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Topological Features of *ic*- Open Sets

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ABSTRACT

Using the idea of *ic*-open sets, we introduce and investigate the topological qualities of an *ic*-closure, *ic*-interior, *ic*-limit points, *ic*-derived, *ic*-border, *ic*-frontier, and *ic*-exterior of a set. Introduce the concepts of "*ic*-continuous mappings," "*ic*-open mappings," "*ic*-irresolute mappings," "*ic*-totally continuous mappings," and "*ic*-homeomorphism," and then look into some of the properties of these mappings.

1. Introduction

Askandar in [2] "using the idea of *i*- open sets, he introduces and examines the topological features of *i*-derivatives, *i*- terms and *i*- set outward appearances. Using *ic*-open sets, we introduce and investigate the same notions in this research. a portion H of "is known as *ic*-open set[1] if there exists a closed set $F \neq \phi, X \in \tau^c$ such that: $F \cap H \subseteq Int(H)$, where $Int(H)$ denotes the interior points of H and τ^c denotes the family of closed sets. An *ic*-closed set is the complement of an *ic*-open set.. We denote the family of *ic*-open set in (X, τ) by τ^{ic}. Let(X, τ^{ic}) be a topological space. This property allows us to prove similar properties *i*- open set. Also, we define *ic*-continuous mappings, *ic*-open mappings, *ic*-totally continuous mappings, *ic*- homeomorphism and investigate some properties of these mappings. The topological spaces (X, τ) and (Y, σ) are denoted here by X and Y, respectively, topological spaces, open sets (as opposed to closed sets) by (*os*), (*cs*), TS. Throughout this paper, topological spaces are referred to as (X, τ) and (Y, σ). $Cl(H)$ and $Int(H)$ denote the closure and interior of a space's subset H, respectively. The following definitions come to mind; they are helpful in the follow-up.

Definition 1.1. A mapping $f: X \to Y$ is named

1. Continuous denoted by (*conm*) [4] if $f^{-1}(U)$ is (os) in X for each (os) U in Y.
2. totally -continuous is denoted by (*t conm*)if [4] $f^{-1}(U)$ is (cl-os) in X for each (os) U in Y.

3. *ic*- continuous is denoted by (*ic*- *conm*)if [1] $f^{-1}(U)$ is (*ic*-os) in X for each (os) U in Y.

Theorem 1.2. [1]

1. Each (*os*) in TS is (*ic*-*os*).
2. Each (*conm*) is (*ic*- *conm*).

2. Applications of *ic*- Open Sets.

Definition 2.1. Assume X be a TS and let $H \subseteq X$. The *ic*- interior of H is defined as the union of all (*ic*- *os*) in X content in H, and is denoted by $Int_{ic}(H)$. It is clear that $Int_{ic}(H)$ is (*ic*-*os*) for any subset H of X.

Proposition 2.2. Assume (X,τ) be a TS and if $H \subseteq K \subseteq X$. Then

1. $Int_{ic}(H) \subseteq Int_{ic}(K)$;
2. $Int_{ic}(H) \subseteq H$;
3. H is *ic*- open iff $H= Int_{ic}(H)$.

Definition 2.3. Assume X be a TS and let $H \subseteq X$. The *ic*-closer of H is defined as The intersection of all (*ic*-*cs*) in X containing H, and is denoted by $CL_{ic}(H)$. It is clear that $CL_{ic}(H)$ is (*ic*-*cs*) for any subset H of X.

Proposition 2.4. Assume (X,τ) be a TS and if $H \subseteq K \subseteq X$. Then

1. $CL_{ic}(H) \subseteq CL_{ic}(K)$;
2. $H \subseteq CL_{ic}(H)$;
3. H is *ic*- closed if and only if $H= CL_{ic}(H)$.

Example 2.5. If $X = \{1,3,5\}$ and $\tau = \{\emptyset, X, \{3\}, \{1,3\}\}$ Then

$\tau^{ic} = \{\emptyset, X, \{1\}, \{3\}, \{1,3\}\}$

Let $H=\{3\}$, $K=\{1,3\}$ and $\{3\} \subseteq \{1,3\} \subseteq X$. Then

1. $Int_{ic}(H)= \{3\} \subseteq Int_{ic}(K)=\{1,3\}$;
2. $Int_{ic}(H) = \{3\} \subseteq H = \{3\}$;

3.$H=\{3\}$ is *ic*- open if and only if $H=\{3\}= Int_{ic}(H)=\{3\}$.

$C(\tau^{ic})= \{\emptyset, X, \{3,5\}, \{1,5\}, \{5\}\}$

Let $H=\{3\}$, $K=\{1,3\}$ and $\{3\} \subseteq \{1,3\} \subseteq X$. Then

1. $CL_{ic}(H)= \{3,5\} \subseteq CL_{ic}(K)=X$;
2. $H= \{3\} \subseteq CL_{ic}(H) = \{3,5\}$;

3.$H=\{5\}$ is *ic*- closed if and only if $H=\{5\}= CL_{ic}(H)=\{5\}$.

Definition 2.6. Let H be a subset of a TS X. A point $n \in X$ is named *ic* – limit point of H if it satisfies the following assertion:

$(\forall G \in \tau^{ic})(n \in G \Rightarrow G \cap (H \setminus \{n\}) \neq \phi)$

The set of all *ic*-limit points of H is named *ic*-derived set of H and is denoted by $D_{ic}(H)$ Note that for a subset H of X, a point $n \in X$ is not *ic*- limit point of H iff there exists (*ic* - os) G in X s.t. $n \in G$ & $G \cap (H \setminus \{n\}) = \phi$

or equivalently,

$n \in G$ and $G \cap H = \phi$ or $G \cap H = \{n\}$

or equivalently,

$n \in G$ and $G \cap H \subseteq \{n\}$

Theorem 2.7. let H be a subset X, and $n \in X$. Then the following are equivalent:

(1) $(\forall G \in \tau^{ic})(n \in G \Rightarrow G \cap H \neq \phi)$.

(2) $n \in CL_{ic}(H)$

Proof. (1)$\Rightarrow$(2) if $n \notin CL_{ic}(H)$, then there exists (*ic*-*cs*) F s.t. $H \subseteq F$ and $n \notin F$. Hence $X \setminus F$ is (*ic*-*os*) containing n and $H \cap (X \setminus F) \subseteq H \cap (X \setminus H)=\phi$. This is contradiction, and hence (2) is valid.

(2) $\Rightarrow$(1) straightforward. ■

Theorem 2.8. If (X,τ) be a TS and let $A \subseteq B \subseteq X$. Then

1. $CL_{ic}(A) = A \cup D_{ic}(A)$.
2. A is ic-closed iff $D_{ic}(A) \subseteq A$
3. $D_{ic}(A) \subseteq D_{ic}(B)$
4. $D_{ic}(A) \subseteq D(A)$
5. $CL_{ic}(A) \subseteq CL(A)$.

Proof. Let $n \notin CL_{ic}(A)$. Then there is (*ic*-*cs*) F in X s.t. $A \subseteq F$ and $n \notin F$. Hence $G=X-F$ is (*ic*-*os*) s.t. $n \in G$ and $G \cap A = \emptyset$. Therefore $n \notin A$ and $n \notin D_{ic}(A)$, then $n \notin A \cup D_{ic}(A)$.

Thus $A \cup D_{ic}(A) \subseteq CL_{ic}(A)$. On the other hand, $n \notin A \cup D_{ic}(A)$ implies that there exists (*ic*-*os*) G in X s.t. $n \in G$ and $G \cap A = \emptyset$. Hence $F=X-G$ is (*ic*- *cs*) in X s.t. $A \subseteq F$ and $n \notin F$. Hence $\notin D_{ic}(A)$. Thus $CL_{ic}(A) \subseteq A \cup D_{ic}(A)$. Therefore; $CL_{ic}(A) = A \cup D_{ic}(A)$. ■

For (2), (3), (4) and (5) the proof is easy.

Example 2.9. Let $X = \{1,2,3\}$ and $\tau = \{\emptyset, X, \{1\}, \{1,2\}\}$ Then

1. $\tau \subseteq \tau^{ic} = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$
2. If $H=\{1,3\}$, then $D(H)=\{3\}$ and $D_{ic}(H) = \emptyset$
3. If $K=\{1,2\}$, then $D(K)=\{2,3\}$ and $D_{ic}(K) = \{3\}$

Theorem 2.10. let τ_1 and τ_2 be topologic on X s.t. $\tau_1^{ic} \subseteq \tau_2^{ic}$. For any subset H of X, each *ic*–limit point of H with respect to τ_2 is *ic*- limit point of H with respect to τ_1.

Proof. Assume n be *ic* -limit point of H with respect to τ_2. Then $G \cap (H \setminus \{n\}) \neq \phi$ for each $G \in \tau_2^{ic}$ s.t. $n \in G$. But $\tau_1^{ic} \subseteq \tau_2^{ic}$, so in particular, $G \cap (H \setminus \{n\}) \neq \phi$ for each $G \in \tau_1^{ic}$ s.t. $n \in G$. Hence n is *ic*-limit point of H with respect to τ_1.■

Theorem 2.11. If H is a subset of a discrete topological space X, then $D_{ic}(H)= \phi$

Proof. Assume n be any element of X. Recall that each subset of X is (os) and so (*ic*-*os*). In particular the singleton set $G:=\{n\}$ is (*ic*-*os*). But $n \in G$ & $G \cap H= \{n\} \cap H \subseteq \{n\}$. Hence n is not *ic*-limit point of H, and so $D_{ic}(H)= \phi$. ■

Theorem 2.12. Let H and K be subsets of X. If $H \in \tau^{ic}$ and τ^{ic} is a topology on X, then $H \cap CL_{ic}(K) \subseteq CL_{ic}(H \cap K)$.

Proof. Assume $n \in H \cap CL_{ic}(K)$. Then $n \in H$ and $n \in CL_{ic}(K) = K \cup D_{ic}(K)$. If $n \in K$, then $n \in H \cap K \subseteq CL_{ic}(H \cap K)$. If $n \notin K$, then $n \in D_{ic}(K)$ and so $G \cap K \neq \phi$ for all $(ic-os)$ G containing n. Since $H \in \tau^{ic}$, $G \cap H$ is also $(ic\text{-}os)$ containing n. Hence $G \cap (H \cap K) = (G \cap H) \cap K \neq \phi$, and consequently $n \in D_{ic}(K \cap H) \subseteq CL_{ic}(H \cap K)$.Therefore $H \cap CL_{ic}(K) \subseteq CL_{ic}(H \cap K)$.

Definition 2.13. For any subset H of X, the set $b_{ic}(H) = H \setminus Int_{ic}(H)$ is called the *ic*- border of H

Proposition 2.14. For a subset A of a space X, the following statements hold:

1. $b_{ic}(A) \subset b(A)$ where $b(A)$ denotes the border of A;
2. $A = Int_{ic}(A) \cup b_{ic}(A)$;
3. $Int_{ic}(A) \cap b_{ic}(A) = \phi$;
4. A is an *ic*- open set if and only if $b_{ic}(A) = \phi$;
5. $b_{ic}(Int_{ic}(A)) = \phi$;
6. $Int_{ic}(b_{ic}(A)) = \phi$;
7. $b_{ic}(b_{ic}(A)) = b_{ic}(A)$;

Proof.

(1) Since $Int(A) \subset Int_{ic}(A)$, we have $b_{ic}(A) = A \setminus Int_{ic}(A) \subseteq A \setminus Int(A) = b(A)$.

(2) & (3). Straightforward.

(4) Assume $Int_{ic}(A) \subseteq A$, it follows from proposition 2.2 (3). That A is $(ic\text{-os}) \Leftrightarrow A = Int_{ic}(A) \Leftrightarrow b_{ic}(A) = A \setminus Int_{ic}(A) = \phi$.

(5) Assume $Int_{ic}(A)$ is (*ic*-os), it follows from (4) that $b_{ic}(Int_{ic}(A)) = \phi$.

(6) If $n \in Int_{ic}(b_{ic}(A))$, then $n \in b_{ic}(A)$. On the other hand, since $b_{ic}(A) \subset A$, $n \in Int_{ic}(b_{ic}(A)) \subset Int_{ic}(A)$. Hence, $n \in Int_{ic}(A) \cap (b_{ic}(A))$, which contradicts (3). Thus $Int_{ic}(b_{ic}(A)) = \phi$.

(7) Using (6), we get $b_{ic}(b_{ic}(A)) = b_{ic}(A) \setminus Int_{ic}(b_{ic}(A)) = b_{ic}(A)$.

Example 2.15. From example 2.5. If $A = \{1, 5\}$ be a subset of X. Then $Int_{ic}(A) = \{1\}$, and so $b_{ic}(A) = A \setminus Int_{ic}(A) = \{1, 5\} \setminus \{1\} = \{5\}$, and $b(A) = A \setminus Int(A) = \{1, 5\} \setminus \phi = \{1, 5\}$. Hence, $b(A) \not\subset b_{ic}(A)$, Therefore, the converse of proposition 2.14 (1) may not always be true.

Definition2.16. $Fr_{ic}(H) = CL_{ic}(H) \setminus Int_{ic}(H)$ is called the *ic*- frontier of H.

Not that if H is (*ic-cs*) of X, then $b_{ic}(H) = Fr_{ic}(H)$.

proposition 2.17. These propositions are true for a subset A of a space X:

1. $Fr_{ic}(A) \subset Fr(A)$ where $Fr(A)$ denotes the frontier of A;
2. $CL_{ic}(A) = Int_{ic}(A) \cup Fr_{ic}(A)$;
3. $Int_{ic}(A) \cap Fr_{ic}(A) = \phi$;
4. $b_{ic}(A) \subset Fr_{ic}(A)$;
5. $Fr_{ic}(A) = b_{ic}(A) \cup D_{ic}(A)$;
6. If A is an *ic*- open set then $Fr_{ic}(A) = D_{ic}(A)$;
7. $Fr_{ic}(A) = CL_{ic}(A) \cap CL_{ic}(X \setminus A)$;
8. $Fr_{ic}(A) = Fr_{ic}(X \setminus A)$;
9. $Fr_{ic}(A)$ is *ic*-closed;
10. $Fr_{ic}(Fr_{ic}(A)) \subset Fr_{ic}(A)$;
11. $Fr_{ic}(Int_{ic}(A)) \subset Fr_{ic}(A)$;
12. $Fr_{ic}(CL_{ic}(A)) \subset Fr_{ic}(A)$;
13. $Int_{ic}(A) = A \setminus Fr_{ic}(A)$.

Proof.

(1) Since $CL_{ic}(A) \subseteq Cl(A)$ and $Int(A) \subseteq Int_{ic}(A)$, it follows that $Fr_{ic}(A) = CL_{ic}(A) \setminus Int_{ic}(A) \subseteq Cl(A) \setminus Int_{ic}(A) \subseteq Cl(A) \setminus Int(A) \subseteq Fr(A)$.

(2) $Int_{ic}(A) \cup Fr_{ic}(A) = Int_{ic}(A) \cup (CL_{ic}(A)) \setminus Int_{ic}(A) = CL_{ic}(A)$.

(3) $Int_{ic}(A) \cap Fr_{ic}(A) = Int_{ic}(A) \cap (CL_{ic}(A)) \setminus Int_{ic}(A)) = \phi$.

(4) Since $A \subseteq CL_{ic}(A)$, we have $b_{ic}(A) = A \setminus Int_{ic}(A) \subseteq CL_{ic}(A) \setminus Int_{ic}(A) = Fr_{ic}(A)$

(5) Since $Int_{ic}(A) \cup Fr_{ic}(A) = Int_{ic}(A) \cup b_{ic}(A) \cup D_{ic}(A)$, $Fr_{ic}(A) = b_{ic}(A) \cup D_{ic}(A)$.

(6) Assume that A is (*ic-os*). Then $Fr_{ic}(A) = b_{ic}(A) \cup (D_{ic}(A) \setminus Int_{ic}(A)) = \phi \cup (D_{ic}(A) \setminus A) = D_{ic}(A) \setminus A = b_{ic}(X \setminus A)$, by using (5), proposition 2.2 (3), proposition2.14(4)

(7) $Fr_{ic}(A) = CL_{ic}(A) \setminus Int_{ic}(A) = CL_{ic}(A) \cap (CL_{ic}(X \setminus A))$.

(8) It follows from (7).

(9) $CL_{ic}(Fr_{ic}(A)) = CL_{ic}(CL_{ic}(A)) \cap (CL_{ic}(X \setminus A)) \subset CL_{ic}(CL_{ic}(A)) \cap CL_{ic}(CL_{ic}(X \setminus A)) = Fr_{ic}(A)$. Hence, $Fr_{ic}(A)$ is *ic*- closed.

(10) $Fr_{ic}(Fr_{ic}(A)) = CL_{ic}(Fr_{ic}(A)) \cap (CL_{ic}(X \setminus Fr_{ic}(A)) \subset CL_{ic}(Fr_{ic}(A)) = Fr_{ic}(A)$.

(11) Since $Int_{ic}(Int_{ic}(A)) = Int_{ic}(A)$, we get $Fr_{ic}(Int_{ic}(A)) = CL_{ic}(Int_{ic}(A)) \setminus Int_{ic}(Int_{ic}(A)) \subseteq CL_{ic}(A) \setminus Int_{ic}(A) = Fr_{ic}(A)$.

(12) $Fr_{ic}(CL_{ic}(A)) = CL_{ic}(CL_{ic}(A)) \setminus Int_{ic}(CL_{ic}(A)) = CL_{ic}(A) \setminus Int_{ic}(CL_{ic}(A)) = CL_{ic}(A) \setminus Int_{ic}(A) = Fr_{ic}(A)$.

(13) $A \setminus Fr_{ic}(A) = (A \setminus CL_{ic}(A)) \setminus Int_{ic}(A) = Int_{ic}(A)$. ■

Example 2.18. Assume that TS (X, τ) provided in Example 2.5, If $A = \{1, 3\}$ be a subset of X. Then $Int_{ic}(A) = \{1, 3\}$, and so $b_{ic}(A) = A \setminus Int_{ic}(A) = \{1, 3\} \setminus \{1, 3\} = \phi$. Since $A = \{5\}$ is *ic*-closed, $CL_{ic}(A) = \{5\}$ and thus $Fr_{ic}(A) = CL_{ic}(A) \setminus Int_{ic}(A) = \{5\} \setminus \{1, 3\} = \phi$.

Theorem 2.19. For a subset H of X, H *is (ic-cs)* iff $Fr_{ic}(H) \subseteq H$

***Proof*.** Assume that H is *(ic-cs)*. Then $Fr_{ic}(H) = CL_{ic}(H) \setminus Int_{ic}(H) = H \setminus Int_{ic}(H) \subseteq H$.

Conversely suppose that $Fr_{ic}(H) \subseteq H$. Then $CL_{ic}(H) \setminus Int_{ic}(H) \subseteq H$, and so $CL_{ic}(H) \subseteq H$. Since $Int_{ic}(H) \subseteq H$. Noticing that $H \subseteq CL_{ic}(H)$, we have $H = CL_{ic}(H)$. Therefore; H is (*ic-cs*). ■

Definition 2.20. For a subset H of X, $Ext_{ic}(H) = Int_{ic}(X \setminus H)$ is said to be an *ic*-exterior of H.

Example 2.21. Assume (X, τ) be a TS in Example 2.9 For subset $H = \{2\}$ and $K = \{1\}$ of X , we have $Ext_{ic}(H) = \{1\}$ and $Ext_{ic}(K) = \{2\}$.

Proposition 2.22. These propositions are true for a subset A of a space X:

1. $Ext_{ic}(A)$ is *ic*- open;
2. $Ext_{ic}(A) = Int_{ic}(X \setminus A) = X \setminus CL_{ic}(A)$;
3. If $A \subset B$, then $Ext_{ic}(A) \supset Ext_{ic}(B)$;
4. $Ext_{ic}(A \cup B) \subset Ext_{ic}(A) \cap Ext_{ic}(B)$;
5. $Ext_{ic}(A \cap B) \supset Ext_{ic}(A) \cup Ext_{ic}(B)$;
6. $Ext_{ic}(X) = \phi$;
7. $Ext_{ic}(\phi) = X$;
8. $Ext_{ic}(A) = Ext_{ic}(X \setminus Ext_{ic}(A))$;
9. $X = Int_{ic}(A) \cup Ext_{ic}(A) \cup Fr_{ic}(A)$.

Proof. (1) and (2) straightforward.

(3) Assume that $A \subseteq B$. Then $Ext_{ic}(B) = Int_{ic}(X \setminus B) \subseteq Int_{ic}(X \setminus A) = Ext_{ic}(A)$

(4) $Ext_{ic}(A \cup B) = Int_{ic}(X \setminus (A \cup B)) = Int_{ic}((X \setminus A) \cap (X \setminus B)) \subseteq Int_{ic}((X \setminus A)) \cap Int_{ic}((X \setminus B)) = Ext_{ic}(A) \cap Ext_{ic}(B)$.

(5) $Ext_{ic}(A \cap B) = Int_{ic}(X \setminus (A \cap B)) = Int_{ic}((X \setminus A) \cup (X \setminus B)) \supset Int_{ic}((X \setminus A)) \cup Int_{ic}((X \setminus B)) = Ext_{ic}(A) \cup Ext_{ic}(B)$.

(6) and (7) Straightforward.

(8) $Ext_{ic}(X \setminus Ext_{ic}(A)) = Ext_{ic}(X \setminus Int_{ic}(X \setminus A)) = Int_{ic}(X \setminus (X \setminus Int_{ic}(X \setminus A))) = Int_{ic}(Int_{ic}(X \setminus A)) = Int_{ic}(X \setminus A) = Ext_{ic}(A)$.

(9) Straightforward.

Example 2.23. If $X = \{1,2,3\}$ and $\tau = \{\emptyset, X, \{1\}, \{1,2\}\}$ Then $\tau^{ic} = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$

1. If $H = \{1\}$, $K = \{2\}$. Then $Ext_{ic}(H \cup K) = \emptyset$, $Ext_{ic}(H) = \{2\}$, $Ext_{ic}(K) = \{1\}$, $Ext_{ic}(H) \cap Ext_{ic}(K) = \emptyset$, so $Ext_{ic}(H \cup K) \subset Ext_{ic}(H) \cap Ext_{ic}(K)$
2. If $H = \{1, 2\}$, $K = \{2\}$. Then $Ext_{ic}(H \cap K) = \{1\}$, $Ext_{ic}(H) = \emptyset$, $Ext_{ic}(K) = \{1\}$, $Ext_{ic}(H) \cup Ext_{ic}(K) = \{1\}$, so $Ext_{ic}(H \cap K) \supset Ext_{ic}(H) \cup Ext_{ic}(K)$.

3. *ic*- Continuous Mappings and *ic*-Homeomorphism

This section is devoted to introduce *ic*-open map, *ic*-irresolute map, *ic*-totally continuous map, *ic*-homeomorphism and discussed the relationships between the other known existing map.

Definition 3.1. A mapping $f : X \rightarrow Y$ is named *ic*-open denoted by (*ic-om*), if $f(U)$ is (*ic-os*) in Y for each (*os*) U in X.

Example 3.2. Let $X = Y = \{3,5,7\}$ and $\tau = \{\emptyset, X, \{3,5\}\}$, $\sigma = \{\emptyset, Y, \{3\}, \{3,5\}\}$ Then $\tau^{ic} = \{\emptyset, Y, \{3\}, \{5\}, \{3,5\}\}$. Clearly, the identity mapping $f: X \rightarrow Y$ is (*ic-om*)

Proposition 3.3. Any (om) is *(ic-om)* but not conversely.

***Proof*.** Assume $f : X \rightarrow Y$ be (om) and H be (os) in X. Since, f is open, then $f(H)$ is (os) in Y. Since, each (os) is (*ic-os*) then, $f(H)$ is (*ic-os*) in Y. Therefore, f is (*ic-om*). ■

If $X = Y = \{1,2,3\}$ and $\tau = \{\emptyset, X, \{2\}, \{1,2\}\}$, $\sigma = \{\emptyset, Y, \{1\}, \{1,2\}\}$ Then
$\tau^{ic} = \{\emptyset, Y, \{1\}, \{2\}, \{1,2\}\}$. Clearly, the identity mapping $f: X \to Y$ is (*ic-om*) but not (*om*).

Theorem 3.4. If $f: X \to Y$ is open & $g: Y \to Z$ is *ic*-open, then $gof: X \to Z$ is *ic*-open.

Proof. Suppose that $f: X \to Y$ be open & $g: Y \to Z$ is ic-open. Let *G* be an (*os*) in *X*. Since, *f* is an open, then *f(G)* is an (*os*) in *Y*. Since, each (*os*) is (*ic-os*), then *f(G)* is (*ic- os*) in *Y*. Since, *g* is (*ic-os*), then $(gof)(G) = g(f(G))$ is (*ic-os*) in Z. Therefore; $gof: X \to Z$ is *ic*-open. ■

Theorem 3.5. If $f: X \to Y$ is (*ic-conm*) and $g: Y \to Z$ is (*conm*), then $gof: X \to Z$ is (*ic-conm*).

Proof. Assume $f: X \to Y$ be (*ic-conm*) & $g: Y \to Z$ is (*contm*). Let *G* be an (*os*) in *Z*. Since, *g* is (*conm*), then $g^{-1}(G)$ is an (*os*) in *Y*. Since, *f* is (*ic-conm*), then $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is (*ic- os*) in *X*. Therefore; $gof: X \to Z$ is (*ic-conm*). ■

Definition 3.6. Amapping $f: X \to Y$ is named *ic*-irresolute is denoted by (*ic-irrem*), if the inverse image of every (*ic-os*) of *Y* is (*ic-os*) in *X*

Example 3.7. If $X = Y = \{2,4,6\}$ and $\tau = \{\emptyset, X, \{2\}, \{2,4\}\}$, $\sigma = \{\emptyset, Y, \{2\}\}$ Then
$\tau^{ic} = \{\emptyset, X, \{2\}, \{4\}, \{2,4\}\}$.
$\sigma^{ic} = \{\emptyset, Y, \{2\}\}$
Clearly, the identity mapping $f: X \to Y$ is (*ic-irrem*)

Proposition 3.8. Each *(ic-irrem)* is *(ic-conm).*

Proof: Suppose that $f: X \to Y$ be (*ic-irrem*) & *V* any (os) in *Y*. Since each (os) is (*ic-os*) and since *f* is *ic*-irresolute, then $f^{-1}(V)$ is (*ic-os*) in *X*. Therefore; *f* is (*ic-conm*). ■

Theorem 3.9. Each *(conm)* is *(ic-irrem)* but not conversely.

Proof. Suppose that $f: X \to Y$ be (*conm*) & *V* any (*ic-*os) in *Y*. Since *f* is (*conm*), then $f^{-1}(V)$ is (*os*) in *X*. Since each (*os*) is (*ic-os*), then $f^{-1}(V)$ is (*ic-os*) in *X* . Therefore; *f* is (*ic-irrem*). ■

Let $X = Y = \{2,4,6\}$ and $\tau = \{\emptyset, X, \{2\}, \{2,4\}\}$, $\sigma = \{\emptyset, Y, \{4\}\}$ Then
$\tau^{ic} = \{\emptyset, X, \{2\}, \{4\}, \{2,4\}\}$.
$\sigma^{ic} = \{\emptyset, Y, \{4\}\}$
Clearly, the identity mapping $f: X \to Y$ is (*ic-irrem*) but not (*conm*)

Theorem 3.10. If $f: X \to Y$ is (*ic-irrem*) & $g: Y \to Z$ is (*ic-conm*), then $gof: X \to Z$ is (*ic-irrem*).

Proof. Let $f: X \to Y$ is (*ic-irrem*) and $g: Y \to Z$ is (*ic-conm*). Let *U* be an (*os*) in *Z*. Then *U* is (ic-os) because each (*os*) is (*ic-os*). Since, *g* is (*ic-conm*), then $g^{-1}(U)$ is (*ic-os*) in *Y*. Since, *f* is (*ic-irrem*), then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is (*ic- os*) in *X*. Therefore; $gof: X \to Z$ is *(ic-irrem)*. ■

Theorem 3.11. The composition of two *(ic-irrem)* is also *(ic-irrem).*

Proof. Assume $f: X \to Y$ & $g: Y \to Z$ any two *(ic-irrem)*. Suppose that *U* be any (ic-*os*) in *Z*. Since, *g* is (*ic-irrem*), then $g^{-1}(U)$ is (*ic-os*) in *Y*. Since, *f* is (*ic-irrem*), then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is (*ic-os*) in $X. Therefore;\ gof: X \to Z\ is\ (ic-irrem)$.■

Definition 3.12. Let *X* and *Y* be TS , a bijective map $f: X \to Y$ is named *ic*-homeomorphism is denoted by (*ic-homm*) if *f* is (*ic-conm*) and (*ic-om*).

Theorem 3.13. If $f: X \to Y$ is *(homm),* then f is *(ic-homm)* but not conversely.

Proof: Since each (*conm*) is (*ic-conm*) by Theorem 1.2 (2). Also, since each (*om*) is (*ic-om*) by proposition (3.3) Further, since *f* is bijective. Therefore, *f* is (*ic-homm*). ■

Let $X = Y = \{1,2,3\}$ and $\tau = \{\emptyset, X, \{1\}, \{2,3\}\}$, $\sigma = \{\emptyset, Y, \{2\}, \{1,3\}\}$ Then
$\tau^{ic} = \{\emptyset, X, \{1\}, \{3\}, \{2\}, \{1,2\}, \{2,3\}, \{1,3\}\}$.
$\sigma^{ic} = \{\emptyset, Y, \{1\}, \{3\}, \{2\}, \{1,2\}, \{2,3\}, \{1,3\}\}$.
Clearly, the identity mapping $f: X \to Y$ is (*ic-homm*) but not (*homm*)

Definition 3.14. Amapping $f: X \to Y$ is named *ic*-totally continuous is denoted by (*ic-tconm*), If each's reverse, (*ic-os*) of *Y* is (*cl-os*) in *X*.

Theorem 3.15. Each *(ic-tconm)* is totally continuous but not conversely.

Proof. Suppose that $f: X \to Y$ be (*ic-tconm*) and *V* be (os) in *Y*, since each (os) is (*ic-os*), then *V* is (*ic-os*) in *Y*. Since *f* is (*ic-tconm*), then, $f^{-1}(V)$ is (*cl- os*) in *X*. Therefore, *f* is ($tconm$). ■

Let $X = Y = \{1,2,3\}$ and $\tau = \{\emptyset, X, \{1\}, \{2,3\}\}$, $\sigma = \{\emptyset, Y, \{2,3\}\}$ Then
$\sigma^{ic} = \{\emptyset, Y, \{2\}, \{3\}, \{2,3\}\}$.
Clearly, the identity mapping $f: X \to Y$ is (*tconm*) but not (*ic-tconm*)

Theorem 3.16. Each *(ic-tconm)* is *(ic-irrem)* but not conversel.

Proof: Assume that $f: X \to Y$ be (*ic-tconm*) and *V* be (*ic-os*) in *Y*. Since *f* is (*ic-tconm*), then $f^{-1}(V)$ is (*cl-os*) in *X*, which implies, $f^{-1}(V)$ is (os), it follows $f^{-1}(V)$ is (*ic-os*) in *X*. Therefore; *f* is (*ic-irrem*). ■

Let $X = Y = \{1,3,5\}$ and $\tau = \{\emptyset, X, \{1\}, \{1,3\}\}$, $\sigma = \{\emptyset, Y, \{3\}\}$ Then
$\tau^{ic} = \{\emptyset, X, \{1\}, \{3\}, \{1,3\}\}$.
$\sigma^{ic} = \{\emptyset, Y, \{3\}\}$
Clearly, the identity mapping $f: X \to Y$ is (*ic-irrem*) but not (*ic-tconm*)

Theorem 3.17. The two's *(ic-tconm)* composition is also *(ic-tconm).*

Proof: Suppose that $f: X \to Y$, $g: Y \to Z$ be any two (*ic-tconm*). Assume *V* be any (*ic-os*) in *Z*. Since, *g* is (*ic-tconm*), then $g^{-1}(V)$ is ($cl-os$) in *Y*, which implies $f^{-1}(V)$ is (os), it follows $f^{-1}(V)$ is (*ic-os*). Since, *f* is (*ic-tconm*), then, $f^{-1}(g^{-1}(V)) =$

$(gof)^{-1}(V)$ is ($cl-os$) in X. Therefore, $gof: X \to Z$ is (*ic-tconm*). ■

Theorem3.18. If $f: X \to Y$ be (*ic-tconm*) and $g: Y \to Z$ *be (ic-irrem),* then $g \circ f: X \to Z$ *is (ic-tconm).*

Proof: Assume that $f: X \to Y$ be (*ic-tconm*) and $g: Y \to Z$ is (*ic-irrem*). Let *V* be (*ic-os*) in *Z*. Since g is (*ic-irrem*) then $g^{-1}(V)$ is (*ic-os*) in *Y*. Since *f* is (*ic-tconm*), then $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is ($cl-os$) in *X*. Therefore, $g \circ f: X \to Z$ is (*ic-tconm*). ■

Theorem3.19. If $f: X \to Y$ *is (ic-tconm)*and $g: Y \to Z$ *is(ic-conm),* then $g \circ f: X \to Z$ *is* ($tconm$).

Proof: Let $f: X \to Y$ be (*ic-tconm*) and $g: Y \to Z$ is (*ic-conm*), let *V* be(os) in *Z*. Since, *g* is (*ic-conm*), then, $g^{-1}(V)$ is (*ic-os*) in *Y*. Since, *f* is (*ic-tconm*), then, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is ($cl-os$) in *X*. Therefore, $g \circ f: X \to Z$ is ($tconm$). ■

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السمات التبولوجية لمجموعات مفتوحة من النمط-ic

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الملخص

باستخدام فكرة المجموعات المفتوحة من النمط, *ic* - نقدم ونتحقق من الصفات التبولوجية للانغلاق من النمط– *ic* الداخل من النمط–*ic* ، نقاط الحد من النمط– ic المشتقة من النمط– ic الحدود من النمط– - ic والجزء الخارجي من النمط- *ic* للمجموعة . قدم مفاهيم التطبيقات المستمرة من النمط, *ic*- والتطبيقات المفتوحة من النمط– ic التطبيقات المذبذبة من النمط– ic التطبيقات المستمرة تماما من النمط– *ic* والتشاكل من النمط– *ic* ثم ننظر في بعض خصائص هذه التطبيقات.