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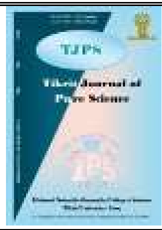

SOME NEW SEPARATION AXIOMS VIA gr-b-I-OPEN SETS

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Abstract

The purpose this paper is to introduce gr-b-I- open sets, via this concept we study some weak separation axioms in ideal topological spaces. The implications of these axioms among themselves and with the known axioms are investigated.

1. Introduction

The notion of regular b-closed sets is introduced and studied recently by Nagaveni and Narmadha [1,2], introduce b-open sets and Andrijevi'c [3], respectively. Levine [4] (resp. Bhattacharya, Palaniappan and Rao [5], introduced and investigated generalized closed sets (resp. regular generalized closed sets ,The subject of ideals in topological spaces has been introduced and studied by Kuratowski [6] and Vaidyanathasamy [7], where an ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) A ∈ I and B ⊂ A implies B ∈ I and (ii) A ∈ I and B ∈ I implies A ∪ B ∈ I.

Given a topological space (X, τ) with an ideal I on X and the P(X) is the set of all subsets of X. The set operator (.)*: P(X)→P(X), called the local function[7] of A with respect to τ and I, is defined as follows: For each A ⊂ X, A*(τ, I) = {x ∈ X| U ∩ A ∉ I for every open neighborhood U of x}. A Kuratowski closure operator Cl*(.) for a topology τ*(τ, I) is called the ⋆-topology, finer than τ is defined by Cl*(A) = A ∪ A*(τ, I) where there is no chance of confusion, A*(I) is denoted by A*. If I is an ideal on X, then (X, τ, I) is called an ideal topological space.

In this paper via gr-b-I- open sets we define some weak separation axioms and study some of their basic properties. The implications of these axioms among themselves and with the known axioms are investigated.

2. Preliminares

Let A be a subset of a topological space (X, τ). A regular open sets if A= int cl(A) (for short, r-open) [8] where cl(A) and int(A) refer to the closure operator and the interior operator of the set A, respectively. The δ-interior [9] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by δ-int (A) and the subset A is called δ-open [9] if A=δ-int(A). i.e., a set is δ-open if it is the union of regular open sets. The complement of δ-open set is δ-closed [10]. Alternatively, a set A ⊂ (X, τ) is called δ-closed [10] if A = δ-cl(A), where δ-cl(A) = {x ∈ X : int (cl(U)) ∩ A ≠ φ, U ∈ τ and x ∈ U}. The family of δ-open sets forms a topology on X and is denoted by τ_δ . It is well known that $\tau_\delta = \tau_s$. A subset A of X is called b-open [3] if A ⊂ int(cl(A)) ∪ cl(int(A)). This notion has been studied extensively in recent years by many topologists (see [11,12,13]) because b-open sets are only natural generalization of open sets. A subset S of an ideal topological space (X, τ, I) is called b-I-open [14] iff S ⊂ int(cl*(S)) ∪ cl*(int(S)). The complement of a b-I-open set is called a b-I-closed set [14]. The intersection of all b-I-closed sets containing S is called the b-I-closure of S and denoted by b-I cl(S). The b-I-interior of S is defined by the union of all b-I-open sets contained in S and denoted by b-I int(S). The set of all b-I-open sets of (X, τ, I) is denoted by BIO(X). Also, the set of all b-I-open sets of (X, τ, I) containing a point x ∈ X is denoted by BIO(X, x).

Definition 2.1: [15] For any subset A of (X,τ),the

1- rcl(A) =∩{B:B⊇A, B is a regular closed subset of X}.

2- r int(A) = ∪{O:O⊆A, A is a regular open subset of X}.

Definition 2.2: A subset A of a space X is said to be a.

1- regular b-closed (briefly r-b-closed) [9] if rcl (A) ⊂ U whenever A ⊂ U and U is b-open in X.

2- generalized closed set (briefly g-closed) [4] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X ,τ).

3- regular generalized closed set (briefly rg-closed) [5] if cl(A) ⊆ U whenever A ⊆ U and U is regular open in (X ,τ).

4- generalized regular closed set (briefly gr-closed) [8] if rcl(A) ⊆ U whenever A ⊆ U and U is open in (X ,τ).The complements of the mentioned closed sets are open sets.

Remark 2.3: [8] rcl(A) is a generalized regular closed set since rcl(A) is a regular closed set and from **Theorem 3.6** in [8] it is a generalized regular closed subset of X for any subset A of X. Therefore we a have a relational structure between the sets as

r-closed set ==⇒δ-closed set ==⇒ δg-closed set ==⇒ gr-closed set ==⇒ g-closed set ==⇒rg- closed set.

Proposition 2.4: let (X, τ) be a topological space, if A is a gr-closed set ,then A is rg-closed set

Proof: Let A be any gr-closed set in X ,such that A ⊆ U where U is open set ,so

rcl(A)=∩{B:B⊇A, B is a regular closed subset of X}-----(1)

But, every regular closed set is closed ,where cl(A)=∩{F:F⊇A, F is closed subset of X}-----(2)

Then, from(1) and(2) we get rcl(A) ⊆ cl(A),so A is a rg-closed set.

Remark 2.5: the convers of proposition **2.4** is not true in general .

Example 2.6: Let X={α, β, μ }and corresponding topological space be τ ={∅,X, { α, β }}. Let A ={α, β}. A is a generalized regular closed set but not a regular generalized closed set.

Definition 2.7 : A subset A of an ideal topological space (X, τ, I) is said to be

1- regular I-open (for short, r-I-open) if A = int (cl*(A)) [16].

2- regular generalized I-closed set (briefly rg-I-closed) [17] if cl*(A) ⊆ U whenever A ⊆ U and U is r-I-open in X. The complement of the mentioned closed set is open set.

Definition 2.8:[9] A function *f*: (X, τ, I) → (Y, σ, J) is said to be b-I-continuous (resp. b-I-irresolute) if the inverse image of every open (resp. b-I-open) set in Y is b-I-open in X.

Definition 2.9:[9] An ideal topological space (X, τ, I) is said to be b-I-regular if for each closed set F of X and each point x ∈ X\F, there exist disjoint b-I-open sets U and V such that F ⊂ U and x ∈ V .

Definition 2.10: [19] Let A and X_0 be subsets of an ideal topological space (X, τ, *I*) such that A ⊂ X_0 ⊂ X. Then (X_0, $\tau_{|X0}$, $I_{|X0}$) is an ideal topological space with an ideal $I_{|X0}$ = {I ∈ *I*|I ⊂ X_0} = {I ∩ X_0|I ∈ I}.

Definition 2.11: [11] A topological space (X, τ) is said to be:

1. b-T_0 if to each pair of distinct points x, y of X there exists a b-open
set A containing x but not y or a b-open set B containing y but not x.

2. b-T_1 if to each pair of distinct points x, y of X, there exists a pair of
b-open sets, one containing x but not y and the other containing y but not x.

3. b-T_2 if to each pair of distinct points x, y of X, there exists a pair of disjoint b-open sets, one containing x and the other containing y.

Definition 2.12: [20] A subset A of an ideal topological space (X, τ, I) is said to be:

1- b-I-T_0 if for any pair of points in X, there is a b-I-open set A containing one points but not the other.

2- b-I-T_1 if for each pair of distinct points x, y of X, there exists a pair of b-I-open sets, one containing x but not y and the other containing y but not x.

3- b-I-T_2 if for each pair of distinct points x, y of X, there exists a pair of disjoint b-I-open sets, one containing x and the other containing y.

3. gr-b-I-T_0 Spaces

Now, in this section we define some new set as following

Definition 3.1: A subset A of a topological space (X, τ) is said to be

1- regular generalized b-closed set (briefly rg-b-closed) if cl(A) ⊆ U whenever A ⊆ U and U is r-b-open in X.

2- generalized regular b- closed set (briefly gr-b-closed) if rcl(A) ⊆ U whenever A ⊆ U and U is b-open in X. The complements of the mentioned closed sets are open sets.

The set of all rg-b-open sets of (X, τ) is denoted by RGBO(X). respectively the set of all gr-b-open sets of (X, τ) is denoted by GRBO(X).

Definition 3.2: A subset A of an ideal topological space (X, τ, I) is said to be:

1- Regular b-I-closed set (briefly, r-b-I-closed) if rcl* (A) ⊆ U, whenever A ⊆ U and U is b-I-open in X. The complements of the mentioned closed set is open sets.

2- Generalized regular b-I- closed set (briefly, gr-b-I-closed) if rcl*(A) ⊆ U, whenever A ⊆ U and U is b-I-open in X. The complements of the mentioned closed set is open sets.

3- Regular generalized b-I-closed set (briefly, rg-b-I-closed) if cl*(A) ⊆ U, whenever A ⊆ U and U is r-b-I-open in X.

The set of all gr-b-I-open sets of (X, τ, *I*) is denoted by GRBIO(X).

From definitions 3.1 and 3.2 Therefore we a have a relational structure between the sets as

gr-b-I set ⇒ rg-b-I set ⇒ gr-b- set

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Definition 3.3: An ideal topological space (X, τ, I) is gr-b-I-T_0 if for any distinct pair of points in X, there is a gr-b-I-open set containing one of the points but not the other.

Theorem 3.4: An ideal topological space (X, τ, I) is gr-b-I-T_0 if and only if for each pair of distinct points x, y of X, gr-b-I cl({x}) ≠ gr-b-I cl({y}).

Proof: Let (X, τ, I) be a gr-b-I-T_0 space and x, y be any two distinct points of X. There exists a gr-b-I-open set G containing x or y, say, x but not y. Then X\G is a gr-b-I-closed set which does not contain x but contains y. Since y ∈ gr-b-I cl({y}) is the smallest gr-b-I-closed set containing y, gr-b-I cl({y}) ⊂ X\G, and so x ∉ gr-b-I cl({y}). Consequently, gr-b-I cl({x}) ≠ gr-b-I cl({y}).

Conversely, let x, y ∈ X, x ≠ y and gr-b-I cl({x}) ≠ gr-b-I cl({y}). Then there exists a point z ∈ X such that z belongs to one of the two sets, say, gr-b-I cl({x}) but not to gr-b-I cl({y}). If we suppose that x ∈ gr-b-I cl({y}), then z ∈ gr-b-I cl({x}) ⊂ gr-b-I cl({y}), which is a contradiction. So x ∈ X\ gr-b-I cl({y}), where X\ gr-b-I cl({y}) is a gr-b-I-open set and does not contain y. This shows that (X, τ, I) is gr-b-I-T_0 .

Lemma 3.5: Let A and X_0 be subsets of an ideal topological space (X, τ, I). If A ∈ GRBIO(X) and X_0 is open in (X, τ, I), then A ∩ X_0 ∈ GRBIO(X_0).

Theorem 3.6: Every open subspace of a gr-b-I-T_0 space is gr-b-I-T_0 .

Proof: Let Y be an open subspace of a gr-b-I-T_0 space (X, τ, I) and x, y be two distinct points of Y . Then there exists a gr-b-I-open set A in X containing x or y, say, x but not y. Now by(**Lemma 3.5**), A ∩ Y is a gr-b-I-open set in Y containing x but not y. Hence (Y, $\tau_{|Y}$, $I_{|Y}$) is gr-b-$I_{|Y}$-T_0.

Definition 3.7: A function *f*: (X, τ, I) → (Y, σ) is said to be point gr-b-I-closure one-to-one if and only if x, y ∈ X such that gr-b-I cl({x}) ≠ gr-b-I cl({y}), then gr-b-I cl({*f*(x)}) ≠ gr-b-I cl({*f*(y)}).

Theorem 3.8: If *f*: (X, τ, I) → (Y, σ) is point gr-b-I-closure one-to-one and (X, τ, I) is gr-b-I-T_0, then *f* is one-to-one.

Proof: Let x and y be any two distinct points of X. Since (X, τ, I) is gr-b-I-T_0,
gr-b-I cl({x}) ≠ gr-b-I cl({y}) by **Theorem 3.4.** But *f* is point gr-b-I-closure one-to-one
implies that gr-b-I cl({f(x)}) ≠ gr-b-I cl({*f*(y)}). Hence *f*(x) ≠ *f*(y). Thus, *f* is one-to-one.

Theorem 3.9: Let f: (X, τ, I) → (Y, σ) be a function from gr-b-I-T_0 space (X, τ, I) into a topological space (Y, σ). Then *f* is point gr-b-I-closure one-to-one if and only if *f* is one-to-one.

Proof: The proof follows from **Theorem 3.8**.

Theorem 3.10: Let *f*: (X, τ, I) → (Y, σ, I) be an injective gr-b-I-irresolute function. If Y is gr-b-I-T_0, then (X, τ, I) is gr-b-I-T_0.

Proof: Let x, y ∈ X with x ≠ y. Since f is injective and Y is gr-b-I-T_0, there exists a gr-b-I-open set Vx in Y such that *f*(x) ∈ Vx and *f*(y) ∉ Vx or there exists a gr-b-I-open set Vy in Y such that *f*(y) ∈ Vy and *f*(x) ∉Vy with *f*(x) ≠ *f*(y). By gr-b-I-irresoluteness of *f* , f^{-1}(Vx) is gr-b-I-open set in (X, τ, I) such that x ∈ f^{-1}(Vx) and y ∉ f^{-1} (Vx) or f^{-1} (Vy) is gr-b-I-open set in (X, τ, I) such that y ∈ f^{-1}(Vy) and x ∉ f^{-1}(Vy). This shows that (X, τ, I) is gr-b-I-T_0.

Definition 3.11: A function *f* : (X, τ, I) → (Y, σ) is called gr-b-I-continuous if the inverse image of each open set in Y is gr-b-I-open set in X

4. gr-b-I-T_1 Spaces

Definition 4.1: An ideal topological space (X, τ, I) is gr-b-I-T_1 if to each pair of distinct points x, y of X, there exists a pair of gr-b-I-open sets, one containing x but not y and the other containing y but not x.

Theorem 4.2: For an ideal topological space (X, τ, I), each of the following statements are equivalent:

1. (X, τ, I) is gr-b-I-T_1 ;
2. Each one point set is gr-b-I-closed in X ;
3. Each subset of X is the intersection of all gr-b-I-open sets containing it ;
4. The intersection of all gr-b-I-open sets containing the point x ∈ X is the set{x}.

Proof: (1)⇒(2): Let x ∈ X. Then by (1), for any y ∈ X, y ≠ x, there exists a gr-b-I-open set Vy containing y but not x. Hence y ∈ Vy ⊂ X\{x}. Now varying y over X\{x} we get X\{x} = ∪ {Vy: y ∈ X\{x}}. So X\{x} being a union of gr-b-I-open set. Accordingly {x} is gr-b-I-closed. (2)⇒(1): Let x, y ∈ X and x ≠ y. Then by (2), {x} and {y} are gr-b-I-closed sets. Hence X\{x} is a gr-b-I-open set containing y but not x and X\{y} is a gr-b-I-open set containing x but not y. Therefore, (X, τ, I) is gr-b-I-T_1.

(2)⇒(3): If A ⊂ X, then for each point y∉ A, there exists a set X\{y} such that A ⊂ X\{y} and each of these sets X\{y} is gr-b-I-open. Hence A = ∩ { X\{y}: y ∈ X\A} so that the intersection of all gr-b-I-open sets containing A is the set A itself.

(3)⇒(4): Obvious.

(4)⇒(1): Let x, y ∈ X and x ≠ y. Hence there exists a gr-b-I-open set Ux such that x ∈ Ux and y∉ Ux. Similarly, there exists a gr-b-I-open set Uy such that y ∈ Uy and x ∉ Uy. Hence (X, τ, I) is gr-b-I-T_1.

Theorem 4.3: Every open subspace of a gr-b-I-T_1 space is gr-b-I-T_1.

Proof: Let A be an open subspace of a gr-b-I-T_1 space (X, τ, I). Let x ∈ A.
Since (X, τ, I) is gr-b-I-T_1, X\{x} is gr-b-I-open in (X, τ, I). Now, A being open, A ∩ (X\{x}) = A\{x} is gr-b-I-open in A by **Lemma 3.5.** Consequently, {x} is gr-b-I-closed in A. Hence by **Theorem 4.2**, A is r-b-I-T_1.

Theorem 4.4: Let X be a T_1 space and *f*: (X, τ) → (Y, σ, I) a gr-b-I-closed surjective function. Then (Y, σ, I) is gr-b-I-T_1.

Proof: Suppose y ∈ Y . Since *f* is surjective , there exists a point x ∈ X such that y = *f*(x). Since X is T_1, {x} is closed in X. Again by hypothesis, *f*({x}) = {y} is gr-b-I-closed in Y . Hence by **Theorem 4.2**, Y is gr-b-I-T_1.

Definition 4.5: A point x ∈ X is said to be a gr-b-I-limit point of A if and only if for each V ∈

GRBIO(X), $U \cap (A \setminus \{x\}) \neq \emptyset$ and the set of all gr-b-I-limit points of A is called the gr-b-I-derived set of A and is denoted by gr-b-Id(A).

Theorem 4.6: If (X, τ, I) is gr-b-I-T_1 and $x \in$ gr-b-Id(A) for some $A \subset X$, then every gr-b-I-neighbourhood of x contains infinitely many points of A.

Proof: Suppose U is a gr-b-I-neighbourhood of x such that $U \cap A$ is finite. Let $U \cap A = \{x_1, x_2, ... x_n\} = B$. Clearly B is a gr-b-I-closed set. Hence $V = (U \cap A) \setminus (B \setminus \{x\})$ is a gr-b- I neighbourhood of point x and $V \cap (A \setminus \{x\}) = \emptyset$, which implies that $x \in$ gr-b-Id(A), which contradicts our assumption. Therefore, the given statement in the theorem is true.

Theorem 4.7: In a gr-b-I-T_1 space (X, τ, I), gr-b-Id(A) is gr-b-I-closed for any subset A of X.

Proof: As the proof of the theorem is easy, it is omitted.

Theorem 4.8: Let *f*: (X, τ, I) → (Y, σ, I) be an injective and gr-b-I-irresolute function. If (Y, σ, I) is gr-b-I-T_1, then (X, τ, I) is gr-b-I-T_1.

Proof: Proof is similar to **Theorem 3.10** .

Definition 4.9: An ideal topological space (X, τ, I) is said to be gr-b-I-R_0 if and only if for every gr-b-I-open sets contains the gr-b-I-closure of each of its singletons.

Theorem 4.10: An ideal topological space (X, τ, I) is gr-b-I-T_1 if and only if it is gr-b-I-T_0 and gr-b-I-R_0 .

Proof: Let (X, τ, I) be a gr-b-I-T_1 space. Then by definition and as every gr-b-I-T_1 space is gr-b-I-R_0, it is clear that (X, τ, I) is gr-b-I-T_0 and gr-b-I-R_0 space. Conversely, suppose that (X, τ, I) is both gr-b-I-T_0 and gr-b-I-R_0. Now, we show that (X, τ, I) is gr-b-I-T_1 space. Let $x, y \in X$ be any pair of distinct points. Since (X, τ, I) is gr-b-I-T_0, there exists a gr-b-I-open set G such that $x \in G$ and $y \notin G$ or there exists a gr-b-I-open set H such that $y \in H$ and $x \notin H$. Suppose $x \in G$ and $y \notin G$. As $x \in G$ implies the gr-b-I cl({x}) $\subset$ G. As $y \notin G$, $y \notin$ gr-b-I cl({x}). Hence $y \in H = X \setminus$ gr-b-I cl({x}) and it is clear that $x \notin H$. Hence, it follows that there exist gr-b-I-open sets G and H containing x and y respectively such that $y \notin G$ and $x \notin H$. This implies that (X, τ, I) is gr-b-I-T_1 .

5. gr-b-I-T_2 Spaces

Definition 5.1: An ideal topological space (X, τ, I) is said to gr-be b-I-T_2 space if for each pair of distinct points x, y of X, there exists a pair of disjoint gr-b-I-open sets, one containing x and the other containing y.

Theorem 5.2 For an ideal topological space (X, τ, I), the following statements are equivalent:

1. (X, τ, I) is gr-b-I-T_2;

2. Let $x \in X$. For each $y \neq x$, there exists $U \in$ GRBIO(X, x) and $y \in$ gr-b-I cl(U).

3. For each $x \in X$, $\cap${gr-b-I cl(U_x) : U_x is a gr-b-I-neighbourhood of x} = {x}.

4. The diagonal $\Delta = \{(x, x) : x \in X\}$ is gr-b-I-closed in $X \times X$.

Proof: **(1)⇒(2):** Let $x \in X$ and $y \neq x$. Then there exist disjoint gr-b-I-open sets U and V such that $x \in U$ and $y \in V$. Clearly, $X \setminus V$ is gr-b-I-closed, gr-b-I cl(U) $\subset X \setminus V$ and therefore $y \notin$ gr-b-I cl(U).

(2)⇒(3): If $y \neq x$, then there exists $U \in$ GRBIO(X, x) and $y \notin$ gr-b-I cl(U). So $y \notin \cap$\{gr-b-I cl(U) : $U \in$ GRBIO(X, x)\}.

(3)⇒(4): We prove that $X \setminus \Delta$ is gr-b-I-open. Let $(x, y) \notin \Delta$. Then $y \neq x$ and since $\cap$\{gr-b-I cl(U) : $U \in$ GRBIO(X, x)\} = {x}, there is some $U \in$ GRBIO(X, x) and $y \notin$ gr-b-I cl(U). Since $U \cap X \setminus$gr-b-I cl(U) = $\emptyset$, $U \times (X \setminus$ gr-b-I cl(U)) is gr-b-I-open set such that $(x, y) \in U \times (X \setminus$ gr-b-I cl(U)) $\subset X \setminus \Delta$.

(4)⇒(5): If $y \neq x$, then $(x, y) \notin \Delta$ and thus there exist $U, V \in$ GRBIO(X) such that $(x, y) \in U \times V$ and $(U \times V) \cap \Delta = \emptyset$. Clearly, for the gr-b-I-open sets U and V we have $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

Corollary 5.3: An ideal topological space is (X, τ, I) gr-b-I-T_2 if and only if each singleton subsets of X is gr-b-I-closed.

Corollary 5.4: An ideal topological space (X, τ, I) is gr-b-I-T_2 if and only if two distinct points of X have disjoint gr-b-I-closure.

Theorem 5.5: Every gr-b-I-regular T_0-space is gr-b-I-T_2.

Proof: Let (X, τ, I) be a gr-b-I-regular T_0 space and $x, y \in X$ such that $x \neq y$. Since X is T_0, there exists an open set V containing one of the points, say, x but not y. Then $y \in X \setminus V$, $X \setminus V$ is closed and $x \notin X \setminus V$. By gr-b-I-regularity of X, there exist gr-b-I-open sets G and H such that $x \in G$, $y \in X \setminus V \subset H$ and $G \cap H = \emptyset$. Hence (X, τ, I) is gr-b-I-T_2.

Theorem 5.6: Every open subspace of a gr-b-I-T_2 space is gr-b-I-T_2.

Proof: Proof is similar to **Theorem 4.3**

Theorem 5.7: If *f*: (X, τ, I) → (Y, σ) is injective, open and gr-b-I-continuous and Y is T_2, then (X, τ, I) is grb-I-T_2.

Proof: Since *f* is injective, $f(x) \neq f(y)$ for each $x, y \in X$ and $x \neq y$. Now Y being T_2, there exist open sets G, H in Y such that $f(x) \in G$, $f(y) \in H$ and $G \cap H = \emptyset$. Let $U = f^{-1}(G)$ and $V = f^{-1}(H)$. Then by hypothesis, U and V are r-b-I-open in X. Also $x \in f^{-1}(G) = U$, $y \in f^{-1}(H) = V$ and $U \cap V = f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Hence (X, τ, I) is gr-b-I-T_2.

Definition 5.8: A function *f* : (X, τ, I) → (Y, σ, J) is called strongly gr-b-I-open if the image of every gr-b-I-open subset of (X, τ, I) is gr-b-J-open in (Y, σ, J).

Theorem 5.9: Let (X, τ, I) be an ideal topological space, R an equivalence relation in X and p : (X, τ, I) → X|R the identification function. If $R \subset (X \times X)$ and p is a strongly gr-b-I-open function, then X|R is gr-b-I-T_2

Proof: Let p(x) and p(y) be the distinct members of X|R. Since x and y are not related, $R \subset (X \times X)$ is gr-b-I-closed in $X \times X$. There are gr-b-I-open sets U and V such that $x \in U$ and $y \in V$ and $U \times V \subset X \setminus R$. Thus p(U) and p(V) are disjoint gr-b-I-open sets in X|R since p is strongly gr-b-I-open.

Definition 5.10: An ideal topological space (X, τ, I) is said to be gr-b-I-R_1 if for x, y in X with gr-b-I cl({x}) ≠gr-b-I cl({y}), there exists disjoint gr-b-I-open sets U and V such that gr-b-I cl({x}) is a subset of U and gr-b-I cl({y}) is a subset of V .

Theorem 5.11: The ideal topological space (X, τ, I) is gr-b-I-T_2 if and only if it is gr-b-I-R_1 and gr-b-I-T_0.

Proof: The proof is similar to **Theorem 4.10** and thus omitted.

Remark 5.12: In the following diagram we denote by arrows the implications between the separation axioms which we have introduced and discussed in this paper and examples show that no other implications hold between them.

T_2	⇒	gr-b-I-T_2	⇒	rg-b-I-T_2	⇒	gr-b-T_2
⇓		⇓		⇓		⇓
T_1	⇒	gr-b-I-T_1	⇒	rg-b-I-T_1	⇒	gr-b-T_1
⇓		⇓		⇓		⇓
T_0	⇒	gr-b-I-T_0	⇒	rg-b-I-T_0	⇒	gr-b-T_0

Example 5.13: consider the topological space Y = { α , β, μ , ω}, τ = {∅, Y,{ α }, { β, μ },{ α, β, μ } and I ={∅, { α },{ μ },{ α, μ }}. Then (Y, τ, I) is gr-b-I-T_i (i = 0, 1, 2) but not T_i (i = 0, 1, 2).

Example 5.14: consider the topological space Y={α , β, μ }, τ = {∅,{ α },{ β },{ α, β },{ β, μ }, Y} and I = {∅, { α }}. Then (Y, τ, I) is gr- b-I-T_0 but not gr- b-I-T_1.

Example 5.15: Let consider the topological space X={α , β, μ }τ = {∅, { α }, { β, μ }, X} and I = {∅, { β } { μ }, { β, μ }}. Then (X, τ, I) is gr- b-T_i (i = 0, 1, 2) but not gr-b-I-T_i (i = 0, 1, 2).

Theorem 5.16: (1). An ideal topological space (X, τ, {∅}) is gr-b-I-T_0 (resp. gr-b-I-T_1, gr-b-I-T_2) if and only if it is gr-b-T_0 (resp. gr-b-T_1, gr-b-T_2).

(2). An ideal topological space (X, τ, N) is gr-b-I-T_0 (resp. gr-b-I-T_1, gr-b-I-T_2) if and only if it is gr-b-T_0 (resp. gr-b-T_1, gr-b-T_2) (N is the ideal of all nowhere dense sets of X).

(3). An ideal topological space (X, τ, P(X)) is gr-b-I-T_0 (resp. gr-b-I-T_1, gr-b-I-T_2) if and only if it is T_0 (resp. T_1, T_2).

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بعض بديهيات الفصل الجديدة من خلال المجموعات المفتوحة gr-b-I

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الملخص

الغرض من البحث هو تقديم المجموعات المفتوحة **gr-b-I** من خلال هذا المفهوم درسنا بعض انواع بديهيات الفصل الضعيفة في الفضاءات التبولوجية المثالية، التطبيقات لهذه البديهيات مع بعضها البعض ومع البديهيات المعرفة قد نوقشت.