Topological Features of \textit{ic}- Open Sets

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\section{ABSTRACT}


\section{1. Introduction}

Askandar in [2] using the idea of \textit{ic}-open sets, he introduces and examines the topological features of \textit{ic}-derivatives, \textit{ic}-terms and \textit{ic}-set outward appearances. Using \textit{ic}-open sets, we introduce and investigate the same notions in this research. a portion \textit{H} of \textit{X} is known as \textit{ic}-open set[1] if there exists a closed set \textit{F} = \emptyset, \textit{X} \in \tau^i such that: \textit{F} \cap \textit{H} \subseteq \text{Int}(\textit{H}), where \text{Int}(\textit{H}) denotes the interior points of \textit{H} and \tau^c denotes the family of closed sets. An \textit{ic}-closed set is the complement of an \textit{ic}-open set. We denote the family of \textit{ic}-open set in (\textit{X}, \tau) by \tau^i. Let(\textit{X}, \tau^i) be a topological space. This property allows us to prove similar properties \textit{ic}-open set. Also, we define \textit{ic}-continuous mappings, \textit{ic}-open mappings, \textit{ic}-totally continuous mappings, \textit{ic}-homeomorphism and investigate some properties of these mappings. The topological spaces (\textit{X}, \tau) and (\textit{Y}, \sigma) are denoted here by \textit{X} and \textit{Y}, respectively, topological spaces, open sets (as opposed to closed sets) by (os), (cs), TS. Throughout this paper, topological spaces are referred to as (\textit{X}, \tau) and (\textit{Y}, \sigma). \text{Cl}(\textit{H}) and \text{Int}(\textit{H}) denote the closure and interior of a space's subset \textit{H}, respectively. The following definitions come to mind; they are helpful in the follow-up.

\textbf{Definition 1.1.} A mapping \textit{f}: \textit{X} \rightarrow \textit{Y} is named
1. Continuous denoted by (\textit{comm}) [4] if \textit{f}^{-1}(\textit{U}) is (os) in \textit{X} for each (os) \textit{U} in \textit{Y}.
2. totally -continuous is denoted by (t \textit{comm}) if [4] \textit{f}^{-1}(\textit{U}) is (cl-os) in \textit{X} for each (os) \textit{U} in \textit{Y}.
3. \textit{ic-} continuous is denoted by (\textit{ic-} \textit{comm}) if [1 ]\n
\( f^{-1}(U) \) is (\textit{ic-os}) in \( X \) for each (os) \( U \) in \( Y \).

\textbf{Theorem 1.2.} [1]

1. Each (os) in TS is (\textit{ic-os}).
2. Each (\textit{comm}) is (\textit{ic-comm}).

\textbf{2. Applications of \textit{ic-} Open Sets.}

\textbf{Definition 2.1.} Assume \( X \) be a TS and let \( H \subseteq X \). The \textit{ic-} interior of \( H \) is defined as the union of all (\textit{ic-} os) in \( X \) content in \( H \), and is denoted by \( \text{Int}_i(H) \). It is clear that \( \text{Int}_i(H) \) is (\textit{ic-os}) for any subset \( H \) of \( X \).

\textbf{Proposition 2.2.} Assume \((X, \tau)\) is a TS and if \( H \subseteq K \subseteq X \). Then
1. \( \text{Int}_i(H) \subseteq \text{Int}_i(K) \);
2. \( \text{Int}_i(H) \subseteq H \);
3. \( H \) is \textit{ic-} open iff \( H = \text{Int}_i(H) \).

\textbf{Definition 2.3.} Assume \((X, \tau)\) be a TS and let \( H \subseteq X \). The \textit{ic-} closer of \( H \) is defined as \( \text{Int}_i(H) \). The intersection of all (\textit{ic-} os) in \( X \) containing \( H \), and is denoted by \( \text{CL}_i(H) \). It is clear that \( \text{CL}_i(H) \) is (\textit{ic-os}) for any subset \( H \) of \( X \).

\textbf{Proposition 2.4.} Assume \((X, \tau)\) be a TS and if \( H \subseteq K \subseteq X \). Then
1. \( \text{CL}_i(H) \subseteq \text{CL}_i(K) \);
2. \( H \subseteq \text{CL}_i(H) \);
3. \( H \) is \textit{ic-} closed if and only if \( H = \text{CL}_i(H) \).

\textbf{Example 2.5.} If \( X = \{1, 3, 5\} \) and \( \tau = \{\emptyset, X, \{1\}, \{1, 3\}\} \) Then
\( \tau^i = \{\emptyset, X, \{1\}, \{1, 3\}\} \)
Let \( H = \{3\} \), \( K = \{1, 3\} \) and \( \{3\} \subseteq \{1, 3\} \subseteq X \). Then
1. \( \text{Int}_i(H) = \{3\} \subseteq \text{Int}_i(K) = \{1, 3\} \);
2. \( \text{Int}_i(H) = \{3\} \subseteq H = \{3\} \);
3. \( H \) is \textit{ic-} open if and only if \( H = \{3\} = \text{Int}_i(H) = \{3\} \).

\( C(\tau^i) = \{\emptyset, X, \{3\}, \{1, 5\}, \{5\}\} \)
Let \( H = \{3\} \), \( K = \{1, 3\} \) and \( \{3\} \subseteq \{1, 3\} \subseteq X \). Then
1. \( \text{CL}_i(H) = \{3\} \subseteq \text{CL}_i(K) = X \);
2. \( H = \{3\} \subseteq \text{CL}_i(H) = \{3\} \);
3. \( H = \{3\} \) is \textit{ic-} closed if and only if \( H = \{3\} = \text{CL}_i(H) = \{3\} \).

\textbf{Definition 2.6.} Let \( H \) be a subset of a TS \( X \). A point \( n \in X \) is named \textit{ic-} limit point of \( H \) if it satisfies the following assertion:
\( (\forall G \in \tau^i) (n \in G \Rightarrow G \cap (H(n)) \neq \emptyset) \)
The set of all \textit{ic-} limit points of \( H \) is named \textit{ic-derived} set of \( H \) and is denoted by \( \text{D}_i(H) \) Note that for a subset \( H \) of \( X \), a point \( n \in X \) is not \textit{ic-} limit point of \( H \) iff there exists (\textit{ic-} os) \( G \) in \( X \) s.t. \( n \in G \) & \( G \cap (H(n)) = \emptyset \) or equivalently,
\( n \in G \) and \( G \cap H = \emptyset \) or \( G \cap H = \{n\} \) or equivalently,
\( n \in G \) and \( G \cap H = \emptyset \) or \( G \cap H = \{n\} \)

\textbf{Theorem 2.7.} Let \( H \) be a subset \( X \), and \( n \in X \). Then the following are equivalent:
(1) \( (\forall G \in \tau^i) (n \in G \Rightarrow G \cap H \neq \emptyset) \)
(2) \( n \in \text{CL}_i(H) \)

\textbf{Proof.} (1) \( \Rightarrow \) (2) if \( n \notin \text{CL}_i(H) \), then there exists (\textit{ic-} os) \( F \) s.t. \( H \subseteq F \) and \( n \notin F \). Hence \( X \cap F \) is (\textit{ic-os}) containing \( n \) and \( H \cap (X \cap F) \subseteq H \cap (X \cap F) = \emptyset \). This is contradiction, and hence (2) is valid.

(2) \( \Rightarrow \) (1) straightforward. \( \blacksquare \)

\textbf{Theorem 2.8.} If \((X, \tau)\) be a TS and let \( A \subseteq B \subseteq X \). Then
1. \( \text{CL}_i(A) = A \cup D_i(A) \);
2. \( A \) is \textit{ic-closed} iff \( D_i(A) \subseteq A \);
3. \( D_i(A) \subseteq D_i(B) \);
4. \( D_i(A) \subseteq D(A) \);
5. \( \text{CL}_i(A) \subseteq \text{CL}(A) \);

\textbf{Proof.} Let \( n \notin \text{CL}_i(A) \). Then there is (\textit{ic-os}) \( F \) in \( X \) s.t. \( A \subseteq F \) and \( n \notin F \). Hence \( G = X \cap F \) is (\textit{ic-os}) \( n \notin G \) and \( G \cap A = \emptyset \). Therefore \( n \notin A \) and \( n \notin D_i(A) \) implies that there exists (\textit{ic-os}) \( G \) in \( X \) s.t. \( n \notin G \) and \( G \cap A = \emptyset \). Hence \( F = X \cap G \) is (\textit{ic-} os) \( n \notin F \) and \( n \notin F \). Hence \( n \notin D_i(A) \) thus \( \text{CL}_i(A) \subseteq A \cup D_i(A) \). Therefore \( \text{CL}_i(A) \subseteq A \cup D_i(A) \). \( \blacksquare \)

For (2), (3), (4) and (5) the proof is easy.

\textbf{Example 2.9.} Let \( X = \{1, 2, 3\} \) and \( \tau = \{\emptyset, X, \{1\}, \{1, 2\}\} \)
1. \( \tau \subseteq \tau^i = \{\emptyset, X, \{1\}, \{1, 2\}\} \)
2. \( n \subseteq D_i(H) \), then \( D(H) = \{3\} \) and \( D_i(H) = \emptyset \)
3. \( K = \{1, 2\} \), then \( D(K) = \{2, 3\} \) and \( D_i(K) = \{3\} \)

\textbf{Theorem 2.10.} Let \( \tau_1 \) and \( \tau_2 \) be topologic on \( X \). Then \( \tau_1^i \subseteq \tau_2^i \). For any subset \( H \) of \( X \), each \textit{ic-} limit point of \( H \) with respect to \( \tau_2 \) is \textit{ic-} limit point of \( H \) with respect to \( \tau_1 \).

\textbf{Proof.} Assume \( n \in \text{ic } - \text{lim point of } H \) with respect to \( \tau_2 \). Then \( G \cap (H(n)) \neq \emptyset \) for each \( G \in \tau_2^i \) s.t. \( n \in G \). But \( \tau_1^i \subseteq \tau_2^i \), so in particular, \( G \cap (H(n)) \neq \emptyset \) for each \( G \in \tau_1^i \). Hence \( n \) is \textit{ic-} limit point of \( H \) with respect to \( \tau_1 \). \( \blacksquare \)

\textbf{Theorem 2.11.} If \( H \) is a subset of a discrete topological space \( X \), then \( D_i(H) = \emptyset \)

\textbf{Proof.} Assume \( n \) be any element of \( X \). Recall that each subset of \( X \) is (os) and so (ic-os). In particular the singleton set \( G = \{n\} \) is (ic-os). But \( n \in G \) & \( G \cap H = \{n\} \cap H \subseteq \{n\} \). Hence \( n \) is not \textit{ic-} limit point of \( H \), and so \( D_i(H) = \emptyset \). \( \blacksquare \)
Theorem 2.12. Let $H$ and $K$ be subsets of $X$. If $H \in \tau^*$ and $\tau^*$ is a topology on $X$, then

$$H \cap \text{CL}_{ic}(K) \subseteq \text{CL}_{ic}(H \cap K).$$

Proof. Assume $n \in H \cap \text{CL}_{ic}(K)$. Then $n \in H$ and $n \in \text{CL}_{ic}(K) = K \cup D_c(K)$. If $n \in K$, then $n \in H \cap K \subseteq \text{CL}_{ic}(H \cap K)$. If $n \notin K$, then $n \in D_c(K)$ and so $G \cap K \neq \emptyset$ for all $(ic-\text{os}) G$ containing $n$. Since $H \in \tau^*$, $G \cap H$ is also $(ic-\text{os})$ containing $n$. Hence $G \cap (H \cap K) = (G \cap H) \cap K \neq \emptyset$, and consequently $n \in D_c(K \cap H) \subseteq \text{CL}_{ic}(H \cap K)$. Therefore $H \cap \text{CL}_{ic}(K) \subseteq \text{CL}_{ic}(H \cap K)$.

Definition 2.13. For any subset $H$ of $X$, the set $b_{ic}(H) = H \cap \text{Int}_{ic}(H)$ is called the $ic$- border of $H$.

Proposition 2.14. For a subset $A$ of a space $X$, the following statements hold:

1. $b_{ic}(A) \subset b(A)$ where $b(A)$ denotes the border of $A$.
2. $A = \text{Int}_{ic}(A) \cup b_{ic}(A)$.
3. $\text{Int}_{ic}(A) \cap b_{ic}(A) = \emptyset$.
4. $A$ is an $(ic)$-open set if and only if $b_{ic}(A) = \emptyset$.
5. $b_{ic}(\text{Int}_{ic}(A)) = \emptyset$.
6. $\text{Int}_{ic}(b_{ic}(A)) = \emptyset$.
7. $b_{ic}(b_{ic}(A)) = b_{ic}(A)$.

Proof. (1) Since $\text{Int}(A) \subset \text{Int}_{ic}(A)$, we have $b_{ic}(A) = A \setminus \text{Int}_{ic}(A) \subseteq A \setminus \text{Int}(A) = b(A)$.

(2) & (3). Straightforward.

(4) Assume $\text{Int}_{ic}(A) \subset A$, it follows from proposition 2.2 (3). That $A$ is $(ic)$-open $\iff A = \text{Int}_{ic}(A) \iff b_{ic}(A) = A \setminus \text{Int}_{ic}(A) = \emptyset$.

(5) Assume $\text{Int}_{ic}(A)$ is $(ic)$-open, it follows from (4) that $b_{ic}(\text{Int}_{ic}(A)) = \emptyset$.

(6) If $n \in \text{Int}_{ic}(b_{ic}(A))$, then $n \in b_{ic}(A)$. On the other hand, since $b_{ic}(A) \subset A$, $n \in \text{Int}_{ic}(b_{ic}(A)) \subset \text{Int}_{ic}(A)$. Hence, $n \in \text{Int}_{ic}(A) \cap (b_{ic}(A))$, which contradicts (3). Thus $\text{Int}_{ic}(b_{ic}(A)) = \emptyset$.

(7) Using (6), we get $b_{ic}(b_{ic}(A)) = b_{ic}(A \setminus \text{Int}_{ic}(b_{ic}(A))) = b_{ic}(A)$.

Example 2.15. From example 2.5. If $A = \{1, 5\}$ be a subset of $X$. Then $\text{Int}_{ic}(A) = \{1\}$, and so $b_{ic}(A) = A \setminus \text{Int}_{ic}(A) = \{1, 5\}$.

Proof. (1) Since $\text{CL}_{ic}(A) \subset \text{Cl}(A)$ and $\text{Int}(A) \subset \text{Int}_{ic}(A)$, it follows that $\text{Fr}_{ic}(A) = \text{CL}_{ic}(A) \setminus \text{Int}_{ic}(A)$, and $\text{Int}_{ic}(A) \subset \text{Cl}(A) \setminus \text{Int}_{ic}(A)$.

(2) $\text{Int}_{ic}(A) \cap \text{Fr}_{ic}(A) = \text{Int}_{ic}(A) \cap (\text{CL}_{ic}(A) \setminus \text{Int}_{ic}(A)) = \emptyset$.

(3) $\text{Int}_{ic}(A) \cap \text{Fr}_{ic}(A) = \text{Int}_{ic}(A) \cap (\text{CL}_{ic}(A) \setminus \text{Int}_{ic}(A)) = \emptyset$.

(4) If $A \subset \text{CL}_{ic}(A)$, we have $b_{ic}(A) = A \setminus \text{Int}_{ic}(A) \subset \text{CL}_{ic}(A) \setminus \text{Int}_{ic}(A) = \text{Fr}_{ic}(A)$.

(5) Since $\text{Int}_{ic}(A) \cup \text{Fr}_{ic}(A) = \text{Int}_{ic}(A) \cup b_{ic}(A) \cup D_c(A)$, $\text{Fr}_{ic}(A) = b_{ic}(A) \cup D_c(A)$.

(6) Assume that $A$ is $(ic)$-open. Then $\text{Fr}_{ic}(A) = b_{ic}(A) \cup D_c(A)$.

(7) $\text{Fr}_{ic}(A) = \text{CL}_{ic}(A) \setminus \text{Int}_{ic}(A) = \text{CL}_{ic}(A) \cap (\text{CL}_{ic}(X \setminus A))$.

(8) It follows from (7).
(9) $\text{CL}_e(A) \cap \text{CL}_o(A) \subseteq \text{CL}_e(X)$

(10) $\text{Fr}_e(A) \cap \text{CL}_o(A) \subseteq \text{CL}_e(X)$

(11) Since $\text{Int}_e(A) \cap \text{Int}_o(A) \subseteq \text{CL}_e(A)$, we get

(12) $\text{Fr}_e(A) \cap \text{CL}_o(A) \subseteq \text{CL}_e(A)$

(13) $\forall X \subseteq \text{Int}_e(A) \cap \text{Int}_o(A) = \text{Int}_e(A)$.

**Example 2.18.** Assume that the TS $(X, \tau)$ provided in Example 2.5, If $A = \{1, 2\}$ be a subset of $X$. Then

$\text{Int}_e(A) = \{1, 3\}$, and so $\text{Fr}_e(A) = \{1, 3\} \neq \emptyset$. Then $A = \{5\}$ is ic-closed, $\text{CL}_e(A) = \{5\}$ and thus $\text{Fr}_e(A) = \text{CL}_e(A)$.

**Theorem 2.19.** For a subset $H$ of $X$, $H$ is (ic-cs) iff $\text{Fr}_e(H) \subseteq H$.

**Proof.** Assume that $H$ is (ic-cs). Then $\text{Fr}_e(H) = \text{CL}_e(H) \cap \text{Int}_e(H) = H \cap \text{Int}_e(H) \subseteq H$.

Conversely suppose that $\text{Fr}_e(H) \subseteq H$. Then $\text{CL}_e(H) \cap \text{Int}_e(H) \subseteq H$, and so $\text{CL}_e(H) \subseteq H$. Since $\text{Int}_e(H) \subseteq H$, noticing that $H \subseteq \text{CL}_e(H)$, we have $H = \text{CL}_e(H)$. Therefore, $H$ is (ic-cs). ■

**Definition 2.20.** For a subset $H$ of $X$, $\text{Ext}_e(H) = \text{Int}_o(X \setminus H)$ is said to be an ic-exterior of $H$.

**Example 2.21.** Assume $(X, \tau)$ be a TS in Example 2.9. For subset $H = \{2\}$ and $K = \{1\}$ of $X$, we have $\text{Ext}_e(H) = \{1\}$ and $\text{Ext}_e(K) = \{2\}$.

**Proposition 2.22.** These propositions are true for a subset $A$ of a space $X$:

1. $\text{Ext}_e(A)$ is ic-open;
2. $\text{Ext}_o(X \setminus A) = X \setminus \text{CL}_e(A)$;
3. If $A \subseteq B$, then $\text{Ext}_e(A) \supseteq \text{Ext}_e(B)$;
4. $\text{Ext}_e(A \cup B) \subseteq \text{Ext}_e(A) \cup \text{Ext}_e(B)$;
5. $\text{Ext}_e(A \cap B) \supseteq \text{Ext}_e(A) \cap \text{Ext}_e(B)$;
6. $\text{Ext}_e(A) = \emptyset$;
7. $\text{Ext}_e(\emptyset) = X$;
8. $\text{Ext}_o(A) = \text{Ext}_o(X \setminus \text{Ext}_e(A));$
9. $X = \text{Int}_e(A) \cup \text{Ext}_e(A) \cup \text{Fr}_e(A)$.

**Proof.** (1) and (2) straightforward.

(3) Assume that $A \subseteq B$. Then $\text{Ext}_e(B) = \text{Int}_e(X) \cap \text{Ext}_e(A)$.

(4) $\text{Ext}_e(A \cup B) = \text{Int}_e(X) \cap \text{Ext}_e(A \cup B)$, $\text{Ext}_e((X \setminus A) \cap \text{Ext}_e(B)) = \text{Ext}_e(A) \cap \text{Ext}_e(B)$.

(5) $\text{Ext}_e(A \cap B) = \text{Int}_e(X) \cap \text{Ext}_e(A) \cup \text{Int}_e(X \setminus B)$.

(6) and (7) Straightforward.

(8) $\text{Ext}_e(X) = \text{Ext}_e(X \setminus \text{Ext}_e(A)) = \text{Int}_e(X) \setminus \text{Int}_e(A)$.

(9) Straightforward.

**Example 2.22.** Assume $\tau = \{1, 2, 3\}$ and $\tau = \emptyset, X, \{1\}, \{1, 2\}$ Then $\text{Ext}_e(H) = \{1\}$, $\text{Ext}_e(H) = \{2\}$, $\text{Ext}_e(H) = \{1\}$, $\text{Ext}_e(H) = \{2\}$, $\text{Ext}_e(H) = \{1\}$, $\text{Ext}_e(H) = \{2\}$.

(1) If $H = \{1\}$, $K = \{2\}$. Then $\text{Ext}_e(H) = \emptyset$.

(2) If $H = \{1\}$, $K = \{2\}$. Then $\text{Ext}_e(H \cap K) = \{1\}$.

3. **ic-Continuous Mappings and ic-Homeomorphism**

This section is devoted to introduce ic-open map, ic-irresolute map, ic-totally continuous map, ic-homeomorphism and discussed the relationships between the other known existing map.

**Definition 3.1.** A mapping $f : X \to Y$ is named ic-open denoted by (ic-om), if $f(U)$ is (ic-os) in $Y$ for each $(os) U$ in $X$.

**Example 3.2.** Let $X = Y = \{3, 5, 7\}$ and $\tau = \emptyset, X, \{3, 5\}$, $\sigma = \emptyset, Y, \{3, 5\}$ Then $\tau = \emptyset, Y, \{3, 5\}$. Clearly, the identity mapping $f : X \to Y$ is (ic-om).

**Proposition 3.3.** Any $(om)$ is (ic-om) but not conversely.

**Proof.** Assume $f : X \to Y$ be $(om)$ and $H$ be $(os)$ in $X$. Since, $f$ is open, then $f(H)$ is $(os)$ in $Y$. Since, each $(os)$ is (ic-os) then, $f(H)$ is (ic-os) in $Y$. Therefore, $f$ is (ic-om). ■
If $X = Y = \{1,2,3\}$ and $\tau = \emptyset, X, \{2\}, \{1,2\}$, $\sigma = \emptyset, Y, \{1\}, \{1,2\}$ Then $\tau^ic = \emptyset, Y, \{1\}, \{2\}, \{1,2\}$ Clearly, the identity mapping $f: X \to Y$ is (ic-om) but not (om).

**Theorem 3.4.** If $f: X \to Y$ is open & $g: Y \to Z$ is ic-open, then $gof: X \to Z$ is ic-open.

**Proof.** Suppose that $f: X \to Y$ be open & $g: Y \to Z$ is ic-open. Let $G$ be an (os) in $Y$. Since, $f$ is an open, then $f(G)$ is an (os) in $Y$. Since, each (os) is (ic-os), then $f(G)$ is (ic-os) in $Y$. Since, $g$ is (ic-os), then $(gof)^{-1}(G) = f(G)$ is (ic-os) in $Z$. Therefore, $gof: X \to Z$ is ic-open. ■

**Theorem 3.5.** If $f: X \to Y$ is (ic-comm) and $g: Y \to Z$ is (comm), then $gof: X \to Z$ is (ic-comm).

**Proof.** Assume $f: X \to Y$ be (ic-comm) & $g: Y \to Z$ is (comm). Let $G$ be an (os) in $Z$. Since, $g$ is (comm), then $g^{-1}(G)$ is an (os) in $Y$. Since, $f$ is (ic-comm), then $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is (ic-os) in $X$. Therefore, $gof: X \to Z$ is (ic-comm). ■

**Definition 3.6.** A mapping $f: X \to Y$ is named ic-irresolute is denoted by (ic-irem), if the inverse image of every (ic-os) of $Y$ is (ic-os) in $X$.

**Example 3.7.** If $X = Y = \{2,4,6\}$ and $\tau = \emptyset, X, \{2\}, \{2,4\}, \sigma = \emptyset, Y, \{2\}$ Then $\tau^{ic} = \emptyset, X, \{2\}, \{4\}, \{2,4\}, \sigma^{ic} = \emptyset, Y, \{2\}$ Clearly, the identity mapping $f: X \to Y$ is (ic-irem).

**Proposition 3.8.** Each (ic-irem) is (ic-comm).

**Proof.** Suppose that $f: X \to Y$ be (ic-irem) & $V$ any (os) in $Y$. Since each (os) is (ic-os) and since $f$ is ic-irresolute, then $f^{-1}(V)$ is (ic-os) in $X$. Therefore, $f$ is (ic-comm). ■

**Theorem 3.9.** Each (comm) is (ic-comm) but not conversely.

**Proof.** Suppose that $f: X \to Y$ be (comm) & $V$ any (ic-os) in $Y$. Since $f$ is (comm), then $f^{-1}(V)$ is (os) in $X$. Since each (os) is (ic-os), then $f^{-1}(V)$ is (ic-os) in $X$. Therefore, $f$ is (ic-comm).

Let $X = Y = \{2,4,6\}$ and $\tau = \emptyset, X, \{2\}, \{2,4\}, \sigma = \emptyset, Y, \{4\}$ Then $\tau^{ic} = \emptyset, X, \{2\}, \{4\}, \{2,4\}, \sigma^{ic} = \emptyset, Y, \{4\}$ Clearly, the identity mapping $f: X \to Y$ is (ic-comm) but not (comm).

**Theorem 3.10.** If $f: X \to Y$ is (ic-irem) & $g: Y \to Z$ is (ic-comm), then $gof: X \to Z$ is (ic-irem).

**Proof.** Let $f: X \to Y$ be (ic-irem) and $g: Y \to Z$ be (ic-comm). Let $U$ be an (os) in $Z$. Then $U$ is (ic-os) because each (os) is (ic-os). Since, $g$ is (ic-comm), then $g^{-1}(U)$ is (ic-os) in $Y$. Since, $f$ is (ic-irem), then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is (ic-os) in $X$. Therefore, $gof: X \to Z$ is (ic-irem). ■

**Theorem 3.11.** The composition of two (ic-irem) is also (ic-irem).

**Proof.** Assume $f: X \to Y$ & $g: Y \to Z$ any two (ic-irem). Suppose that $U$ be any (ic-os) in $Z$. Since, $g$ is (ic-irem), then $g^{-1}(U) = (ic-os)$ in $Y$. Since, $f$ is (ic-irem), then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is (ic-os) in $X$. Therefore, $gof: X \to Z$ is (ic-irem). ■

**Definition 3.12.** Let $X$ and $Y$ be $TS$, a bijective map $f: X \to Y$ is named ic-homeomorphism is denoted by (ic-homm) if $f$ is (ic-comm) and (ic-om).

**Theorem 3.13.** If $f: X \to Y$ is (homm), then $f$ is (ic-homm) but not conversely.

**Proof.** Since each (comm) is (ic-comm) by Theorem 1.2. (2). Also, since each (om) is (ic-om) by proposition (3.3). Further, since $f$ is bijective. Therefore, $f$ is (ic-homm). ■

Let $X = Y = \{1,2,3\}$ and $\tau = \emptyset, X, \{1\}, \{2,3\}, \sigma = \emptyset, Y, \{1\}, \{2\}$ Then $\tau^{ic} = \emptyset, X, \{1\}, \{3\}, \{2,3\}, \{1,3\}, \sigma^{ic} = \emptyset, Y, \{1\}, \{3\}, \{2\}, \{3\}$. Clearly, the identity mapping $f: X \to Y$ is (ic-homm) but not (homm).

**Definition 3.14.** A mapping $f: X \to Y$ is named ic-totally continuous is denoted by (ic-tcomm), if each reverse, (ic-os) of $Y$ is (cl-os) in $X$.

**Theorem 3.15.** Each (ic-tcomm) is totally continuous but not conversely.

**Proof.** Suppose that $f: X \to Y$ be (ic-tcomm) and $V$ be (os) in $Y$, since each (os) is (ic-os), then $V$ is (ic-os) in $Y$. Since $f$ is (ic-tcomm), then $f^{-1}(V)$ is (cl-os) in $X$. Therefore, $f$ is (ic-tcomm). ■

Let $X = Y = \{1,2,3\}$ and $\tau = \emptyset, X, \{1\}, \{2,3\}, \sigma = \emptyset, Y, \{2\}$ Then $\sigma^{ic} = \emptyset, Y, \{2\}$. Clearly, the identity mapping $f: X \to Y$ is (tcomm) but not (ic-tcomm).

**Theorem 3.16.** Each (ic-tcomm) is (ic-irem) but not conversely.

**Proof.** Assume that $f: X \to Y$ be (ic-tcomm) and $V$ be (ic-os) in $Y$. Since $f$ is (ic-tcomm), then $f^{-1}(V)$ is (cl-os) in $X$, which implies, $f^{-1}(V)$ is (os), it follows $f^{-1}(V)$ is (ic-os) in $X$. Therefore, $f$ is (ic-irem). ■

Let $X = Y = \{1,3,5\}$ and $\tau = \emptyset, X, \{1\}, \{1,3\}, \sigma = \emptyset, Y, \{3\}$ Then $\sigma^{ic} = \emptyset, Y, \{3\}$. Clearly, the identity mapping $f: X \to Y$ is (ic-irem) but not (ic-tcomm).

**Theorem 3.17.** The two’s (ic-tcomm) composition is also (ic-tcomm).

**Proof.** Suppose that $f: X \to Y$, $g: Y \to Z$ be any two (ic-tcomm). Assume $V$ be any (ic-os) in $Z$. Since, $g$ is (ic-tcomm), then $g^{-1}(V)$ is (cl-os) in $Y$, which implies $f^{-1}(V)$ is (os), it follows $f^{-1}(V)$ is (ic-os). Since, $f$ is (ic-tcomm), then $f^{-1}(g^{-1}(V)) =$
(gof)^{-1}(V) \text{ is } (cl-os) \text{ in } X. \text{ Therefore, } \\text{gof}: X \to Z \text{ is (ic-tcomm).} 
\textbf{Theorem 3.18.} If \ f: X \to Y \text{ be (ic-tcomm) and } g: Y \to Z \text{ be (ic-irrem), then } g \circ f: X \to Z \text{ is (ic-tcomm).} 
\textbf{Proof:} Assume that \ f: X \to Y \text{ be (ic-tcomm) and } g: Y \to Z \text{ is (ic-irrem). Let } V \text{ be (ic-os) in } Z. \text{ Since } g \text{ is (ic-irrem) then } g^{-1}(V) \text{ is (ic-os) in } Y. \text{ Since } f \text{ is (ic-tcomm), then } f^{-1}(g^{-1}(V)) = (gof)^{-1}(V) \text{ is (cl-os) in } X. \text{ Therefore, } g \circ f: X \to Z \text{ is (ic-tcomm).} 

\textbf{References} 

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