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ON PARETO SET FOR A BI-CRITERIA SINGLE MACHINE SCHEDULING PROBLEM

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ABSTRACT

This paper considers a bi-criteria planning problems on a single machine, with the goal of minimizing total square time duration and maximizing earliness. To solve this problem we have to find the Pareto set. We introduced a strong relation between lower bound, upper bound of the problem and the number of efficient solutions via a theorem which shows also that the lower bound is near to optimal solution if the number of efficient solutions is small.

1. Introduction

During recent years many approaches for solving multi-objective scheduling have been analyzed [7]. T'kindt and Billaut offered a comprehensive assessment of the over a hundred multi-objective scheduling problems, They concentrated on single-machine scheduling, parallel-machine scheduling, flow shop scheduling, and fuzzy scheduling issues [13]. The hierarchical problems and the simultaneous problems are two alternative structures of bi-criteria scheduling issues. Simultaneous difficulties lead to the discovery of a collection of non-dominated solutions (the Pareto set), which gives the decision maker more information about which solution to choose [11]. Several researchers paid attention on finding Pareto set for bi-criteria problems. Hoogeveen and Velde [5] simultaneously decreased maximum completion time and cost. Lazarev et al. the Pareto set was discovered for jobs with similar processing times according to the criteria Lmax and Cmax [9]. Nguyen and Bao used a genetic algorithm to tackle the mixed store scheduling issue [12]. For the scheduling problems which involving quadratic measure of performance, there are relatively little works done in this form. Townsend posed an issue using a quadratic optimization method for completion times. [14]. Bagga and Kalra modified in some sense the Townsend's algorithm [2]. Gupta and Sen improved branching procedure of quadratic penalty function of completion times [3]. Abdul-Razaq and Kawi combined their efforts to solve a function of square completion time and high tardiness [1].

The total square completion time and greatest earliest completion time were the two criteria we focused on in this paper $\sum_{j=1}^{n} c_j^2$ and Emax. This problem was solved by Hoogeveen and Van de Velde simultaneously and they found all the efficient solutions of the problem [4]. We introduced a theorem which found a strong relation between optimal solution, lower bound and number of efficient solutions. This theorem is also can be applied for all the problems that have the same structure.

2. Fundamental Ideas and Definitions

We describe the following in this section:
N: set of jobs {1, ..., n},
Pj: processing time for job j,
dj: due date for job j,
cj: completion time for job j,
Ej: earliness of job j, Ej = dj- cj,
MST: (minimal slack periods) Minimum slack times are sequenced in a non-descending order sj, where sj = dj - pj,
SPT: jobs with the quickest processing time are ordered in non-descending order by pj.
LB: A value of the objective function that is less than or equal to the optimal value is known as the (lower bound),
UB: (upper bound) an objective function value that is more than or equal to the optimal value,
opt: optimal value.
Definition (1): [1]. A schedule S is said to be efficient (Pareto optimal) if there does not exist another schedule S^* satisfying $f_i(S^*) \le f_i(S)$, i= 1, 2, ..., k with at least one of the above holding as a strict inequality. Otherwise S is said to be dominated by S^*.

We will analyze a bi-criteria Problem with scheduling a single machine, with complete square completion times as the performance metric $\sum_{j=1}^{n} c_j^2$ and maximum cost fmax. i.e., the issue is in simultaneous form. The cost function f may be regular or irregular function. The issue is as described in the following:

Assume n jobs (j=1, ..., n) must be scheduled on a single machine which can only handle one task at a time. A positive process time is required for each job pj and has a due date dj. The maximum cost function in this paper is maximum earliness Emax.

3. Problem Approaches for Multi-Criteria Scheduling

In scheduling problems multi-criteria relates to the issue in which there are more than one performance criteria. The hierarchical and simultaneous problems are two sorts of issues. One of the criteria is regarded a fundamental criterion in the hierarchical scenario, while the other is called a secondary criterion, whereas in the simultaneous case both criteria have the same importance, and in this case the solution leads to generate all the efficient solutions. Lee and Vairaktarakis [10] reviewed computational complexity results of hierarchical minimization problems. Hoogeveen [11] introduced a survey on multi-criteria scheduling problems containing simultaneous approximation. Hoogeveen and Van de Velde [5] solved a simultaneous minimization problem in a polynomial time. It's important to mention that the specific instance of the simultaneous scenario 1// F(f, g) is hierarchical scheduling problem 1// Lex (f, g) Where f is the fundamental criterion and the secondary criterion is g, and the simultaneous case is likewise NP-hard if the hierarchical issue is NP-hard.

4. Pareto Set and Optimal Solution

The Pareto set for the simultaneous situation was discovered by Hoogeveen and Van de Velde [6] 1// F $(\sum_{j=1}^{n} c_{j,}\ E_{max})$, By employing a genetic algorithm, Kurz and Canterbury [8] discovered the Pareto set for the similar problem.

We will give a theorem that finds a relation between the Pareto set, The optimal solution, as well as the lowest bounds for total square completion time and greatest earliness. It is important to mention that this theorem can be applied to all the problems in this structure, this means the bi-criteria problems in simultaneous case with f_{max} Here in this paper $f_{max} = E_{max}$.

Let the lower bound $LB = \sum_{j=1}^{N} c_j^2\,(SPT) + E_{max}(MST)$ and the upper bound $UB = \sum_{j=1}^{N} c_j^2\,(SPT) + E_{max}(SPT)$.

Theorem 4.1:

There exists an integer M and non-negative such that $LB + M = optimal\ value$ and $M \in [N_1 - 1, N_2 + 1]$ where N_1 = number of effective solutions and $N_2 = E_{max}(SPT)$-$E_{max}(MST)$.

Proof:

Since LB is less the optimal value, so there exists an integer M and non-negative so that $LB + M = optimal\ value$ The first section of the theorem is proved by this. It is still to demonstrate M ∈ [N1-1, N2+1] or to demonstrate N1-1 $\le M \le$ N2+1. We have M = optimum rate - LB ≤ UB - LB
$= \sum_{j=1}^{N} c_j^2\,(SPT) + E_{max}(SPT)$ -$\sum_{j=1}^{N} c_j^2\,(SPT) - E_{max}(MST)$
$= E_{max}(SPT)$ - $E_{max}(MST)$ $= N_2 \le N_2 + 1$.
Hence $M \le N_2 + 1$.

To prove $N_1 - 1 \le M$ we will use mathematical induction on N_1 . If $N_1 = 1$, That seems to be, there is just one effective solution, SPT. then M = $\sum_{j=1}^{N} c_j^2\,(SPT) + E_{max}(SPT)$ -$\sum_{j=1}^{N} c_j^2\,(SPT) - E_{max}(MST)$ =0, because $E_{max}(SPT)\_E_{max}(MST)$ =0.

This is a special case where the SPT sequence is the same as MST sequence and M ∈ [0, 1]. Thus $N_1 - 1 \le M \le N_2 + 1]$, As a result, the theorem is correct for $N_1 = 1$.

If $N_1 = 2$, that is, SPT and σ are the only two effective options, say. Since $N_1 = 2$, so $N_1 - 1 = 1$. The two options are as follows:

a- If SPT is optimal then $\sum_{j=1}^{N} c_j^2\,(SPT) + E_{max}(SPT) - \sum_{j=1}^{N} c_j^2\,(SPT) - E_{max}(MST)$, implied that
$E_{max}(SPT)\_E_{max}(MST) \ge 1 = N_1 - 1$.

b- If σ is optimal then $\sum_{j=1}^{N} c_j^2\,(\sigma) + E_{max}(\sigma) - \sum_{j=1}^{N} c_j^2\,(SPT) - E_{max}(MST) \ge 1$,
because $\sum_{j=1}^{N} c_j^2\,(\sigma) - \sum_{j=1}^{N} c_j^2\,(SPT) \ge 1$. Thus $N_1 - 1 \le M \le N_2 + 1$ and hence the theorem is true for $N_1 = 2$.

If $N_1 = 3$, that is, SPT, σ and σ_1 are the three most efficient solutions, say.

Since $N_1 = 3$, so $N_1 - 1 = 2$.. The three cases are as follows:

a- If SPT is optimal then $\sum_{j=1}^{N} c_j^2 (SPT) + E_{max}(SPT)$ $- \sum_{j=1}^{N} c_j^2 (SPT) - E_{max}(MST)$, implies that

$E_{max}(SPT) - E_{max}(MST) \geq 2 = N_1 - 1$.

b- If σ is optimal then $M = \sum_{j=1}^{N} c_j^2 (\sigma) + E_{max}(\sigma)$ $- \sum_{j=1}^{N} c_j^2 (SPT) - E_{max}(MST)$, implies that

$\sum_{j=1}^{N} c_j^2 (\sigma) - \sum_{j=1}^{N} c_j^2 (SPT) + E_{max}(\sigma)$ $- E_{max}(MST) \geq 1 + 1 = 2 = N_1 - 1$.

c- If σ1 is optimal then $M = \sum_{j=1}^{N} c_j^2 (\sigma_1) + E_{max}(\sigma_1)$ $- \sum_{j=1}^{N} c_j^2 (SPT) - E_{max}(MST)$, implies that

$\sum_{j=1}^{N} c_j^2 (\sigma_1) - \sum_{j=1}^{N} c_j^2 (SPT) \geq 2$. So, $N_1 - 1 \leq M \leq N_2 + 1$ as a result, the theorem is correct for $N_1 = 3$.

Assume the theorem is correct for $N_1 = k$, that is, for the k efficient solutions, the theorem holds true SPT, σ, σ_1, ... , σ_{k-2} . Let $N_1 = k + 1$، this implies, there are $k + 1$ solutions which are effective. If either of the first k efficient approaches is the optimal, as well as the theorem is correct, then for $N_1 = k$ we obtain $N_1 - 1 \leq M$ and hence $N_1 - 1 \leq M \leq N_2 + 1$. If σ_{k-1} final effective solution is optimal then

$M = \sum_{j=1}^{N} c_j^2 (\sigma_{k-1}) + E_{max}(\sigma_{k-1})$ $- \sum_{j=1}^{N} c_j^2 (SPT) - E_{max}(MST)$, implies that

$\sum_{j=1}^{N} c_j^2 (\sigma_{k-1}) + E_{max}(\sigma_{k-1})$ $- \sum_{j=1}^{N} c_j^2 (SPT) - E_{max}(MST) \geq k$. Thus M ∈[N1-1, N2+1] the theorem is correct for $N_1 = k + 1$.

4.1 Illustrated Example

Consider the following example with three jobs

j	1	2	3
p_j	2	1	5
d_j	1	6	12

At first we find all the possible sequences for 3 jobs which are 3! = 6. The results are as follows:

Sequences	Note	$\sum_{j=1}^{N} c_j^2$	E_{max}	$\sum_{j=1}^{N} c_j^2 + E_{max}$	Efficient solutions
(1, 2, 3)	MST-rule	77	4	81	Efficient
(1, 3, 2)		117	5	122	
(2, 1, 3)	SPT-rule	74	6	80	Efficient and optimal
(2, 3, 1)		101	6	107	
(3, 1, 2)		138	7	145	
(1, 2, 3)		125	7	132	

Here, the optimal solution is the sequence ((2, 1, 3) with the cost 80. To use the theorem we find

$LB = \sum_{j=1}^{N} c_j^2 (SPT) + E_{max}(MST) = 74 + 4 = 78$

$UB = \sum_{j=1}^{N} c_j^2 (SPT) + E_{max}(SPT) = 74 + 6 = 80$

$N_2 = E_{max}(SPT) - E_{max}(MST) = 6 - 4 = 2$

N_1 = number of efficient solution -1 = 2 – 1 = 1, so $M \in [N_1 - 1, N_2 + 1] = [0, 2]$. Therefore M = 2 because LB + 2 = 80.

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الملخص

تأخذ هذا البحث في الاعتبار مشاكل التخطيط لجدولة دالة ثنائية الاهداف على ماكنة واحدة بهدف تصغير هدفيين : مجموع مربع أوقات الاتمام واعظم تبكير. لحل هذه المشكلة نقوم بايجاد مجموعة باريتو. قدمنا علاقة قوية بين الحد الأدنى والحد الأعلى للمسالة وكذلك عدد الحلول الكفؤة عبر نظرية توضح أيضًا أن الحد الأدنى قريب من الحل الأمثل إذا كان عدد الحلول الكفؤة صغيرة.