'_4\}$.

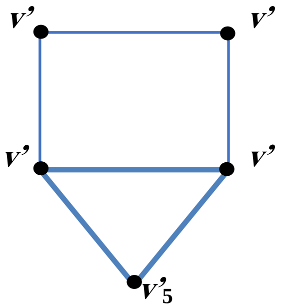

Fig. 8: A Hamiltonian circuit

Thin, a subbase of a supra topologygraph $S_G=\{\{v'_2, v'_3\}, \{v'_1, v'_4\}, \{v'_2, v'_3, v'_5\}, \{v'_4\}\}$.and the base is of a supra topology graph,$B_G = \{\{\phi, V'(G), \{v'_2\}, \{v'_1, v'_4\}, \{v'_4\}, \{v'_2, v'_3, v'_5\}\}$.Therefore, the supra topology graph:$\mu_G = \{\phi, V'(G), \{v'_1\}, \{v'_2\}, \{v'_4\}, \{v'_1, v'_4\}, \{v'_2, v'_3\}, \{v'_2, v'_4\}, \{v'_2, v'_3, v'_5\}, \{v'_1, v'_2, v'_4\}, \{v'_2, v'_4\}, \{v'_1, v'_2, v'_3, v'_4\}, \{v'_2, v'_3, v'_4\}, \{v'_2, v'_3, v'_4, v'_5\}\}$.Then the Supra Separation axiom on Hamiltonian circuit. The post classes of the vertices of Hamiltonian circuit graph,$v'_1R = \{v'_2\}$, $v'_2R = \{v'_3\}$,$v'_3R=\{v'_5\}$,$v'_4R=\{v'_1\}$,$v'_5R=\{v'_4\}$$v_1, v_2 \in V'(G)$ and $v_1 \in v'_4R$,but $v_2 \notin v'_4R$.$v_1, v_3 \in V'(G)$ and $v_3 \in v'_2R$, but $v_1 \notin v'_2R$.$v_1, v_4 \in V'(G)$ and $v_4 \in v'_5R$, but $v_1 \notin v'_5R$. $v_1, v_5 \in V'(G)$ and $v_5 \in v'_3R$, but $v_1 \in v'_3R$. $v_2, v_3 \in V'(G)$ and $v_2 \in v'_1R$, but $v_3 \notin v'_1R$.$v_2, v_4 \in V'(G)$ and $v_4 \in v'_5R$, but $v_2 \notin v'_5R$. $v_2, v_5 \in V'(G)$and $v_5 \in v'_3R$, but $v_2 \notin v'_3R$. $v_3, v_4 \in V'(G)$ and $v_3 \in v'_2R$, but $v_4 \notin v'_2R$.$v_3, v_5 \in V'(G)$ and $v_5 \in v'_3R$, but $v_3 \notin v'_3R$. $v_4, v_5 \in V'(G)$ and $v_4 \in v'_5R$, but $v_5 \notin v'_5R$. then G is supra gT_0. Now $v_1, v_2 \in V'(G)$,$v_1 \in v'_4R$and$v_2 \in v'_1R$.
$v_1, v_3 \in V'(G)$,$v_1 \in v'_4R$ and $v_3 \in v'_2R$.$v_1, v_4 \in V'(G)$,$v_4 \in v'_4R$ and $v_4 \in v'_5R$.$v_1, v_5 \in V'(G)$,$v_1 \in v'_4R$ and $v_5 \in v'_3R$. $v_2, v_3 \in V'(G)$,$v_2 \in v'_1R$and$v_3 \in v'_2R$.$v_2, v_4 \in V'(G)$, $v_2 \in v'_1R$ and $v_5 \in v'_5R$. $v_2, v_5 \in V'(G)$, $v_2 \in v'_1R$ and $v_5 \in v'_3R$.$v_3, v_4 \in V'(G)$,$v_3 \in v'_2R$ and $v_4 \in v'_5R$. $v_3, v_5 \in V'(G)$, $v_3 \in v'_2R$ and $v_5 \in v'_3R$. $v_4, v_5 \in V'(G)$,$v_4 \in v'_5R$ and $v_5 \in v'_3R$. Then G is supra gT_1. N0w $v_1, v_2 \in V'(G)$, $v_1 \in v'_4R$ and $v_2 \in v'_1R$, $v'_2R \cap v'_1R = \emptyset$, $v_1, v_3 \in V'(G)$, $v_1 \in v'_4R$ and$v_3 \in v'_2R$,v'_4R $\cap v'_4R$ = $\emptyset$,$v_1, v_4 \in V'(G)$, $v_1 \in v'_4R$ and $v_4 \in v'_5R$, v'_4R $\cap v'_5R$ = $\emptyset$, $v_1, v_5 \in V'(G)$, $v_1 \in v'_4R$ and $v_5 \in v'_3R$, v'_4R $\cap v'_3R = \emptyset$, $v_2, v_3 \in V'(G)$, $v_2 \in v'_1R$ and $v_3 \in v'_2R$, $v'_1R \cap v'_2R=\emptyset$, $v_2, v_4 \in V'(G)$, $v_2 \in v'_1R$ and $v_4 \in v'_5R$, $v'_1R \cap v'_3R = \emptyset$, $v_2, v_5 \in V'(G)$, $v_2 \in v'_1R$ and $v_5 \in v'_3R$, $v'_1R \cap v'_5R = \emptyset$, $v_3, v_4 \in V'(G)$, $v_3 \in v'_2R$ and $v_4 \in v'_5R$, v'_2R $\cap v'_5R$ = $\emptyset$, $v_3, v_5 \in V'(G)$, $v_3 \in v'_2R$ and $v_5 \in v'_3R$, v'_2R $\cap v'_3R = \emptyset$, $v_4, v_5 \in V'(G)$, $v_4 \in v'_5R$ and $v_5 \in v'_3R$, $v'_5R \cap v'_3R = \emptyset$. Then G is supra gT_2. $v'_2 \in v'_1R$, F =

$\{v'_3\}$. Let W = v'_2R, R= v'_1R, then R ∩ W = ∅, then G is supra gT_3

Example: The Hamiltonian path is $\{v'_5, v'_1, v'_2, v'_3, v'_4\}$,

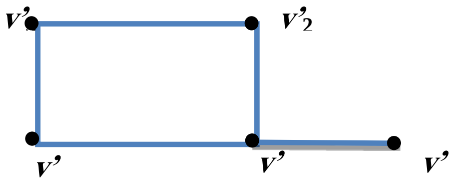

Figure 9

the post classes of the vertices of Hamiltonian path is :v'_1R = $\{v'_2\}$,v'_2R=$\{v'_3\}$,v'_3R=$\{v'_4\}$,v'_5R=$\{v'_1\}$.v_1, v_2 ∈ V'(G),v_1 ∈v'_5R, but$v_2 \notin v'_5$R. v_1, v_3 ∈V'(G),v_1 ∈v'_5R and $v_3 \notin v'_5$R .v_1, v_4 ∈V'(G),v_1 ∈v'_5R and $v_4 \notin v'_5$R.v_1, v_5 ∈V'(G),v_1 ∈v'_5R and $v_5 \notin v'_5$R.v_2, v_3 ∈V'(G),v_2 ∈v'_1R and $v_3 \notin v'_1$R .v_2, v_4 ∈V'(G), v_2 ∈v'_1R and $v_4 \notin v'_1$R.v_2, v_5 ∈V'(G),v_2 ∈v'_1R and $v_5 \notin v'_1$R .v_3, v_4 ∈V'(G), v_3 ∈v'_2R and $v_4 \notin v'_2$R. v_3, v_5 ∈V'(G),v_3 ∈v'_2R and $v_5 \notin v'_2$R .v_4, v_5 ∈V'(G), v_4 ∈v'_3R and $v_5 \notin v'_3$R . Then G is supra gT_0.This graph is not supra gT_1 v_2 and v_5 ∈v'_1R.

3. Main Results

Proposition 3.1. Let G be a connected graph contains only one cycle with a supra topological graph (V'(G), μ_G), $|G| = |E|$, thin G is supra gT_0.

Proof : Let G be a graph with a supra topological graph (V'(G), μ_G) and G is connected with one cycle & $|G| = |E|$. Then any vertex in V'(G) has two others vertices in his neighborhoods intersected in one vertex. Therefore, G is supra gT_0.

Remark 3.2. The **Proposition 3.1** is not true, when V'(G)=E(G)= 4.

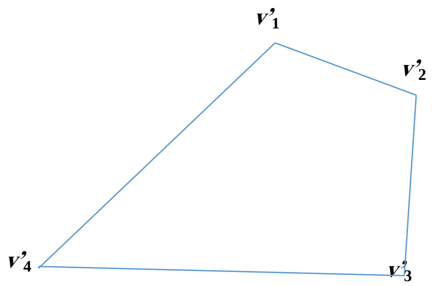

Fig. 10: A graph G which is not supra gT_0

Example 3.3. Let G be a connected graph contains only one cycle with a supra topological space (V'(G), μ_G) as shown in Figure 10.Where,

V'(G)=$\{v_i$:/1 ≤ i≤4$\}$,

E(G)=$\{v_1 v_i$:/i=2,4$\}$∪$\{v_2 v_i$:/i=1,3$\}$∪ $\{v_3 v_i$:/i=2,4$\}$∪$\{v_4 v_i$:/i=1,3$\}$.

The post class of all vertices are: v'_1R = $\{v'_2, v'_4\}$, v'_2R = $\{v'_1, v'_3$,v'_3R = $\{v'_2, v'_4\}$, v'_4R = $\{v'_1, v'_3\}$.Then it's easy to show that a graph G is not supra gT_0

Remark 3.4 In general, supra topology, every subspace of a supraT_0-space is supraT_0- space. This can no terrify in supra. topological graph the following example. We see it is not necessary that every sub graph of a supra gT_0 is also supra gT_0.

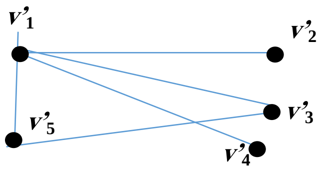

Fig. 11: A sub graph H which is n0t supragT₀.

Example 3.5 Let H be a graph with a supra topological space (V'(H), μ_H) as shown in Figure 11, a sub graph of a graph G on Example 2.3.

The post class for every vertices are: v'_1R = $\{v'_2, v'_3, v'_4, v'_5\}$, v'_2R = $\{v'_1\}$, v'_3R = $\{v'_1, v'_5\}$,v'_4R = $\{v'_1\}$, v'_5R = $\{v'_1, v'_3\}$. Then it's easy to show that a graph H is not supra gT_0.

Theorem 3.6. Let G = (V',E) be a graph with a supra topological graph (V'(G), μ_G). Then the following statements are equivalents:

I. G is supra gT_0.

II. ∀x, y∈ V'(G) and x≠y then supra $Cl_G(x)$ ≠ supra $Cl_G(y)$.

III. ∀x ∈ V'(G), N(x) is subset or equal to a supra closed post class.

Proof : (i) → (ii): If G is supra gT_0, then if x, y∈ V'(G) are not adjacent , x≠ y ∃ a post class U such that x∈ U, y ∈ V'(G) -U. Then x ∉ N (y) and x∉ supra Cl_G (y). Since supra Cl_G (y)={y} ∪ N(y) and supra Cl_G (x)={x} ∪ N(x). Also, y ∉ supra Cl_G(x). Thin supra Cl_G (x) ≠ supra Cl_G (y).

(ii). → (iii): For any two particular not neighboring vertices x, y∈ V'(G) and x≠ y. We have x∉ supra Cl_G (y) and y∉ supra Cl_G (x). F0r all x∈ V'(G), x∉ N(x), then there exists a vertex z not adjacent to x, z≠x and by (ii), Cl_G (z) ≠ supra Cl_G (x) and s0 x∉ supra Cl_G (z), supra Cl_G (z) ∩ N(x) ≠ ∅. Hence N(x)= ∪{ supra Cl_G (z) : supra Cl_G (z) ∩N(x) ≠ ∅}, i.e, N (x) is ⊆ C(x) .

(iii) → (i) For every x, y∈ V'(G) and x≠ y and x≠y. Thin y ∉ N(x) and y∉ supra Cl_G (x).S0 y ∈ V'(G) - supra Cl_G (x) which is a post class. Therefore x∉ V'(G) - supra Cl_G (x). Then G is gT_0.

Theorem 3.7. Let G= (V', E) be a graph with a supra topological graph (V'(G), μ_G). Then the statements are equivalents:

I. G is supra gT_1.

II. ∀x∈ V'(G), then x⊆supraC(∪U_y).

III. ∀x ∈ V'(G), N(x) = ∅.

Proof. :(i) → (ii): For all x∈ V'(G), y∈ supra C (x), this means x∉ supra C (x). Since G is supra gT_1, then ∃ a post class U_y such that x ∉ U_y. Then supraC(x)=∪{U_y:y∈ supra C(x)}. C(x)g. So supra

C(supraC(x)) ⊆ supraC(∪U_y).Therefore x⊆ supra C(∪U_y).

(ii). →(iii) : For all x∈ V'(G), by (ii), x =∩(∪U_y)=$\cap_y$ supra C (U_y),N(x) = ∅. distinct not adjacent vertices x, y∈ V'(G) and x≠ y. We have x∉ supra Cl_G (y) and y∉ supra Cl_G(x). For all x∈ V'(G), x∉ N(x), then there exists *a* vertex z not adjacent to x, z≠x and by (ii), Cl_G(z) ≠ supra Cl_G (x) and s0 x∉ supra Cl_G(z), supra Cl_G(z) ∩ N(x) ≠ ∅. Hence N(x)= ∪ { supra Cl_G (z): supra Cl_G (z) ∩N(x) ≠ ∅}, i.i, N(x) is a subset or equal to a closed post class.

(iii) → (i) For all x, y∈ V'(G) and x≠ y and x≠y, by (z), x= ∩supra C (U_y) and y= ∩supra *C* (U_x). Thin x ∉ (U_y) and y ∉ (U_x). Therefore, G is supra gT_1.

Remark 3.8. In general, supra topology, Theorem 3.7 is necessary to be satisfied. But in supra topological graph the result is verified although N(x) is not empty as in the following example.

Example 3.9 From example 2.17., The latest remark is achieved.

Theorem 3.10. Let G be a graph with a supra topological graph (V'(G), μ_G). Then the following statements are equivalents:

I. G is a supra gT_2.

II. ∀x, y∈ V'(G) and x≠y, ∃ a post class *U* C V'(G), such that x ∈ *U*, y ∈supra C(supra Cl(*U*)).

III. ∀x, y∈ V'(G) and x≠y, ∃ a post class F such that x∈ F, y ∈ supra C (F).

Proof. (i)→ (ii): If G is a supra gT_2, thin by Definition2.7. x, y∈ V'(G), x≠y there exist two post classis U, R such that x ∈*U*, y ∈ R and *U* ∩ R=∅. Thin x∈*U* and y ∈ supraC(supra Cl_G(*U*)) =W which is a post class.

(ii) → (iii) : Since F is closed post class of x, then F is in the nhd of x.

∃ post class U with the end goal that , x∈*U* ⊆ F. Then supra Cl_G (*U*) =F. By (ii), x∈ F and y ∈supra C (F).

(iii) → (i) : Let for all x, y∈ V'(G), x≠y and F is closed post class b of x. Then by (iii), x ∈ F and y ∈ supraC (F). So ∃ a post class *U* such that x∈*U* ⊆ F. So x∈ U and y ∈supra C (supra Cl_G(*U*)). Supra C(supra Cl_G(*U*)) is a post class V', x∈*U*, y ∈ V' and *U*∩V'=∅ Then G is supra gT_2.

Theorem 3.11: Let G be a graph with a supra topological graph (V'(G),μ_G). Then the following are equivalent:

I. G is supra gT_3.

II. For every p ∈ V'(G), F is supra closed post class and F ⊆ supra C(p) there exists a post class *U* such that p ∈*U*⊆ supra Cl_G(*U*) ⊆ supra C(F).

Proof. (i) →(ii) :Lit G supra gT_3,F is supra C(x) and p supra C(F). Then there exists *a* post class *U* such that p∈*U*⊆supra Cl_G(*U*). Put V'=supraC (supra Cl_G(*U*)). Thin F⊆ supraC(supra Cl_G(*U*)) and supra Cl_G(*U*)⊆supraC(F). Therefore p∈*U*⊆ supra Cl_G(*U*)⊆supra C(F).

(ii) → (i) : Let F be supra closed post class, p∈ supra C (F), ∃ a post class *U*, such that p ∈ U ⊆ supra Cl_G(U) ⊆ supra C (F). Then p ∈*U*, supra Cl_G(*U*) ⊆supra C (F).So F ⊆ supra C (supra Cl_G (*U*)) = V' which is a post class *U*∩V'= ∅. Therefore, G is supra gT_3.

Theorem 3.12: Let (. V' (G), μ_G). be a supra topological. Let G is a supra gT_3, x, y∈ V'(G) and x≠ y. Then either supra Cl_G (x) = supra Cl_G (y). or supra Cl_G (x) ∩ supra Cl_G (y) =∅

Proof: Let G be as supra gT_3, and x, y∈ V'(G), x≠ y. Suppose that supra Cl_G (x) ≠supra Cl_G (y). Then (y) ∉ supra Cl_G (x) and (x) ∉ supra Cl_G (y). Since y ∉ supra Cl_G(x), then bydifinition2.8, there exist *a* post class H such that Cl_G (x) ⊆H. S0y∈ supraC(supra Cl_G (x) and y ∈ supraC(H), which is a supra closed post class. So supra Cl_G (y) ⊆supraC (H). Therefore, supra Cl_G (x)∩ supra Cl_G (y) ⊆ (H)∩ supraC(H) =∅and so supra Cl_G (x) ∩ supraCl_G(y) =∅

4. Conclusion

In this paper we were can some results on supra separation axioms via graph thi0ry we studied the supra siparati0n axioms on graph simple and complete graph and Hamilton graph and Euler graph with results. So, this research is c0nsidirid a starting point of many works in the Real-life applications**.**

References

[1] Mahmoud, B. K. (2018). On supra–Separation Axioms for Supra Topological Spaces. *Tikrit journal of pure science*, *22*(2), 117-120.

[2] Chartrand, G., Lesniak, L., & Zhang, P. (2016). Textbooks in mathematics (graphs and digraphs).

[3] AI-Mashhour. H. S. (1983). On supra to topological spaces.

[4] Bondy, J. A. (2008). Murty. USR: Graph Theory.

[5] Thulasiraman, K., & Swamy, M. N. (2011). Graphs: theory and algorithms. John Wiley & Sons.

[6] Rahman, M. S. (2017). Basic graph theory. Springer International Publishing.

[7] Willson R. J. (1996). Introduction to Graph Theory. 14th edition. Addison Wisliy Longman Limitid. Harlow. United Kingdom.

[8] Al-Shami, T. M. (2016). Some results related to supra topological spaces. J. Adv. Stud. Topol, 7(4), 283-294.

Tikrit Journal of Pure Science Vol. 27 (1) 2022

TJPS

بعض النتائج على بديهيات الفصل العلوي عبر نظرية الرسم البياني

فاطمة ابراهيم فتحي[1]، نبيل عزالدين عارف[1]، لمى سعد عبد الباقي[2]

[1] قسم الرياضيات، كلية التربية للعلوم الصرفة ، جامعة تكريت ، تكريت ، العراق

[2]قسم الرياضيات، كلية التربية للبنات ، جامعة تكريت ، تكريت ، العراق

الملخص

تم دراسة مفاهيم الرسم البياني فوق الطوبولوجي وتقديم البديهيات فوق الفصل على الرسم البياني فوق الطوبولوجي(supra gT0 ، supra gT1، supra gT2، supra gT3، supra gT4). وأعطت نتائج لهم.