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# A Relation between $\pi$ Generalized Pre Connectedness and $\pi$ Generalized Supra Connectedness In Intuitionistic Fuzzy Topological Space

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# Abstract

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#### **1. Introduction**

In1965, the concept of " fuzzy set" was introduced by Zadeh in his classical paper [1]. Chang [2] introduced the concept of "fuzzy topological space" by using concept of fuzzy sets. After that, in 1986, Atanassov [3] introduced the " intuitionistic fuzzy sets" . in the 1997 Coker [4] defined the concept of "intuitionistic fuzzy topological spaces". Recently appeared many concepts of fuzzy topological such as fuzzy connectedness have been generalized in intuitionistic fuzzy topological spaces ,also S. Özça g and D. Coker [5] introduced the  $\pi$ -generalized Semi Connectedness in "Intuitionistic Fuzzy Topological Spaces connectedness" .In this paper we introduce concept of intuitionistic fuzzy  $\pi$  generalized pre connectedness with it relation intuitionistic fuzzy  $\pi$ generalized supra connectedness and investigates some them properties .

## 2. Preliminaries

**Definition 2.1:** [3] Let  $X \neq \emptyset$ . An "intuitionistic fuzzy set" ("IFS", in short) M in X is an object having the form  $M = \{\langle x, \sigma_M(x), \delta_M(x) \rangle : x \in X\}$  where the functions  $\sigma_M(x): X \rightarrow [0, 1]$  and

$$\begin{split} \delta_M(x): \, X &\to [0,1] \text{ denote the degree of membership} \\ (namely " \, \mu_A(x)") \text{ and the degree of non-} \end{split}$$

L he aim of this paper is to give the concepts of intuitionistic fuzzy  $\pi$  generalized pre connectedness with study its properties. Finally we study the relation between intuitionistic fuzzy  $\pi$  generalized pre connectedness and intuitionistic fuzzy  $\pi$  generalized supra connectedness in intuitionistic fuzzy topological space.

membership (namely " $\gamma_A(x)$ ")  $\forall x \in X$  to the set M, respectively, and  $0 \le \sigma_M(x) + \delta_M(x) \le 1$ 

 $\forall x \in X$ . Denote by "IFS(X)", the set of all "intuitionistic fuzzy sets" in X.

**Definition 2.2:** [7] Let M and N be "IFSs" of the form  $M = \langle x, \sigma_M, \delta_M \rangle$  and

 $N = \langle x, \sigma_N, \delta_N \rangle$ . Then

 $\begin{array}{ll} 1 \text{-} \ M \ \subseteq \ N \ \text{iff} \quad \sigma_M(x) \leq \sigma_N(x) \ \text{and} \ \delta_M(x) \geq \delta_N(x) \\ \forall \ x \in X. \end{array}$ 

- 2- M = N iff M  $\subseteq$  N and N  $\subseteq$  M.
- 3-  $M^c = \{ \langle x, \delta_M(x), \sigma_M(x) \rangle : x \in X \}.$
- 4-  $M \cap N = \{(x, \sigma_M(x) \land \sigma_N(x), \delta_M(x) \lor \delta_N(x)) : x \in X\}$ .

5- M ∪ N ={(x,  $\sigma_M(x)$  ∨  $\sigma_N(x)$ ,  $\delta_M(x) \land \delta_N(x)$ ): x ∈ X}.

6-  $\tilde{0} = \{(x,0,1): x \in X\}, \tilde{1} = \{(x,1,0): x \in X\}.$ we use this notation:

 $\begin{array}{ll} M &= \mbox{ } \langle x, (\sigma_M\,, \sigma_N), (\delta_M\,, \delta_N)\,\rangle & \mbox{instead} & \mbox{of} & M \\ \langle x, (M/\sigma_M\,, N/\sigma_M), (M/\delta_M\,, N/\delta_N)\,\rangle \,. \end{array}$ 

**Definition 2.3:** [3] Let  $X \neq \emptyset$ . An "intuitionistic fuzzy topology" ("IFT", for short) on X is a collection  $\rho$  of "IFSs" in X with this conditions:

1)  $\tilde{0}, \tilde{1} \in \rho$ .

2)  $j_1 \cap j_2 \in \rho$  for any  $j_1, j_2 \in \rho$ .

3)  $\cup j_i \in \rho$  for any arbitrary family  $\{G_i : i \in J\} \subseteq \rho$ . The pair  $(X, \rho)$  is called an "intuitionistic fuzzy topological space" ("IFTS", for short), any "IFS" in  $\rho$  is "intuitionistic fuzzy open set" ("IFOS", for short) in X. The complement M<sup>c</sup> of "IFOS M" in an "IFTS" (X,  $\rho$ ) is called

"intuitionistic fuzzy closed set" ("IFCS", for short) in X.

**Definition 2.4:** [3] Let  $(X, \rho)$  be an "IFTS" and  $M = \langle x, \sigma_M, \delta_M \rangle$  be "IFS" in X. Then :

 $Int(M) = \cup \{ J : J \text{ is IFOSin } X, J \subseteq M \},\$ 

 $Cl(M) = \cap \{ L : L \text{ is IFCSin } X, M \subseteq L \}.$ 

**Definition 2.5:** An "IFS" M

 $\{\langle x\,,\rho_M(x),\delta_M(x)\rangle {:}\, x\in X\}\,$  in an "IFTS"  $(X,\rho)$  is said to be an

1) [4] "intuitionistic fuzzy semi closed set" ("IFSCS", for short) if  $Int(Cl(M)) \subseteq M$ ,

2) [4] "intuitionistic fuzzy  $\alpha$ -closed set" ("If $\alpha$ CS", for short) if Cl(Int(Cl(M)))  $\subseteq$  M,

3) [4] "intuitionistic fuzzy pre-closed set" ("IFPCS", for short) if  $Cl(Int(M)) \subseteq M$ ,

4) [8] "intuitionistic fuzzy generalized closed set" ("IFGCS", for short) if  $Cl(M) \subseteq K$  whenever

 $M \subseteq K$  and K is an "IFOS",

5) [8] "intuitionistic fuzzy  $\pi$  open set" ("IF $\pi$ GSCS", for short) if sCl(M)  $\subseteq$  K and K is "IFO $\beta$ ",

6) [8] "intuitionistic fuzzy  $\pi$  open set" ("IF $\pi$ GSCS", for short) if pCl(M)  $\subseteq$  K, and K is "IFO $\beta$ ",

7) [8] "intuitionistic fuzzy generalized semi closed set" ("IF $\pi$ GSCS", for short) if sCl(M)  $\subseteq$  K, whenever M  $\subseteq$  K and K is an IF $\pi$ OS,

8) [8] "intuitionistic fuzzy generalized pre closed set" ("IF $\pi$ GPCS", for short) if pCl(M)  $\subseteq$  K, whenever

 $M \subseteq K$  and K is an "IF $\pi$ OP".

**Definition 2.6:** [8] An "IFS" M is said to be an "intuitionistic fuzzy  $\pi$  – generalized semi open set" ("IF $\pi$ GSOS", for short) in X if M<sup>c</sup> is "IF $\pi$ GSCS" in

X .The collection of "IF $\pi$ GSCSs" of "IFTS"

 $(X, \rho)$  is write by "IF $\pi$ GSC(X)".

**Result 2.7:**[8] Every "IFCS", "If $\alpha$ CS", "IFGCS", "IFPCS", "IF $\alpha$ GCS" is an "IF $\pi$ GSCS" but the reverse not true in generally.

**Definition 2.8:** [8] Let M be an "IFS" in an "IFTS"  $(X, \rho)$ . Then " $\pi$ -generalized semi closure" M (" $\pi$ gscl(M)", for short) and " $\pi$ -generalized Semi interior" of M (" $\pi$ gsint(M)", for short) are :

 $\begin{aligned} \pi gsInt(M) &= \cup \{ J : J \text{ is "IF} \pi GSOS" \text{ in } X, J \subseteq M \}, \\ \pi gscl(M) &= \cap \{ L : L \text{ is "IF} \pi GSCS" \text{ in } X , M \subseteq L \}. \end{aligned}$ 

**Definition 2.9:** [3] Let a map T:  $(M, \rho) \rightarrow (N, \gamma)$ . If  $N = \{ \langle w, \sigma_N(w), \delta_N(w) \rangle : w \in W \}$  is an "IFS" in W, then the image of N under T denoted by  $T^{-1}(N)$ , is the "IFS" in X defined by

$$\begin{split} T^{-1}(N) &= \left\{ \langle r \,, T^{-1}\big(\sigma_N(r)\big), T^{-1}\big(\delta_N(r)\big) \rangle : r \in R \right\} \text{ . If } \\ M &= \left\{ \langle r \,, \sigma_M(r), \delta_M(r) \rangle : r \in R \right\} \text{ is an "IFS" in } M, \end{split}$$

then the image of M under T denoted by T(M) is "IFS" in W where ,

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 $T(M) = \{ \langle w, T(\sigma_M(w)), \underline{T}(\delta_N(w)) \rangle : w \in W \}$ where  $T(\delta_B(w)) = 1 - T(1 - \delta_N) .$ 

**Definition 2.10:** [8] A map T:  $(X, \rho) \rightarrow (Y, \gamma)$  is "intuitionistic fuzzy  $\pi$  generalized semi irresolute" ("If $\pi$ GSir ,for short) if T<sup>-1</sup>(N) is "IF $\pi$ SGCS" in (X,  $\rho$ )  $\forall$  "IF $\pi$ GSCS" N of (Y,  $\gamma$ ).

**Definition 2.11:** [8] A map T:  $(X, \rho) \rightarrow (Y, \gamma)$  is "intuitionistic fuzzy  $\pi$  generalized semi continuous" ("IF $\pi$ GSC", for short) if T(M) is "IF $\pi$ SGOS" in  $(Y, \gamma) \forall$  "IF $\pi$ GSOS M " of  $(X, \rho)$ .

**Definition 2.12:** A map T:  $(X, \rho) \rightarrow (Y, \gamma)$  is

(a) [4] "intuitionistic fuzzy closed mapping" ("IFCM", for short) if T(M) is "IFCS" in  $Y \forall$  "IFCS" A in X.

(b) [4] "intuitionistic fuzzy  $\alpha$  -closed mapping" ("If $\alpha$ CM", for short) if T(M) is "If $\alpha$ CS" in Y  $\forall$  "IFCS" M in X.

**Definition 2.13:** [6] "IFTS"  $(X, \rho)$  is "IF $\pi T_{1/2}$ " space

if every "IFRWGCS" in X is "IFCS" in X, where

IFRWGCS is brief of Intuitionistic Fuzzy Regular Generalized Closed Set .

**Definition 2.14:** [6] "IFTS"  $(X, \rho)$  is "IF  $\pi g T_{1/2}$ " space if every "IFRWGCS" in X is "IFPCS" in X.

**Definition 2.15:**[5] "IFTS"  $(X, \rho)$  is said to be "intuitionistic fuzzy  $C_5$  - connected space" if

 $\tilde{0} \text{ and } \tilde{1} \text{ are both "IFOS" and "IFCS" only } .$ 

**Definition 2.16:**[5] "IFTS"  $(X, \rho)$  is "intuitionistic fuzzy GO-connected space" if  $\tilde{0}$  and  $\tilde{1}$ 

are both "IFGOS" and "IFGCS" only .

**Definition 2.17:**[5] "IFTS"  $(X, \rho)$  is an "intuitionistic fuzzy C<sub>5</sub>-connected" between "IFS" M, N if  $\nexists$ "IFOS" Q in  $(X, \rho)$  s.t M  $\subseteq$  Q and Q  $\subseteq$  N<sup>c</sup>.

**Definition 2.18:** [5] "IFTS"  $(X, \rho)$  is "IF $\pi$ GS connected space" if  $\tilde{0}$  and  $\tilde{1}$  are both IF $\pi$ GSOS" and "IF $\pi$ GSCS" only .

3. "Intuitionistic fuzzy  $\pi$  generalized pre connected spaces"

In this section, we have introduced "intuitionistic fuzzy  $\pi$  generalized pre connected" ("IF $\pi$ GP connected", for short) space and studied some of its properties.

**Definition 3.1:** An "intuitionistic fuzzy  $\pi$  – generalized pre open sets" ("IF $\pi$ GPOS", for short)(resp. "pre closed sets" ("IF $\pi$ GPCS", for short) in (X,  $\rho$ ) if its complement M<sup>c</sup> is "IF $\pi$ GPCS" in X (resp. if its complement M is "IF $\pi$ GPOS" in (X,  $\rho$ )). The collection of "IF $\pi$ GPOS" (resp. "IF $\pi$ GPCS") of "IFTS" (X,  $\rho$ ) is denoted by "IF $\pi$ GPO(X)" (resp. "IF $\pi$ GPC(X)").

**Definition 3.2:** "IFTS"  $(X, \rho)$  is "IF $\pi$ GP connected space" if  $\tilde{0}$  and  $\tilde{1}$  are both "IF $\pi$ GPOS" and "IF $\pi$ GPCS" only.

**Example 3.3:** Let  $W = \{m, n\}$  and  $\rho = \{\tilde{0}, K, \tilde{1}\}$ 

be "IFT" on X, where  $K = \langle w, (0.3, 0.2), (0.4, 0.2) \rangle$ . Then "IFTS" (X,  $\rho$ ) is "IF $\pi$ GP connected space" between the "IFS"

 $\begin{array}{ll} \mathsf{M} = \langle \mathsf{w}, (0.7, 0.1), (0.3, 0.3) \rangle & \text{and} & \mathsf{N} = \\ \langle \mathsf{w}, (0.1, 0.2), (0.5, 0.3) \rangle \end{array}$ 

**Proposition 3.4:** Each "IF $\pi$ GP connected space" is "IFC<sub>5</sub>-connected space"

**Proof:** Let  $(X, \rho)$  is "IF $\pi$ GP connected space". Suppose that  $(X, \rho)$  is not "intuitionistic fuzzy  $C_5$  – connected space", then  $\exists$  "IFS A" which is both "intuitionistic fuzzy open" and "intuitionistic fuzzy closed" in  $(X, \rho)$ . So M is "IF $\pi$ GPOS" and "IF $\pi$ GPCS" in  $(X, \rho)$ . Hence  $(X, \rho)$  is not an

"IF $\pi$ GP connected space". Thus we get a contradiction. Therefore (X,  $\rho$ ) is "intuitionistic fuzzy C<sub>5</sub> –connected space".

**Remark 3.5:**The converse of above proposition is not true . the example bellow shows the converse is not true .

Example 3.6: Let  $W=\{m,n\}$  ,  $\rho=\{\tilde{0},K,\tilde{1}\}$  be "IFT" on W , where

$$\begin{split} K &= \langle w, (0.2, 0.2), (0.1, 0.8) \rangle. \text{ Then } (X, \rho) \text{ is "IFC}_5\text{-} \\ \text{connected space" because the "intuitionistic fuzzy sets" } \tilde{0} \text{ and } \tilde{1} \text{ are both "IFOS" and "IFCS" , but not an "IF\piGP connected space", since the "IFS K" in $\rho$ is both an "IF\piGPOS" and an "IF\piGPCS" in $(X, \rho)$. \end{split}$$

**Proposition 3.7:** Let  $(X, \rho)$  is an "IFTS". Then  $(X, \rho)$  is "GO-connected" iff  $\forall$  "IF $\pi$ GP connected space". **Proof:**  $\Longrightarrow$  Let  $(X, \rho)$  is "IF $\pi$ GP connected space". suppose that  $(X, \rho)$  "IFGO-connected space", let  $\tilde{0}$  and  $\tilde{1}$  are both "IFGOS" and IFGCS ( $(X, \rho)$  is an "IFGO-connected space") since "IF $\pi$ GPOS" and "IF $\pi$ GPCS" are  $\tilde{0}$  and  $\tilde{1}$ . Hence $(X, \rho)$  is "IF $\pi$ GP connected space".

⇐Suppose (X, ρ) is not an "IFGO –connected space", then ∃ "IFS M" which is both "IFGOS" and "IFGCS" in (X, ρ). So M is both "IFπGPOS" and "IFπGPCS" in (X, ρ). Thus (X, ρ) is not

"IF $\pi$ GP connected space". Hence ,we get a contradiction. Thus  $(X, \rho)$  is "IFGO –connected space".

**Remark 3.8:** Let  $(X, \rho)$  is an "IFTS". The implications are valid :

IFπGP connected space

IFGO-connected space

#### <sup>™</sup>IFC<sub>5</sub>-connected space

Example 3.10: Let W = { m, n } and  $\rho = \{ \tilde{0}, K, \tilde{1} \}$  be "IFT" on X , where

 $K = \langle w, (0.6, 0.1), (0.2, 0.2) \rangle$ . Then "IFTS"  $(X, \rho)$  is IFC<sub>5</sub>-connected space between the "IFS"

 $M = \langle w, (0.5, 0.4), (0.4, 0.5) \rangle \text{ and } N = \langle w, (0.7, 0.3), (0.2, 0.6) \rangle, \text{ but it is not IF} \pi GP \text{ connected}$ 

space . Also  $(X, \rho)$  is IFGO-connected space but it is not IF $\pi$ GP connected space.

**Proposition 3.11:** The "IFTS"  $(X, \rho)$  is "IF $\pi$ GP connected space" iff  $\exists$  no non-zero

"IF $\pi$ GPOS" M and N in (X,  $\rho$ ) s.t M = N<sup>c</sup>.

**Proof:** Let M and N be two "IF $\pi$ GPOS" in (X,  $\rho$ ) such that M  $\neq \tilde{0}$ , N  $\neq \tilde{1}$  and M = N<sup>c</sup>. Therefore

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N<sup>c</sup> is an "IF $\pi$ GPCS". Since M  $\neq \tilde{0}$  N  $\neq \tilde{1}$ . So that N is "IFS" which is both "IF $\pi$ GPOS" and

"IF $\pi$ GPCS" in (X,  $\rho$ ). Thus (X,  $\rho$ ) is not "IF $\pi$ GP connected space", but this a contradiction .

Hence  $\exists$  no non-zero "IF $\pi$ GPOS" M and N in (X,  $\rho$ ) s.t M = N<sup>c</sup>. Let M be both

"IF $\pi$ GPOS" and "IF $\pi$ GPCS" in (X,  $\rho$ ) such that  $\tilde{0} \neq M \neq \tilde{1}$ . Now let N = M<sup>c</sup>. Then N is

"IF $\pi$ GPOS" and B  $\neq$  1~. Hence N<sup>c</sup> = M  $\neq$  0, and its contradiction . Therefore (X,  $\rho$ )

is "IF $\pi$ GP connected space".

**Proposition 3.12:** Let  $(X, \rho)$  be "IF $\pi$  T<sub>1/2</sub> " space, then this implications are equivalent:

(i)  $(X, \rho)$  is "IF $\pi$ GP connected space".

(ii) (X, ρ) is "IFGO-connected space".

(iii)  $(X, \rho)$  is "IFC<sub>5</sub>-connected space".

**Proof:** (i)  $\rightarrow$  (ii): By using Proposition 3.8, we get the result.

(ii)  $\rightarrow$  (iii): By using Remark 3.9, we get the result .

(iii)  $\rightarrow$  (i): Let  $(X, \rho)$  be "IFC<sub>5</sub>-connected space". Suppose  $(X, \rho)$  is not "IF $\pi$ GP

connected space", so  $\exists$  "IFS M " in (X,  $\rho$ ) which is both "IF $\pi$ GPOS" and "IF $\pi$ GPCS" in

(X,  $\rho$ ). But (X,  $\rho$ ) is an "IF $\pi$ a T<sub>1/2</sub> " space, M is both "IFO" and "IFC" in (X,  $\rho$ ). Thus (X,  $\rho$ )

not "IFC5-connected", and its contradiction . Hence  $(X,\rho)$  must be an" IF\piGP connected

space".

**Definition 3.13:** A map  $T: (X, \rho) \rightarrow (Y, \gamma)$  is "intuitionistic fuzzy  $\pi$  generalized pre irresolute" ("IF $\pi$ GPir", for brief) if  $T^{-1}(N)$  is "IF $\pi$ PGCS" in  $(X, \rho) \forall$  "IF $\pi$ GPCS N" of  $(Y, \gamma)$ .

**Proposition 3.14:** If T:  $(X, \rho) \rightarrow (Y, \gamma)$  is "IF $\pi$ GP continuous" and  $(X, \rho)$  is an "IF $\pi$ GP connected space", then $(Y, \gamma)$  is "IFC5 connected space".

**Proof:** Let  $(X, \rho)$  be "IF $\pi$ GP connected space". Suppose that  $(Y, \gamma)$  is not "intuitionistic fuzzy C<sub>5</sub>-

connected space", then  $\exists$  "IFS M" which is both "IFO" and "IFC" in  $(Y, \gamma)$ . Since T is "If $\pi$ GP

continuous mapping", so  $T^{-1}(M)$  is "IF $\pi$ GPOS" and "IF $\pi$ GPCS" in (X,  $\rho$ ), but it is a contradiction.

Therefore  $(Y, \gamma)$  is IFC<sub>5</sub>-connected space.

**Proposition 3.15:** If  $T: (X, \rho) \rightarrow (Y, \gamma)$  is "IF $\pi$ GPir" and  $(X, \rho)$  is an "IF $\pi$ GP connected space", then  $(Y, \gamma)$  is "IF $\pi$ GP connected space".

**Proof:** Suppose that  $(Y, \gamma)$  is not "IF $\pi$ GP connected space", so  $\exists$  "IFS M" s.t M is both

"IF\piGPOS" and "IF\piGPCS" in  $(Y,\gamma)$  . Since T is "IF\piGP irresolute mapping",  $T^{-1}(M)$  is both

"IF $\pi$ GPOS" and "IF $\pi$ GPCS" in (X,  $\rho$ ), we get a contradiction. Therefore (Y,  $\gamma$ ) is

an "IF $\pi$ GP connected space".

**Definition 3.16:** "IFTS"  $(X, \rho)$  is "IF $\pi$ GP connected" between "IFS" M and N if  $\nexists$  "IF $\pi$ GPOS K" in  $(X, \rho)$  s.t M  $\subseteq$  K and K  $\subseteq$  M<sup>c</sup>.

**Example 3.17:** Let  $W = \{ m, n \}$  and  $\rho = \{ \tilde{0}, K, \tilde{1} \}$  be "IFT" on X, where

 $K = \langle w, (0.3, 0.3), (0.4, 0.5) \rangle$ . Then "IFTS" (X,  $\rho$ ) is "IF $\pi$ GP connected" between the "IFS"

 $M = \langle w, (0.6, 0.2), (0.6, 0.4) \rangle \text{ and } N = \langle w, (0.6, 0.2), (0.3, 0.3) \rangle \text{ where }$ 

 $M^{c} = \langle w, (0.06, 0.4), (0.6, 0.2) \rangle$ .

**Proposition 3.18:** An "IFTS"  $(X, \rho)$  is "IF $\pi$ GP connected" between two "IFSs" M and N iff there is no "IF $\pi$ GPOS" and "IF $\pi$ GPCS K" in  $(X, \rho)$  s.t  $M \subseteq K \subseteq N^{c}$ .

**Proof:** Let  $(X, \rho)$  be "IF $\pi$ GP connected" between M and N. Suppose that  $\exists$  an "IF $\pi$ GPOS"

and "IF $\pi$ GPCS K " in  $(X, \rho)$  s.t M  $\subseteq$  K  $\subseteq$  N<sup>c</sup>, then N  $\subseteq$  K<sup>c</sup> and M  $\subseteq$  K. So  $(X, \rho)$ 

not "IF $\pi GP$  connected" between M and N, , so we get a contradiction .

Therefore  $\exists$  no "IF $\pi$ GPOS" and an "IF $\pi$ GPCS K" in  $(X, \rho)$  s.t  $M \subseteq K \subseteq N^c$ .

Conversely suppose that  $(X, \rho)$  is not "IF $\pi$ GP connected" between M and N. So  $\exists$  "IF $\pi$ GPOS

K" in  $(X, \rho)$  s.t M  $\subseteq$  K and K  $\subseteq$  N<sup>c</sup>. So that  $\exists$  an "IF $\pi$ GPOS K" in  $(X, \rho)$  s.t M  $\subseteq$  K  $\subseteq$  N<sup>c</sup>, but we

get a contradiction. Thus  $(X, \rho)$  is "IF $\pi$ GP connected" between M and N .

**Proposition 3.19:** Let  $(X, \rho)$  be an "IFTS" and M and N be "IFS" in  $(X, \rho)$ . Then,  $M \subseteq N$ , iff X is an "IF $\pi$ GP connected" between M and N.

**Proof:** Suppose  $(X, \rho)$  is not "IF $\pi$ GP connected" between M and N. Then  $\exists$  an "IF $\pi$ GPOS K" in  $(X, \rho)$  s.t M  $\subseteq$  K and K  $\subseteq$  N<sup>c</sup>. So that M  $\subseteq$  N<sup>c</sup>, but we get a contradiction (M  $\subseteq$  N). Therefore X is an "IF $\pi$ GP connected" between M, N.

Conversely suppose  $(X, \rho)$  is an "IF $\pi$ GP connected" between M, N.Thus M = N (by definition of

"IF $\pi$ GPOS connected" ). Therefore, M  $\subseteq$  N.

4-The Relations between  $\pi$  Generalized Pre Connectedness and  $\pi$  Generalized supra connectedness in "IFTS"

**Definition 4-1:** An "intuitionistic fuzzy  $\pi$  –generalized supra open sets" ("IF $\pi$ GSUOS", for short)(resp. "supra closed sets" ("IF $\pi$ GSUCS", for short) in (X,  $\rho$ ) if its complement M<sup>c</sup> is

"IF $\pi$ GSUCS" in X (resp. if its complement M is "IF $\pi$ GSUOS" in (X,  $\rho$ )). The collection of

"IF $\pi$ GSUOS" (resp. "IF $\pi$ GSUCS") of "IFTS" (X,  $\rho$ ) is denoted by "IF $\pi$ GSUO(X)" (resp.

"IF $\pi$ GSUC(X)").

**Proposition 4.2:** The relation among  $\pi$  Generalized Pre Connectedness and  $\pi$  Generalized supra connectedness of intuitionistic fuzzy connectedness is given in the following diagram.



#### **Proof** :

IF $\pi$ GS connected space IFC<sub>5</sub>-connected space by definition of "IFC<sub>5</sub>-connected space" and Remark 3.8, we get the result .

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**Remark 4.3:** By transitivity we get  $IF\pi GSU$ 

**Remark 4.4:**The converse of above proposition are not true . the counter examples bellow shows the converse are not true .

**Example 4.5:** Recall Example 3.6, we get  $(X, \rho)$  is an IFC5-connected space because the

"IFS" Õand Ĩ are both "IFOS" and "IFCS", but not an "IF $\pi$ GP connected space", because the "IFS K" in  $\rho$  is both an "IF $\pi$ GPOS" and an "IF $\pi$ GPCS" in (X,  $\rho$ ).

**Example 4.6:** Let W = {m, n, k} and  $\rho = \{ \tilde{0}, K, \tilde{1} \}$  be "IFT" on W, where

$$\begin{split} M &= \langle w, (0.7, 0.3), (0.5, 0.2), (0.2, 0.7) \rangle & . & \text{Then} \\ (X, \rho) \text{ is an "IF}\pi GP \text{ connected space" because the} \\ \text{both "IF}\pi GPOS" \text{ and "IF}\pi GPCS" \text{ are } \tilde{0} \text{ and } \tilde{1} \text{ , but} \\ \text{not an "IF}\pi GSU \text{ connected space", because the "IFS} \\ M" \text{ in } \rho \text{ is not "IF}\pi GSUOS" \text{ and "IF}\pi GSUCS" in \\ (X, \rho) . \end{split}$$

**Example 4.7:** Let  $W = \{m, n, k, h\}$  and  $\rho = \{\tilde{0}, K, \tilde{1}\}$  be an "IFT" on W, where

 $K = \langle w, (0.6, 0.6), (0.3, 0.2), (0.3, 0.5), (0.3, 0.3) \rangle .$ 

Then  $(X, \rho)$  is an "IF $\pi$ GSU connected space" because the both "IF $\pi$ GSUOS" and "IF $\pi$ GSUCS" are  $\tilde{0}$  and  $\tilde{1}$ , but not an "IF $\pi$ GP connected space", because the "IFS K" in  $\rho$  is not an "IF $\pi$ GPOS" and an "IF $\pi$ GPCS" in  $(X, \rho)$ .

**Example 4.8:** Let  $W = \{m, n, k, h\}$  and  $\rho = \{\tilde{0}, K, \tilde{1}\}$  be "IFT" on W, where

 $K = \langle w, (0.1, 0.6), (0.2, 0.6), (0.3, 0.3), (0.5, 0.6) \rangle$ .

Then  $(X, \rho)$  is an "IFC<sub>5</sub>-connected space" because the IFS  $\tilde{0}$  and  $\tilde{1}$  are both "IFOS" and "IFCS", but not "IF $\pi$ GS connected space", since the "IFS K" in  $\rho$  is not an "IF $\pi$ GSUOS" and an "IF $\pi$ GSUCS" in  $(X, \rho)$ .

**Proposition 4.9:** Let  $(X, \rho)$  be "  $IF\pi T_{1/2}$  space", then these relations are given as:



IFGO-connected

IF $\pi$ GP connected space  $\longrightarrow$  IFC<sub>5</sub>-connected space : its prove by transitivity .

IF $\pi$ GSU connected space space : By using Proposition 4.2 we get the result . IF $\pi$ GSU connected space  $\longrightarrow$  IF $\pi$ GP connected space

Suppose  $(X, \rho)$  be "IF $\pi$  T<sub>1/2</sub> " space and  $(X, \rho)$  is "IF $\pi$ GSU connected space" so  $\exists$ 

"IFGOS" and "IFGCS" are  $\tilde{0}$  and  $\tilde{1}$  in  $(X,\rho).$  Since  $\tilde{0}$  and  $\tilde{1}$  are

both "IFGPOS" and "IFGSUCS" in  $(X, \rho)$  because  $(X, \rho)$  be an" IF $\pi a T_{1/2}$ ". Thus  $(X, \rho)$  is "IF $\pi GP$ 

connected space".

**Remark 4.10:**The converse of above proposition are not true . The counter examples

bellow shows the converse are not true .

**Example 4.11:** Let  $W = \{m, n, k\}$  and  $\rho = \rho = \{\tilde{0}, K, \tilde{1}\}$  be "IFT" on W, where

 $K = \langle x, (0.3, 0.7), (0.3, 0.5), (0.2, 0.1), (0.7, 0.3) \rangle$ 

Then  $(X, \rho)$  is an "IFGO-connected" because the "IFS"  $\tilde{0}$  and  $\tilde{1}$ are both "IFOS" and "IFCS", but not an "IF $\pi$ GSU connected space", since the "IFS K" in  $\rho$  is not an "IF $\pi$ GSUOS" and an "IF $\pi$ GSUCS" in  $(X, \rho)$ . Also  $(X, \rho)$  is not an "IF $\pi$ GP connected", since the "IFS K" in  $\rho$  is not an "IF $\pi$ GPOS" and an "IF $\pi$ GPCS" in  $(X, \rho)$ .

**Example 4.12:** Let  $W = \{m, n, k\}$  and  $\rho = \{\tilde{0}, K, \tilde{1}\}$  be "IFT" on W, where

 $K = \langle w, (0.5, 0.6), (0.4, 0.2), (0.5, 0.2), (0.3, 0.7) \rangle$ 

Then  $(X,\rho)$  is an "IFC<sub>5</sub>-connected space" because the "IFS"  $\tilde{0}$  and  $\tilde{1}$  are both "IFOS" and "IFCS", but not "IFGO-connected space", because  $\tilde{0}$  and  $\tilde{1}$  are not "IFOS" and "IFCS" in  $(X,\rho)$ .

#### References

[1] Zadeh, L. A.," Fuzzy sets", Information and control, 8, 338-353, 1965.

[2] Chang C.L, "Fuzzy topological spaces", J. Math. Anal. Appl, 24(1968), 182-190.

[3] Atanassov, K., "Intuitionistic fuzzy sets, Fuzzy Sets and Systems", 20 (1986), 87-96.

[4] Coker D. "An introduction to intuitionistic fuzzy topological spaces", Fuzzy sets and systems, 88(1997), 81-89.

[5] Coker D. and S. Özça<sup>\*</sup>g ., "On connectedness in intuitionistic fuzzy special topological

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**Example 4.13:** Let  $W = \{m, n, k, w\}$  and  $\rho = \{\tilde{0}, K, \tilde{1}\}$  be an "IFT" on W, where K =

 $\langle w, (0.6, 0.4), (0.5, 0.1), (0.4, 0.3), (0.4, 0.6), (0.8, 0.2) \rangle$ . Then (X,  $\rho$ ) is "IF $\pi$ GP connected space"

because the "IFS"  $\tilde{0}$  and  $\tilde{1}$  are both "IFPOS" and "IFPCS", but not "IF $\pi$ GSU connected space",

because  $\tilde{0}$  and  $\tilde{1}$  are not "IFSUOS" and "IFSUCS" in  $(X, \rho)$ .

**Proposition 4.14**: For any "IFTS"  $(X, \rho)$ , we have every "IFOS", "IFSUOS", is an "IF $\pi$ GPOS".

**Proof:** "IFOS"  $\rightarrow$  "IF $\pi$ GPOS" :

Suppose that  $(X, \rho)$  is "IFTS" and  $M \subseteq X$ . Since every "IFOS" is "IFPOS" also every "IFPOS" is "IFGPOS" in  $(X, \rho)$ , thus  $A^c$  is an "IFGPCS" in X, so that  $N^c$  is an "IF $\pi$ GPCS" in X. Thus  $(X, \rho)$  is "IF $\pi$ GPCS".

"IFSOS"  $\rightarrow$  "IF $\pi$ GPOS" : it's clear.

**Remark 4.15:**The converse of above Proposition is not true . the examples bellow shows the converse are not true .

**Example 4.16:** Let  $W = \{m, n\}$  and  $H = \langle x, (0.7, 0.1), (0.5, 0.7) \rangle$ . Then  $\rho = \{\tilde{0}, Q, \tilde{1}\}$  is

in W.

**Example 4.17**: Let  $W = \{r, e\}$  and  $S = \langle w, (0.1, 0.9), (0.9, 0.1) \rangle$ . Then  $\rho = \{\tilde{0}, Q, \tilde{1}\}$  is "IFT" on W. So "IFS

 $M'' = \langle w, (0.2, 0.1), (0.8, 0.2) \rangle$  is "IF $\pi$ GPOS" but not "IFOS" in W.

spaces" Int. J. Math. Math. Sci. 21 (1998), no. 1, 33-40.

[6] Coker D. and A. H. E.s, "On fuzzy compactness in intuitionistic fuzzy topological spaces", J.

Fuzzy Math. 3 (1995), no. 4, 899–909.

[7] Coker D. and M. Demirci, "On intuitionistic fuzzy points", Notes IFS 1 (1995), no. 2, 79–84.

[8] Sarsak, M.S., and Rajesh, N., " $\pi$  – Generalized Semi – Pre closed Sets", International Mathematical Forum, 5 (2010), 73-578.

# العلاقة بين الترابط من النوع Pre – π والترابط من النوع Supra–π في الفضاءات التبولوجية المضبية الحدسية

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#### الملخص

ان الهدف من هذا البحث هو تقديم مفاهيم جديدة للترابط المعمم من النوع pre – π في الفضاء التبولوجي المضبب الحدسي ودراسة بعض خواصها. واخيرا درسنا العلاقة بين الترابط من النوع pre – π مع الترابط شبه المعمم من النوع semi–π في الفضاء التبولوجي المضبب الحدسي.