Abstract

The aim of this paper is to introduce a new classes of supra mappings called Intuitionistic Generalized Pre supra mapping, Intuitionistic Generalized Semi supra mapping, Intuitionistic Generalized α-supra mapping and Intuitionistic Generalized β-supra mapping. At last we studied some of their properties and investigate relationships among this concepts.

Definition 2.2 [5] Let $X \neq \emptyset$, and let $\bar{M} = \{x, M_1, M_2\}$, $\bar{N} = \{x, N_1, N_2\}$ are two Intuitionistic sets respectively. Also, let $\{\bar{M}_s; s \in S\}$ be a collection of "Intuitionistic sets in $X$", and $\bar{M}_1 = (x, M_1^{(1)}, M_2^{(1)})$, the following is valid.

1) $\bar{M} \subseteq \bar{N}$ iff $M_1 \subseteq N_1$ and $N_2 \subseteq M_2$.
2) $\bar{M} = \bar{N}$ iff $\bar{M} \subseteq \bar{N}$ and $\bar{N} \subseteq \bar{M}$.
3) The complement of $\bar{M}$ is denoted by $\bar{M}$ and defined by $\bar{M} = (x, M_2, M_1)$.
4) $\bar{M}_1 = (x, M_1^{(1)}, M_2^{(1)}) \cap M_2^{(2)}$, $\cap \bar{M}_1 = (x, M_1^{(1)}, M_2^{(2)})$.
5) $\emptyset = (x, \emptyset, X)$, $\bar{X} = (x, X, \emptyset)$.

Definition 2.3 [6] Let $X \neq \emptyset$, $w \in X$ and let $\bar{M} = (x, M_1, M_2)$ be an Intuitionistic set. The Intuitionistic point $(\bar{p}, f, \text{for briefly})$ of $w$ is defined by $\bar{w} = (x, \{w\}, \{w\})^{3}$ in $X$. The Is $w$ is said also $\bar{w}$ contains in $\bar{M}$ (briefly) iff $w \in M_1$, also $w \in M_2$.
\((\bar{w} \in \bar{M}, \text{ for short}) \text{ iff } w \in M_2.\)

**Definition 2.4 [5]** Let \(X, Y \neq \emptyset\), and \(r : (X, \mu) \rightarrow (Y, \nu)\) be a mapping.

a) If \(\bar{N} = (y, N_1, N_2)\) is an Is in \(Y\), then the inverse image of \(\bar{N}\) under \(r\) defined by 
\[
r^{-1}(\bar{N}) = (x, r^{-1}(N_1), r^{-1}(N_2)).
\]

b) If \(\bar{M} = (x, M_1, M_2)\) is an Is in \(X\), then \(r(\bar{M}) = (y, r(M_1), r(M_2))\) is an Is in \(Y\) where \(r(\bar{M}) = \left( r(M_2) \right) \).

**Definition 2.5 [7]** Let \(X \neq \emptyset\). An Intuitionistic topology \(\text{ITS, for short}\) on \(X\) is a collection \(\mu\) of an "Intuitionistic sets" in \(X\) satisfying:

1. \(\emptyset, X \in \mu.\)
2. \(\mu\) is closed under finite intersections.
3. \(\mu\) is closed under arbitrary unions.

Each element in \(\mu\) is called "Intuitionistic open set" and denoted by "IOS".

The complement of an "Intuitionistic open set" is called "Intuitionistic closed set" denoted by "ICS".

**Definition 2.6 [7]** Let \((X, \mu)\) be an ITS and let \(\bar{M} = (x, M_1, M_2) \subseteq X\). The "interior" (namely, \(\text{int}(\bar{M})\)) and the "closure" (namely, \(\text{cl}(\bar{M})\)) are defined:

\[
\text{int}(\bar{M}) = \bigcup \{ \bar{V} : \bar{V} \subseteq \bar{M}, \bar{V} \in \mu \},
\]

\[
\text{cl}(\bar{M}) = \bigcap \{ \bar{J} : \bar{J} \subseteq \bar{M}, \bar{J} \in \mu \}.
\]

1. \(\text{sint}(\bar{M}) = \bigcup \{ \bar{V} : \bar{V} \subseteq \bar{M}, \bar{V} \in \text{IOS} \},\)

2. \(\text{pint}(\bar{M}) = \bigcup \{ \bar{V} : \bar{V} \subseteq \bar{M}, \bar{V} \in \text{IPCS} \},\)

3. \(\alpha \text{int}(\bar{M}) = \bigcup \{ \bar{V} : \bar{V} \subseteq \bar{M}, \bar{V} \in \text{IcOX} \},\)

4. \(\beta \text{int}(\bar{M}) = \bigcup \{ \bar{V} : \bar{V} \subseteq \bar{M}, \bar{V} \in \text{IcOS} \},\)

\[
\text{beta}(\bar{M}) = \bigcap \{ \bar{J} : \bar{M} \subseteq \bar{J}, \bar{J} \in \text{IcS} \}.
\]

**Remark 2.7 [7]** This implications are valid:

\[
\text{sint}(\bar{M}) \subseteq \bar{M}, \text{ sccl}(\bar{M}) = \bar{M}, \text{ pint}(\bar{M}) \subseteq \bar{M}, \text{ pcl}(\bar{M}) = \bar{M},
\]

\[
\alpha \text{int}(\bar{M}) \subseteq \bar{M}, \alpha \text{cl}(\bar{M}) = \bar{M}, \beta \text{int}(\bar{M}) \subseteq \bar{M}, \beta \text{cl}(\bar{M}) = \bar{M}.
\]

**Definition 2.8. [8]** Let \((X, \mu)\) be an ITS. IS \(\bar{M}\) of \(X\) is said to be:

1. ISOS if \(\bar{M} \subseteq \text{iol}(\text{int}(\bar{M}))\),

2. IPOS if \(\bar{M} \subseteq \text{iol}(\text{int}(\bar{M}))\),

3. IcOS if \(\bar{M} \subseteq \text{iol}(\text{int}(\text{iocl}(\bar{M})))\),

4. IcOS if \(\bar{M} \subseteq \text{ioli}(\text{icocl}(\bar{M})))\).

The family of all intuitionistic semi-open, pre-open, \(\alpha\)-open and \(\beta\)-open sets of \((X, \mu)\) are denoted by "ISOS(X)". "IPoS(X)". "IcOS(X)" and "IcOS(X)" respectively. Also the complement of all intuitionistic semi-open, pre-open,, \(\alpha\)-open and \(\beta\)-open sets of \((X, \mu)\) are denoted by "ISCS(X)". "IPCS(X)". "IcS(X)" and "IcS(X)" respectively.

**Definition 2.9. [8]** Let \((X, \mu)\) be an ITS. An intuitionistic set \(\bar{M}\) of \(X\) is said to be:

1) Intuitionistic generalizes- open set (IGS, for short) if \(\forall U \subseteq \text{IcOS} \text{ s.t. } U \subseteq \bar{M}\).

2) Intuitionistic generalizes semi-open set (IGSS, for short) if \(\forall U \subseteq \text{IPCS} \text{ s.t. } U \subseteq \bar{M}\).

3) Intuitionistic generalizes open pre-set (IGP, for short) if \(\forall U \subseteq \text{IcOS} \text{ s.t. } U \subseteq \bar{M}\).

4) Intuitionistic generalizes \(\alpha\) -open set (IGS\(\alpha\), for short) if \(\forall U \subseteq \text{IcOS} \text{ s.t. } U \subseteq \bar{M}\).

5) Intuitionistic generalizes \(\beta\) -open set (IGS\(\beta\), for short) if \(\forall U \subseteq \bar{M}\).

**Definition 2.10. [8]** A map \(r : (X, \mu) \rightarrow (Y, \nu)\) is said to be:

1) Intuitionistic continuous if the pre-image \(f^{-1}(\bar{M})\) is IOS in \(X\) for every IOS \(\bar{M}\) in \(Y\).

2) Intuitionistic pre continuous if the pre image \(f^{-1}(\bar{M})\) is IPCS in \(X\) for every IOS \(\bar{M}\) in \(Y\).

3) Intuitionistic semi continuous if the pre image \(f^{-1}(\bar{M})\) is ISOS in \(X\) for every IOS \(\bar{M}\) in \(Y\).

4) Intuitionistic \(\alpha\)-continuous if the pre image \(f^{-1}(\bar{M})\) is IcOS in \(X\) for every IOS \(\bar{M}\) in \(Y\).

5) Intuitionistic \(\beta\)-continuous if the pre image \(f^{-1}(\bar{M})\) is IcOS in \(X\) for every IOS \(\bar{M}\) in \(Y\).

**Section 2 INTUITIONISTIC GENERALIZED PRE, SEMI, \(\beta\) & \(\alpha\) - of SUPRA MAPPINGS**

In this section we have introduced intuitionistic this concepts: generalized pre supra mapping, Intuitionistic generalized semi supra mapping, Intuitionistic generalized \(\beta\) supra mappings, Intuitionistic generalized \(\alpha\)-supra mapping and studied some from its properties.

**Definition 2.1:** A mapping \(r : (X, \mu) \rightarrow (Y, \nu)\) is an "Intuitionistic generalizer pre supra mapping" ("IgPs", for short) (resp., "Intuitionistic generalized semi supra mapping" ("IgSSm", for short), "Intuitionistic generalized \(\alpha\) supra mapping" (Igasm, for short), "Intuitionistic generalized \(\beta\) supra mapping" (IgBsm, for short)) if \(r^{-1}(\bar{M})\) is an "IGPOS" (resp., is an "IGSOS", "Ig\(\alpha\)OS", "Ig\(\beta\)OS")
Proposition 2.2: Let \( r: (E, \mu) \to (D, \gamma) \) and \( p: (D, \gamma) \to (I, \delta) \) be IgPs. Then \( p \circ r: (E, \mu) \to (I, \delta) \) is IgPs.

Proof: Let \( M \) be IgPOS in \( I \). Then \( p^{-1}(M) \) is IgPOS in \( D \), since \( r \) is IgPs, so \( r^{-1}(p^{-1}(M)) \) is IgPOS in \( E \). Therefore \( p \circ r \) is an IgPs.

Proposition 2.3: Let \( r: (E, \mu) \to (D, \gamma) \) and \( p: (D, \gamma) \to (I, \delta) \) be Igsm. Then \( p \circ r: (E, \mu) \to (I, \delta) \) is IgS in \( E \).

Proof: Let \( M \) be IgSOS in \( I \). Then \( p^{-1}(M) \) is IgSOS in \( D \), since \( r \) is Igsm, so \( r^{-1}(p^{-1}(M)) \) is IgS in \( E \). Therefore \( p \circ r \) is IgS.

Proposition 2.4: Let \( r: (E, \mu) \to (D, \gamma) \) be an IgP and \( p: (D, \gamma) \to (I, \delta) \) be IgP continuous suprema mapping, then \( p \circ r: (E, \mu) \to (I, \delta) \) is IgP continuous suprema mapping.

Proof: Let \( M \) be IOS in \( I \). Then \( p^{-1}(M) \) is IgPOS in \( Y \). Since \( r \) is IgPs, then \( r^{-1}(p^{-1}(M)) \) is IgPOS in \( X \). Therefore \( p \circ r \) is IgP continuous suprema mapping.

Proposition 2.5: Let \( r: (E, \mu) \to (D, \gamma) \) be an Igsm and \( p: (D, \gamma) \to (I, \delta) \) be IgS continuous mapping, then \( p \circ r: (E, \mu) \to (I, \delta) \) is IgS continuous mapping.

Proof: Let \( M \) be IOS in \( I \). Then \( p^{-1}(M) \) is an IgPOS in \( D \). Since \( r \) is Igsm, then \( r^{-1}(p^{-1}(M)) \) is IgS in \( E \) and every IgSOS is IgSOS. Hence \( p \circ r \) is IgS continuous mapping.

Proposition 2.6: Let \( r: (E, \mu) \to (D, \gamma) \) be an IgS and \( p: (D, \gamma) \to (I, \delta) \) be IgS continuous mapping, then \( p \circ r: (E, \mu) \to (I, \delta) \) is IgS continuous mapping.

Proof: It's obvious.

Proposition 2.7: Let \( r: (E, \mu) \to (D, \gamma) \) be IgPs and \( p: (D, \gamma) \to (I, \delta) \) be IgP continuous mapping, then \( p \circ r: (E, \mu) \to (I, \delta) \) is IgP continuous mapping.

Proof: It's obvious.

Proposition 2.8: Let \( r: (E, \mu) \to (D, \delta) \) be IgSm. Then this implications are equivalent:

(i) \( r^{-1}(M) \) is IgSOS in \( E \) for each IgSOS \( M \) in \( D \).
(ii) \( r^{-1}a\text{int}(M) \subseteq \text{int}r^{-1}(M) \) for every "Is M" of \( D \).
(iii) \( \text{acl}^{-1}(M) \subseteq r^{-1}\text{cl}(M) \) \( \forall " \text{Is M}" \) of \( D \).

Proof: (i) \( \Rightarrow \) (ii) Let \( M \) be \( \text{Is SOS} \) in \( D \) and \( \text{int}(M) \subseteq M \). Since \( \text{int}(M) \) is \( \text{IgOS} \) in \( D \), and every \( \text{IgSOS} \) is \( \text{IgSOS} \). So \( \text{IgOS} \) in \( D \). Therefore \( r^{-1}\text{int}(M) \) is \( \text{IgSOS} \) in \( E \). and \( r^{-1}\text{int}(M) \) is \( \text{IgSOS} \) in \( E \), since \( r^{-1}\text{int}(M) \subseteq r^{-1}\text{int}(M) \subseteq \text{int}r^{-1}(M) \).

(ii) \( \Rightarrow \) (iii) by taking complement of (ii) we get the result of (iii).
Proposition 2.14: Let \( r : (E, \mu) \to (D, \gamma) \) be IgSsm and \( p: (D, \gamma) \to (J, \delta) \) be Ig continuous mapping , then \( p \circ r : (E, \mu) \to (J, \gamma) \) is IgS continuous mapping .

Proof: Let \( \bar{M} \) be IOS in \( J \) . So that \( p^{-1}(\bar{M}) \) is IgSOS in \( D \) , since \( r \) is IgSsm , then \( r^{-1}(p^{-1}(\bar{M})) \) is IgSOS in \( E \) . Since every IgOS is IgSOS . Thus \( r^{-1}(p^{-1}(\bar{M})) \) is IgSOS in \( E \) . Therefore \( p \circ r \) is IgS continuous mapping .

Proposition 2.15: Let \( r : (E, \mu) \to (D, \gamma) \) be IgPsm and \( p: (D, \gamma) \to (J, \delta) \) be Ig continuous mapping , then \( p \circ r : (E, \mu) \to (J, \gamma) \) is IgS continuous mapping .

Proof: Let \( \bar{M} \) be IPOS in \( J \) . Thus \( p^{-1}(\bar{M}) \) is IgPOS in \( D \) , since \( r \) is IgSsm , then \( r^{-1}(p^{-1}(\bar{M})) \) is IgPOS in \( E \) . Since every IgPOS is IgSOS . Hence \( r^{-1}(p^{-1}(\bar{M})) \) is IgSOS in \( E \) . Therefore \( p \circ r \) is IgS continuous mapping .

Proposition 2.16: Let \( r : (E, \mu) \to (D, \gamma) \) be Igssm and \( p: (D, \gamma) \to (J, \delta) \) be Ig continuous mapping , then \( p \circ r : (E, \mu) \to (J, \gamma) \) is IgS continuous mapping .

Proof: it is obvious .

Proposition 2.17: Let \( r : (E, \mu) \to (D, \gamma) \) be IgSm and \( p: (D, \gamma) \to (J, \delta) \) be Ig continuous mapping , then \( p \circ r : (E, \mu) \to (J, \gamma) \) is IgS continuous mapping .

Proof: Let \( \bar{M} \) be Iisos in \( J \) . Thus \( p^{-1}(\bar{M}) \) is IgSOS in \( D \) , since \( r \) is IgSsm , then \( r^{-1}(p^{-1}(\bar{M})) \) is IgSOS in \( E \) . Since every IgSOS is IgSOS . Hence \( r^{-1}(p^{-1}(\bar{M})) \) is IgSOS in \( E \) . Therefore \( p \circ r \) is IgS continuous mapping .

Proposition 2.18: Let \( r : (E, \mu) \to (D, \gamma) \) be Igssm and \( p: (D, \gamma) \to (J, \delta) \) be Ig continuous mapping , then \( p \circ r : (E, \mu) \to (J, \gamma) \) is IgP continuous mapping .

Proof: it is obvious .

Proposition 2.19: Let \( r : (E, \mu) \to (D, \gamma) \) be Igssm and \( p: (D, \gamma) \to (J, \delta) \) be Ig continuous mapping , then \( p \circ r : (E, \mu) \to (J, \gamma) \) is IgS continuous mapping .

Proof: it is obvious .

Section 3 The RELATIONS AMONG INTUITIONISTIC GENERALIZED PRE SUPER MAPPING, INTUITIONISTIC GENERALIZED \( \beta - \) SUPER MAPPING, INTUITIONISTIC GENERALIZED SEMI SUPER MAPPING AND INTUITIONISTIC GENERALIZED \( \alpha - \) SUPER MAPPING .

Now, we give this important theorem .

Theorem 3.1: The implication among some types of mappings are given by the following diagram.

\[ \begin{align*}
\text{IgPsm} & \quad \text{Igssm} \\
\text{Igsm} & \quad \text{Igssm} \\
\text{Igsm} & \quad \text{Igsms} \\
\text{Igssm} & \quad \text{Igssm} \\
\text{Proof}: \text{IgPsm} & \quad \text{Igssm}
\end{align*} \]

Let \( r : (E, \mu) \to (D, \delta) \) be a mapping and \( \bar{M} \) be IPOS in \( D \) , since \( r \) is IgPsm , then \( r^{-1}(\bar{M}) \) is IgPOS in \( E \) . Since each IPO(Y) is IgO(Y) . Hence \( r^{-1}(\bar{M}) \) is IgSOS in \( E \) for each \( \bar{M} \) in IgSOS in \( D \) . Therefore \( r \) is Igssm .

\[ \begin{align*}
\text{Igsm} & \quad \text{Igssm} \\
\text{Igsm} & \quad \text{Igssm} \\
\text{Igsm} & \quad \text{Igssm} \\
\text{Igssm} & \quad \text{Igssm} \\
\text{Igsm} & \quad \text{Igssm}
\end{align*} \]

Example 3.3: Let \( E = \{a, m, n\} \) with topology \( \mu = \{E, \emptyset, \tilde{S}, \tilde{R}, \tilde{U}\} \) , where \( \tilde{S} = (e, \{w\}, \{z\}) \) , \( \tilde{R} = (e, \{w\}, \emptyset) \) , \( \tilde{U} = (e, \{w\}, \{z\}) \) and \( D = \{5, 6, 7\} \) with topology \( \delta = \{Y, \emptyset, W, Q\} \) , where \( W = (d, \{5\}, \{6, 7\}) \) , \( Q = (d, \{5, 6\}) \) . Let a mapping \( r : (E, \mu) \to (D, \delta) \) defined by \( r([a]) = \{5\} , r([m]) = \{7\} , r([n]) = \{6\} \) . Then

1- \( r \) is Igssm , because \( \forall \bar{M} \text{ be IgSOS in } D , r^{-1}(\bar{M}) \) is IgSOS in \( E \) . But \( r \) is not Igpsm , because \( r^{-1}(\{5, 7\}) = \{a, m\} \) is not IgPOS in \( E \) .

2- Also \( r \) is Igssm , because \( \forall \bar{M} \text{ be IgSOS in } D , r^{-1}(\bar{M}) \) is IgSOS in \( E \). But \( r \) is not IgSsm , because \( r^{-1}(\{6, 7\}) = \{m, n\} \) is not IgSOS in \( E \).
Example 3.4. Let $E = \{c, d, e\}$ with topology $\mu = \{\emptyset, \emptyset, M, N\}$, where $M = \{e, c\}, (d, e)\), $N = \{e, c\}, \emptyset\}$ and $D = \{1, 2, 3\}$ with topology $\delta = \{d, \emptyset, \emptyset, F\}$, where $\emptyset = (d, 1), \{3\}\), $F = (d, 1), \emptyset\). Let a mapping $r : (E, \mu) \to (D, \gamma)$ be $\{\emptyset, 0\}$, $\gamma = \{\emptyset, 0\}$, $\delta = \{\emptyset, \emptyset, 0\}$. Thus $r$ is IgPsm, because for each $\emptyset \in \mu$ be IgPsm in $D$, $r^{-1}(\emptyset)$ is IgPsm in $E$. But $r$ is not IgSm, because $r^{-1}(\{1, 3\}) = \{\emptyset, 0\}$ is not IgOS in $E$.

Example 3.5. Let $E = \{d, v, p, t\}$ with topology $\mu = \{\emptyset, \emptyset, B, J, X, R\}$, where $B = (e, d, v, p, t)$, $J = (e, \emptyset, \emptyset, v, p, t), N = (e, \emptyset, \emptyset, 1, 3, 5)$ with topology $\gamma = \{\emptyset, \emptyset, L, L, \emptyset\}$, where $L = (d, 1, \{3, 5\}), P = (d, 1, \emptyset)$. Let mapping $r : (E, \mu) \to (D, \gamma)$ be $\{\emptyset, 0\}$, $\gamma = \{\emptyset, 0\}$. Therefore $r$ is IgSm, because $\forall \emptyset \in \mu$ be IgSOS in $D$, $r^{-1}(\emptyset)$ is IgOS in $E$.

But $r$ is not IgOS, because $r^{-1}(\{3, 5\}) = \{p, v, t\}$ is not IgOS in $E$.

Example 3.6. Let $E = \{i, j\}$ with topology $\mu = \{\emptyset, \emptyset, M, N\}$, where $M = \{e, i, j\}, N = \{e, i, \emptyset\}$ and $D = \{3, 4\}$ with topology $\gamma = \{\emptyset, \emptyset, M, N\}$, where $\emptyset = (x, \emptyset, \{3\}), M = (x, \emptyset, \emptyset)$. Let a mapping $r : (E, \mu) \to (D, \gamma)$ be $\{\emptyset, 0\}$, $\gamma = \{\emptyset, 0\}$. So that $r$ is IgSm, because $\forall \emptyset \in \mu$ be IgOS in $D$, $r^{-1}(\emptyset)$ is IgOS in $E$.

Example 3.8. Let $E = \{w, r, i\}$ with topology $\mu = \{\emptyset, \emptyset, R, G, \emptyset, \emptyset\}$, where $R = \{e, \{w, r\}, \emptyset\}$. Therefore $r$ is IgSm, because $\forall \emptyset \in \mu$ be IgSOS in $D$, $r^{-1}(\emptyset)$ is IgOS in $E$. But $r$ is not IgPsm, because $r^{-1}(\{w, r, i\}) = \{\emptyset, 0\}$ is not IgOS in $E$.

Example 3.9. Let $E = \{o, p, u\}$ with topology $\mu = \{\emptyset, \emptyset, W, R, K, \emptyset, \emptyset, \emptyset\}$, where $W = (e, \{o, p, u\}), K = (e, \emptyset, \emptyset), R = (e, \emptyset, \emptyset)$. Therefore $r$ is IgSm, because $\forall \emptyset \in \mu$ be IgOS in $D$, $r^{-1}(\emptyset)$ is IgOS in $E$. But $r$ is not IgPsm, because $r^{-1}(\{4, 7\}) = \{\emptyset, u\}$ is not IgOS in $E$.

References

حول تعميم بعض الأشكال الضعيفة للتطبيقات الفوقية في الفضاءات التبولوجية الحدسية

سامر رعد ياسين١، هند فاضل عباس٢

١قسم الرياضيات، كلية علوم الحاسوب والرياضيات، جامعة تكريت، تكريت، العراق
٢متوسطة خالد ابن الوليد، مديرية تربية صلاح الدين، تكريت، العراق

الملخص

في هذا البحث قدمنا صفوف جديدة من التطبيقات اسميناها : intuitionistic generalized Pre supra mapping, intuitionistic generalized Semi supra mapping, intuitionistic generalized α-supra mapping , intuitionistic generalized β-supra mapping ودرسنا بعض خواصها. وأخيرا درسنا واستقصينا العلاقات بين هذه المفاهيم.