Tikrit Journal of Pure Science 23 (9) 2018 *ISSN: 1813 – 1662 (Print)*
E-ISSN: 2415 – 1726 (On Line)

TJPS

TIKRIT JOURNAL OF PURE SCIENCE

Journal Homepage: http://main.tu-jo.com/ojs/index.php/TJPS/index

w-extending and pw-extending Modules

Habit K. Mohammad ali , Mohammad E. Dahsh

Depertment of Mathematics, College of Computer Science and Mathematics, Tikrit University, Tikrit, Iraq

ARTICLE INFO.

Article history:

-Received: 10 / 4 / 2018

-Accepted: 20 / 5 / 2018

-Available online: / / 2018

Keywords: Extending, w-extending, pw-extending.

Corresponding Author:

Name: Mohammad E. Dahsh

E-mail:

mohammadali2013@gmail.com

Tel:

Abstract

In this work, we introduce and study two concepts of modules. The first one is a generalization of extending modules which is called a w-extending module. And the second concept is called pw-extending module which is a stronger property than w-extending. We give basic properties, example and characterization of these concepts. Moreover relationship of w-extending module and pw-extending modules with other types of modules are studied.

1.Introduction

Throughout this paper all rings are commutative with identity and all modules are left unital module. A module X is called extending, if every submodule of X is essential in a direct summand of X. Equivalently that every closed submodule of X is a direct summand of X [1], where a non zero submodule K of a module X is called essential if $K \cap L \neq (0)$ for every non zero submodule L of X [2]. And a submodule K of a module X is called closed if K has no proper essential extension in X [2]. A submodule C of a module X is called weak essential if $C \cap S \neq (0)$ for each non zero semi-prime submodule S of X [3]. A submodule K of a module X is called w-closed , if K has no proper weak essential extension in X [4]. A submodule L of a module X is called pw-closed if for each $m \in M, m \notin L$, there exists a w-closed submodule F of X with $L \leq F \text{ and } m \notin F$ [5]. Every w-closed submodule is a pw-closed submodule, while the converse is not true in general [5], an R-module M is called multiplication if every submodule N of M is of the form $N = IM$ for some ideal I of R [6 . An R-module Y is called fully semi-prime if every proper submodule of Y is a semi-prime [7]. An R-module M is called a completely essential if every non zero weak essential submodule of is essential [7] . Let Y be an R-module $ann(y) = \{r \in R : ry = 0\}$[2],

$Z(y) = \{y \in Y : ann(y) \text{ essential in } R\}$ is called singular submodule of Y if $Z(y) = Y$, then Y is called singular module, if $Z(y) = (0)$, then Y is called nonsingular module [2] .

2. W- extending Modules

In this section , we introduce a new generalization of extending module called a w-extending module.

Definition (2.1)

An R-module Y is called w- extending , if every submodule of Y is a weakly essential in a direct summand of Y. A ring R is a w-extending, if R is a w-extending R-module.

The following is a characterization of w-extending module.

Proposition (2.2)

A module Y is a w- extending if and only if every w-closed submodule of Y is a direct summand of Y.

Proof

($\Longrightarrow$) Let E be a w-closed submodule of Y, then E is a weak essential in a direct summand of Y say L. But E is a w-closed in Y, then by [4, Prop.2.4] E=L. Hence E is a direct summand of Y .

($\Longleftarrow$) Let E be a submodule of Y If $E = \langle 0 \rangle$, then clearly E is a weak essential in a direct summand of Y. If $E \neq \langle 0 \rangle$, then by [4, Prop.2.6], there exists a w-closed submodule L in Y such that E is a weak essential in L. Thus, by hypothesis L is a direct summand of Y, so E is a weak essential in a direct summand of Y . Hence Y is a w- extending module.

Proposition (2.3)

A module Y is a w-extending if and only if every pw-closed submodule of Y is a weak essential in a direct summand of Y.

Proof

(⟹) Let E be a pw-closed submodule of Y, then E is a weak essential in a direct summand of Y.

(⟸) Direct.

Rmarks and Examples (2.4)

1. Every extending module is a w-extendng, however the converse is not true in general. As the following example shows : let $Y = Z_8 \oplus Z_2$ be a "Z-module", Y "is not extending since the submodule" $K = \langle \bar{2}, \bar{1} \rangle$ is closed but not a direct summand of Y. On the other hand Y is a w-extending since all w-closed submodules of Y are $\langle \bar{0}, \bar{1} \rangle$, $\langle \bar{4}, \bar{1} \rangle$ and Y itself and all of them are direct summand of Y. That is $Y = \langle \bar{1}, \bar{0} \rangle \oplus \langle \bar{4}, \bar{1} \rangle$ and $Y = \langle \bar{1}, \bar{0} \rangle \oplus \langle \bar{0}, \bar{1} \rangle$.
2. Z_n as a Z-module is a w-extending for each $n \geq 2$.

Proposition (2.5)

Let Y be a w-extending module and E, F are submodules of Y with $E \cap F$ is a w- closed submodule of Y. Then $E \cap F$ is a direct summand of E and F.

Proof

It is cays so is omitted. $E(Y)$

Proposition (2.6)

Let Y be an R-module, with $T \cap Y$ is a w-closed submodule of Y for all direct summand T of $E(Y)$. Then Y is a w-extending if and only if $T \cap Y$ is a direct summand of Y.

Proof

(⟹) Is direct.

(⟸) Let K be a submodule of Y, and B a relative complement of K in Y, then by [2, Prop.1.3] $K \oplus B$ is an essential submodule of Y, but Y is an essential in $E(Y)$, it follows that by [2, Prop.1.1] $K \oplus B$ is an essential in $E(Y)$, hence $E(Y) = E(K \oplus B) = E(K) \oplus E(B)$. That is $E(K)$ is a direct summand of $E(Y)$. Thus by assumption we have $E(K) \cap Y$ is a direct summand of Y. Now, we have K is essential in $E(K)$ and Y is essential in $E(Y)$, then by [2, Prop.1.1(2)] $K = K \cap Y$ is essential in $E(K) \cap Y$, thus by [3] K is a weak essential in $E(K) \cap Y$ which is a direct summand of Y.

From Proposition (2.2) and Proposition (2.6) we get the following.

Proposition (2.7)

Let Y be an R-module, with $T \cap Y$ is a w-closed submodule of Y for all direct summand T of $E(Y)$. Then the following statements are equivalent :

1. Y is a w- extending module.
2. Every w- closed submodule of Y is a direct summand of Y.
3. $T \cap Y$ is a direct summand of Y.

Proof

(1) ⟺ (2) Follows by Proposition (2.2).

(2) ⟹ (3) Let T be a direct summand of $E(Y)$, then by hypothesis $T \cap Y$ is a w-closed submodule of Y. Thus by (2) $T \cap Y$ is a direct summand of Y.

(3) ⟺ (1) Follows by Proposition (2.6).

Proposition (2.8)

Let Y be a non zero w-extending module, and E, F be a submodule of M such that $M = E \oplus F$ with $ann\,(E) + ann\,(F) = R$, and every weak essential extension of $K \oplus F\ (or\ E \oplus K)$ are completely essential modules, where K is any w-closed submodule in $E\ (or\ F). Then\ E(or\ F)$ is a w-extending modules.

Proof

To prove first E is a w-extending. Let K be a w-closed submodule in E. Since F is a w-closed submodule in F, then by [4, Prop.2.26] we have $K \oplus F$ is a w-closed submodule in Y. But Y is a w-extending module, then $K \oplus F$ is a direct summand of Y. Thus there exists a submodule L of Y such that $Y = (K \oplus F) \oplus L = K \oplus (F \oplus L)$. That is K is adirect summand of Y. Now, K is a submodule of E, then by [2, Prop.1.10] K is a direct summand of E, hence E is a w-extending.

In similar way we can prove that F is a w-extending.

As a generalization of Proposition (2.8) we get the following.

Corollary (2.9)

Let $Y = \oplus_{i=1}^{n} E_i$ be a non zero w-extending module, where E_i is a submodule of Y for each $i = 1,2,...,n$, and every weak essential extensions of $K \oplus E_1 \oplus E_2 \oplus ... \oplus E_{j-1} \oplus E_{j+1} \oplus ... \oplus E_n$ are completely essential modules, where K is a w-closed submodule of E_j and $ann(E_1) \oplus ann(E_2) \oplus ... \oplus ann(E_n) = R$. Then E_j is a w-extending module.

Proposition (2.10)

Let Y be a finitely generated faithful, and multiplication R-module over a w-extending ring R. Then Y is a w-Extending R- module

Proof

Let F be a w-closed submodule of Y, then since Y is a multiplication, then $F = [F{:}Y]\,Y$. Thus by [4, Prop.3.7] we have [F:Y] is a w-closed ideal in R. But R is a w-extending ring, then [F:Y] is a direct summand of R. Thus $R = [F{:}Y] \oplus L$, where L is an ideal of R, hence $Y = R\,Y = ([F{:}Y] \oplus L)\,Y = [F{:}Y]\,Y + L\,Y$. But Y is faithful and multiplication module so by [6, Theo.(1.6)], we have $[F{:}Y]Y \cap L\,Y = ([F{:}Y] \cap L)Y$ and by definition of the direct sum, we have $[F{:}Y] \cap L = (0)$, thus $[F{:}Y]Y \cap L\,Y = (0)M = (0)$, therefore $Y = [F{:}Y]\,Y \oplus L\,Y$. That is $[F{:}Y]\,Y = F$ is a direct of Y.

The following proposition gives the converse of Proposition (2.10).

Recall that for any R-module M and any ideals I and J of R, if I is a semi-prime ideal of J, then IM is a semi-prime submodule of JM. This is called condition (*) [7].

Proposition (2.11)

Let Y be a finitely generated faithful and multiplication R-module such that Y satisfies the condition (*). Then Y is a w-extending module if and only if R is a w- extending ring.

Proof

(⟹) Let L be a w-closed ideal in R . Since Y is a faithful multiplication and satisfies the condition (*), then by [4, Coro.3.8] LY is a w-closed submodule in Y. But Y is a w-extending module, then LY is a direct summand of Y. Thus $Y = LY \oplus N$ for some submodule N of Y. Since Y is a multiplication, then $N = [N{:}Y]Y$, so $Y = LY \cap [N{:}Y]Y = (L + [N{:}Y])Y$. Then by [6, Theo.1.6] $Y = LY \cap [N{:}Y]Y = (0) = (L \cap [N : Y])Y$, it follows that $L \cap [N : Y] \leq ann\, Y$, Since Y is a faithful and finitely generated multiplication modules, then by [6, Theo.3.1] we have $R = L \oplus [N{:}Y]$. Therefore L is a direct summand of R.

The following propositions show that under certain condition w-extending modules implies extending modules.

Proposition (2.12)

Let Y be a w-extending R-module provided that any non zero weak essential extensions of any submodule of Y is a completely essential. Then Y is an extending module.

Proof

Let E be a non zero submodule of Y, then E is a weak essential in a direct summand of Y, say K. That is K is a non zero weak essential extension of E, then K is a completely essential, then E is essential in K, hence Y is extending.

Proposition (2.13)

Let Y be a w-extending and a fully semi-prime module. Then Y is extending module

Proof

Let E be a non zero submodule of Y. Since Y is a w- extending, then E is a weak essential in a direct summand of Y say H. But Y is a fully semi-prime module then, by [7, Prop.2.4] E is essential submodule of H. That is Y is an extending.

As a direct application of Proposition (2.12) and Proposition (2.13) we get the following corollaries .

Corollary (2.14)

Let Y be a w-extending R-module, with every non zero direct summand of Y is completely essential module. Then Y is extending.

Corollary (2.15)

Let Y be a w- Extending module, with every nonzero direct summand of Y is a fully semi-prime module. Then Y is extending.

Proposition (2.16)

Let Y be a w-extending module such that for every submodule L of Y there exists a w-closed submodule K of Y with L is essential submodule of K . Then Y is extending.

Proof

Let L be a submodule of Y, then by assumption, there exists a w-closed submodule K of Y such that L is essential in K. But Y is a w-extending, then K is a direct summand of Y, hence Y is extending.

Proposition (2.17)

Let Y be a fully semi-prime and w-extending module. Then every direct summand of Y is a w-extending module.

Proof

Let N be a direct summand of Y, and let L be a w-closed submodule of N . Since N is a direct summand of Y, then N is a closed submodule of Y. Thus by [4, Prop.2.4] N is a w-closed submodule of Y. Then by [4, Prop.2.17] we have L is a w-closed of Y, but Y is a w-extending, then L is a direct summand of Y. That is $Y = L \oplus E$ for some submodule E of Y. $Y \cap N = N = (L \oplus E) \cap N = L \oplus (E \cap N)$, it folows that L is a direct summand of N . That is N is a w-extending .

3. pw-extending modules

In this part of the paper we introduced the definition of pw-extending module, which is a stronger from of w-extending module and give some of it's basic properties.

Definition (3.1)

A module1 Y is called pseudo w- extending (for short pw- extending), if every pw-closed submodule of Y is a direct summand of Y. A ring R is called a pw-extending, if R is a pw-extending R-module .

Remarks and Examples (3.2)

1. It is clear that every pw- extending module is a w-extending, but the converse is not true in general . For example consider $Y = Z_{16}$ as a Z-module is a w-extending by Remarks and Examples (2.2)(2), while Z_{16} is not pw-extending since the submodule $\langle \bar{2} \rangle$ is a pw- closed submodule of Z_{16}, but not direct summand .
2. Every semi- simple R-module Y is a pw-Extending but the converse not true.

Proof

Suppose that Y is semi-simple module, and let K be a pw-closed submodule of Y then K is a direct summand of M , hence Y is a pw-extending. For the converse consider the following example : let $Y = Z_6 \oplus Z_2$ as a z-module Y is a pw –extending because the only pw-closed submodule of M are $\langle \bar{0}, \bar{1} \rangle$, $\langle \bar{3}, \bar{1} \rangle$ and Y itself and are direct summand of Y. But Y is not semi-simple because not all submodules of Y is a direct summand of Y.

3. Z_{10} as a Z-module is a pw-extending Z-module .
4. Pw-extending module and extending module are in dependent concepts, as the following examples show: The Z-module Z_{16} is not pw-extending but it is extending since every submodule of Z_{16} is essential in a direct summand of Z_{16} , and Z_{16} is indecomposable, that is the only direct summand of Z_{16} is Z_{16} itself and (0). Also, we notes in (2) the Z-module $Z_6 \oplus Z_2$ is a pw-extending , but not extending module since the submodule $\langle \bar{2}, \bar{1} \rangle$ is a closed submodule of $Z_6 \oplus Z_2$, but not direct summand . Hence $Z_6 \oplus Z_2$ is not extending.

Proposition (3.3)

A direct summand of a pw- extending module Y is a pw- extending , provided that Y is a fully semi-prime.

Proof

Let E be a direct summand of Y, then there exists a submodule L of Y such that $Y = E \oplus L$. Let K be a pw-closed submodule of E. Since E is a direct summand of Y, then E is a closed submodule of Y. Then by [4, Prop.2.14] we get E is a w-closed submodule, thus by [5, Prop.2.7] K is a pw- closed submodule of Y. But Y is a pw-extending, then K is a dirct summand of Y. Thus $Y = K \oplus T$ for some submodule T of Y. Now, $E = Y \cap E = (K \oplus T) \cap E = K \oplus (T \cap E)$. Thus K is a direct summand of E, hence E is a pw-extending.

Proposition (3.4)

Let Y be a module such that every weak essential extensions of any submodule of Y is a completely essential, and Y is a pw- extending . Then Y is extending module.

Proof

Let E be a nonzero closed submodule of Y. so by [5, Prop.2.6] E is a pw- closed submodule of Y, but Y is a pw-extending, then E is a direct summand of Y. Hence Y is extending.

Proposition (3.5)

Let Y be a fully semi-prime module, and pw-extending. Then Y is extending.

Proof

Follows by [5, Prop.2.7].

Proposition (3.6)

Let Y be a module, with every submodule E of Y there exists a pw-closed submodule L of Y with E essential submodule of L. If Y is a pw-extending then Y is extending.

Proof

It is easy, so we omitted.

4.Relationships of w-extending modules and pw-extending modules with some types of modules.

The main goal of this section is study the relation between w-extending and pw-extending modules with some types of modules.

“Recall that a module M is called CLS-module, if every y-closed submodule of M is a direct summand [8]”. Where a submodule A of an R-modile M is called y-closed submodule of M, if $\frac{M}{A}$ is non-singular module [2].

Proposition (4.1)

Let Y be a fully semi-prime module . If Y is a w-extending module, then Y is a CLS-module.

Proof

Assume that Y is a w-extending, and E be a non zero y-closed submodule of Y, then by [4, Prop.2.33] E is a w-closed submodule of Y. But Y is a w-extending, then E is a direct summand of Y, hence Y is a CLS-module.

Proposition (4.2)

Let Y be a non-singular R-module. If Y is a CLS-module, then Y is a w-extending module .

Proof

Let E be a w- closed submodule of Y. Since Y is non-singular, then by [4, Prop.2.34] E is a y- closed submodule of Y . But Y is CLS-module, then E is a direct summand of Y, hence Y is a w- extending module .

Proposition (4.3)

Let Y be a fully semi-prime and non-singular R-module. Then the following statements are equivalent:

1. Y is CLS-module.
2. Y is Extending module.
3. Y is a w-Extending module.

Proof

(1)⟺(2)

Let K be a closed submodule of Y, then by [9, Prop.2.1.2] K is a y-closed , hence K is a direct summand of Y. Thus Y is extending module .

(2)⟹(3)

Let E be a w-closed submodule of Y, then by [4, Prop.2.13] E is a closed submodule of Y. Hence E is a direct summand of Y. Thus Y is a w-extending.

(3)⟹(1)

Let L be a y-closed submodule of Y. Then by [4, Prop.2.33] L is w-closed. Hence L is a direct summand of Y . Thus Y is a CLS-module .

“Recall that a module M is called duo if every submodule K of M is a fully invariant. That is $f(N) \leq N$ for each $f \in End(M)$ [8]”.

“Recall that an R-module M is called FI-extending if every fully invariant submodule of M is essential in a direct summand of M [10]”.

Proposition (4.4)

Let Y be a duo R-module. If Y is FI-extending module, then Y is a w-Extending module.

Proof

Let E be a submodule of Y. Since Y is a duo, then E is fully invariant in Y. But Y is FI-extending, then E is an essential in a direct summand of Y. Hence E is a weak essential in a direct summand of Y. Thus Y is a w-extending module.

Proposition (4.5)

Let Y be a duo R-module, with every nonzero direct summand of Y is a completely essential. Then Y is a w-extending if and only if Y is FI-extending module.

Proof

(⟹) Since Y is a w-extending, then by corollary (2.14) Y is extending, hence by [10] Y is FI-extending.

(⟸) Follows by Proposition (4.4).

Theorem (4.6)

Let Y be a duo R-module, with every nonzero direct summand of Y is a completely essential. Then the following statements are equivalent :

1. Y is extending.
2. Y is FI-extending.
3. Y is a w-extending.

Proof

(1) ⟺ (2) direct.

(2) ⟺ (3) Follows by Proposition (4.4).

(3) ⟺ (1) Follows Corollary (2.14).

"Recall that an R-module M is prime-extending if every non zero submodule of M is essential in a prim direct summand [11]".

Proposition (4.7)

Every prime –extending module is a w-extending.

proof

Follows by [11, Remark 1.11]

Theorem (4.8)

Let Y be a fully semi-prime R-module, with every direct summand of Y is a prime. Then the following statements are equivalent :

1. Y is a prime extending.
2. Y is a extending.
3. Y is a w-extending.

Proof

(1) ⟺ (2) by [11, Remark.1.11].

(2) ⟺ (3) direct.

(3) ⟺ (1) :Let K be a submodule of Y. Since Y is a w-extending, then there exists a direct summand say H of Y such that K is a weak essential in H. But M is a fully semi-prime, then by [7, Lemma 2.3] K is an essential in H. By hypothesis H is a prime, thus Y is a prime-extending.

"Recall that a module M is called strongly extending if every submodule N of M is essential stable direct summand of M". "[12]. Where a submodule N is stable if $f(N) \leq N$ For each R-homomorphism $f: N \rightarrow M$".

Proposition (4.9)

Every strongly extending module is a w-extending.

Proof

Let Y be strongly-extending module then by [12, Rem 1.3]. Y is extending hence Y is a w-extending.

Theorem (4.10)

Let Y be a fully semi-prime module, with every direct summand of Y is stable. Then the following statements are equivalent:-

1. Y is strongly extending.
2. Y is extending.
3. Y is w-extending.

Proof

(1) ⟺ (2) by Proposition (4.9).

(2) ⟺ (3) direct.

(3) ⟺ (1) suppose that Y is a w-extending and let K be a submodule of Y, then K is a weak essential in a direct summand of Y say H. But Y is a fully semi-prime, then K is essential in H. Thus by hypothesis H is stable, Hence Y is a strongly extending.

It is well-known sully invariant direct summand is stable [12, Lemma 2.1.6], we get the following result.

Corollary (4.11)

Let Y be a fully semi-prime R-module, with every direct summand of Y is a fully invariant. Then the following statements are equivalents:

1. Y is a strongly extending.
2. Y is an extending.
3. Y is w-extending.

Recall that an R-module M is S-extending if every stable submodule of M is essential in a direct summand of M [2].

Proposition (4.12)

Let Y be a module, with every weak essential extending of any submodule of Y is a completely essential. Then the following statements hold:-

1. If Y is a pw-extending, then Y is FI-extending.
2. If Y is a pw-extending, then M is S-extending.

Proof

(1) By Proposition (3.4) Y is extending, hence Y is FI-extending.

(2) Again by Proposition (3.4) Y is extending hence by [12] Y is S-extending.

Recall that an R-module M is a purely-extending if every submodule of M is essential in a pure submodule [13].

Theorem (4.13)

Let X be a fully semi-prime R-module. Then the following statements hold:

1. If X is a pw-extending, X is FI-extending.
2. If X is a pw-extending, then X is S-extending.
3. If X is a pw-extending, X is CLS-extending.
4. If X is a pw-extending, then X is purely-extending.

Proof

(1) By Proposition (3.5) X is extending, hence by [10] X is IF-extending.

(2) Again by Proposition (3.5) X is extending, then by [12] X is S-extending.

(3) Let E be a non zero y-closed submodule of X. Since X is a fully semi-prime submodule, then by [5, Prop.2.14] E is a pw-closed submodule of X. But X is a pw-extending, then E is a direct summand of X. Hence X is CLS-module.

(4) By Proposition (3.5) X is extending, hence by [13] X is a pure extending.

References

[1] Dangh, N.; Huynh, D.; Smith, p. And Wisbawer R. (1994). Extending Module. Pitman Research Note In Mathematics Series.

[2] Goodear, L, K.R. (1976). Rings Theory, Nonsingular Rings And Modules. Marcel Dekker, New York.

[3] Muna A. A. (2009). Weak Essential submodules. *Um-Salama Science Journal*, **6(1)**: 214-221.

[4] Haibat, K. and Mohammad E. (2018). W-closed submodules and Related concepts. *Ibn Al-Hathem Journal For pure and Applied Science*, 31, 2018.

[5] Haibat, K. and Mohammad E. (2018). Pw-closed submodules and Related concepts. *Journal of AL-Anbar University For pure Science*, to appear.

[6] El-Bast, Z. A. and Smith P.F., (1981). Multiblication Modules. Comm. In Algebra, **70** : 545-560.

[7] Muna, A. A. (2016). Some Results On Weak Essential submodules. *Baghdad science Journal*., **13(3)**:231-242.

[8] Tercan, A. (1995). On Cls-Modules. *Rocky Mountain Journal of Mathematics*, **25(4)**: 1557-1564.

[9] Lmyaa, H. and Bahar, H. (2013). Injectivity And Chaim Conditions on Y-closed submodules. *Iraqi Journal of Science*, **54(3)**: 642-650.

[10] Birkenmeier, G.; Muller, B. and Rizvis, B. (2002). Modules In Which Every Fully Invariant

ISSN: 1813 – 1662 (Print)
E-ISSN: 2415 – 1726 (On Line)

Submodules Is Essential In a Direct summand. *Journal Communications in Algebra*, **30(3):** 1395-1415.

[11] Tamadher, A. (2011). Prime-Extending Modules and S-prime Modules, *Journal of Al-Nahrain University,* **14(4)**: 166-170.

[12] Abdul-kadim, S.S. (2007). S-Extending Module And Related Concepts. Ph.D. Thesis, Al mustansirya University.

[13] Clark, J. (1999). On purely extending modules. In Abelian Groups and Modules .Birkhäuser, Basel. 353-358.

المقاسات الموسعة من النمط – w والمقاسات الموسعة من النمط – pw

هيبة كريم محمد علي ، محمد عيسى دهش

قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

الملخص

في هذا العمل قدمنا ودرسنا مفهومين من المقاسات. الاول هو إعمام المقاسات الموسعة و الذي يدعى المقاس الموسع من النمط –w والثاني والذي يدعى المقاس الموسع من النمط –pw والذي هو أقوى حالة من المقاس الموسع من النمط – w.

أعطينا الصفات الاساسية، الامثلة و المكافئات لهذين المفهومين. بالإضافة لذلك درسنا علاقة المقاسات الموسعة من النمط –w والمقاسات الموسعة من النمط –pw مع بعض انواع المقاسات الاخرى.