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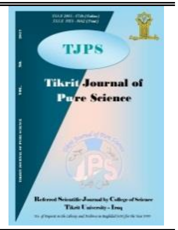

New traveling wave solutions of a nonlinear diffusion–convection equation by using standard tanh method

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Abstract

Exact solutions of traveling wave are acquired by employ a relatively new technique which is called standard tanh method for a nonlinear diffusion–convection equation. The tanh method is applied for the first time for finding travelling wave solutions for the nonlinear diffusion–convection equation $u_t = (u^m)_{xx} + (u^n)_x$, where n and m, are integers, and $n \geq m > 1$, of order (m=4, n=7) and (m=5, n=9). Analytical solutions of a nonlinear diffusion–convection equations are obtained as a polynomial in tanh(x), and the plots for exact solutions are given. The obtained results are compared with F-Expansion method to validate the proposed approach.

1-Introduction

A nonlinear partial differential equations (NLPDEs) have been vastly used to solve many complex physics phenomena accord in different fields of science, which appear in many disciplinaries such as engineering, mathematic, plasma physics, optical, biology, and solid-state physics. Large number of reaches has been conducted researches to find a powerful analytic solution of non-linear PDEs. Some of the proposed solutions have special form in which the solution depends on a single combination of variables. For example, travailing wave variables in travelling wave solution of non-linear PDEs.

numerous methods to find exact solution of nonlinear PDFs, have been suggested in the literature like: the tanh-coth method [1, 2], sine-cosine method [3], homogeneous balance method [4,5], exp-function method [6], first-integral method [7], Jacobi elliptic function method [8], and (G'/G)-Expansion method [9,10,11]. The standard method offered by Malfliet [12] is a strong technique to compute exact traveling waves solutions of non-linear partial differential equations, we will utilize the standard tanh method to find the traveling wave for a nonlinear diffusion–convection equation for different values of m and n [13, 14, 15].

$$\frac{\partial u}{\partial t} = \frac{\partial (u)^n}{\partial x} + \frac{\partial^2 (u^m)}{\partial x^2}, \quad (1)$$

This equation arises in many physical phenomena for different values of n and . For example, Equation (1) turn into classic Burgers' equation [16, 17] which has many applications in gas dynamics, traffic flow, and shock wave phenomena. Eq.1, a arises in the theory of infiltration of water under gravity during a homogeneous and isotropic porous medium for $n \geq m > 1$ [18]. Also, Eq. 1 the flow of a thin viscous sheet models on an inclined bed [19].

The paper is coordinated as follows: in part 2, the summary of the standard method are given. In part 3, the application of the standard method for a nonlinear diffusion–convection equation of order $(m = 4, n = 7)$ and $(m = 5, n = 9)$ are given. In the last part, the conclusions" are given.

2- Summary of the standard tanh method

In this section, we describe the standard tanh method for finding traveling wave solutions of non-linear PDEs. The main step in this method is to use an independent variable to convert the non-linear PDEs to an ordinary differential equation (ODEs) which may or may not be integrable and in the case of integrable ODEs the integration constant will be neglected.

Suppose that a non-linear PDEs in two independent variable x and t is presented by the following formula:

$$P(u, u_t, u_x, u_{xt}, u_{tt}, u_{xx}, \dots) = 0 \quad (2)$$

Where P is a polynomial in $u=(x,)$, and $u=(x,t)$ is an unknown function dependent on x and t variables and it's various partial derivatives. The process of finding traveling wave solution using the tanh method can be illustrated in the following six steps:

Step 1: Looking for travelling wave solutions in equation (2), we assume that the wave variable:

$$u(x,t) = u(\zeta), \zeta = (x - ct) \quad (3)$$

The constant c is termed the wave velocity. By substituting equation (3) into equation (2) we find the following ordinary differential equations (ODEs) in ζ (which illustrates a principal advantage of a travelling wave solution, i.e., a PDE is reduced to ODE):

$$P(u, c\,u', cu'', c^2u'', u'', \dots) = 0 \quad (4)$$

Step 2: In sometimes, we integrate equation (4) and set the constants of integration to be zero for simplicity.

Step 3: We suppose that equation (4) has the following formal solution:

$$u(\zeta) = s(Y) = a0 + \sum_{i=1}^{m} a_i\, y^i, \quad (5)$$

Where m is a positive integer, and $a0$, and, ai are constants, while Y is given by:

$$Y = \tanh(\zeta) \quad (6)$$

The independent variable (6) leads to the following derivatives:

$$\frac{d}{d\zeta} = (1-y^2)\frac{d}{dy} \quad (7)$$

$$\frac{d^2}{d\zeta^2} = (1-y^2)\left\{(1-y^2)\frac{d^2}{dy^2} - 2y\frac{d}{dy}\right\} \quad (8)$$

$$\frac{d^3}{d\zeta^3} = 2(1-y^2)(3y^2-1)\frac{d}{dy} - 6y(1-y^2)^2\frac{d^2}{dy^2} + (1-y^2)^3\frac{d^3}{dy^3} \quad (9)$$

And so on.

Step 4: Usually the positive integer m can be determined by making a balance between the linear terms of highest order derivative and non-linear terms appearing in (4).

Step 5: Solving these presented algebraic equations by using a symbolic computation system such as Maple and Mathematica, we get the values of $a0$, ai and c.

Step 6: Substituting these values into equations (5) and (3), we can obtain the exact travelling wave solutions of equation (2).

3. Applications of The Standard Tanh Method

Considering $m = 4$ and $n = 7$ in equation (1), we get

$$u_t = (u^4)_{xx} + (u^7)_x \quad (10)$$

We can get the traveling wave solutions of (1), by using the transformations

$$u(x,t) = u(\zeta), \quad \zeta = (x - ct) \quad (11)$$

Substituting (11) into (10), we obtain the following ordinary differential equations (ODE):

$$cu' - 4(3)u^2(u')^2 - 4u^3u'' - 7u^6u' \quad (12)$$

where prime denotes differentiation with respect to ζ. Balancing the orders of u' and u^3u'' in equation (12), we obtained integer $m = \frac{1}{3}$

Now, we use the transformation $u(\zeta) = w^{\frac{1}{3}}(\zeta)$ in equation (12)

We get the ODE in the following form.

$$cww' - 7w^3w' - 4w^2w'' - \frac{4}{3}ww'^2 = 0 \quad (13)$$

According to homogeneous balance between ww' and w'' in equation (13), we have $m = 1$

then, we can write solutions of equation (10) in the form

$$u(\zeta) = a_0 + a_1 y \quad (14)$$

Where m is a positive integer, and a_0 and a_1 are constants. Substituting equation (14) into equation (13), with computerized symbolic computation, equating to zero the coefficients of all power y^i (i=1,2,3,4,5), we derive a set of algebraic equations for a_0, a_1 and c

y^1: $7{a_1}^3 - \frac{28}{3}{a_1}^2 = 0$,

y^2: $14a_0{a_1}^2 - 8a_0a_1 = 0$,

y^3: $-ca_1 + 7{a_0}^2a_1 - 7{a_1}^3 + \frac{32}{3}{a_1}^2 = 0$,

y^4: $-14a_0{a_1}^2 + 8a_0a_1 = 0$,

y^5: $ca_1 - 7{a_0}^2a_1 - \frac{4}{3}{a_1}^2 = 0$.

Solving this system by software of Maple, we have the following two sets of solutions:

Case 1:

$a_o = a_o$, $a_1 = 0$, $c = c$

Case 2:

$a_o = 0$, $a_1 = \frac{4}{3}$, $c = \frac{16}{9}$

In view of this, we obtain the following solitons solution:

$$u_1(x,t) = \left(\frac{4}{3}\tanh\left(x - \frac{16}{9}t\right)\right)^{\frac{1}{3}} \quad (15)$$

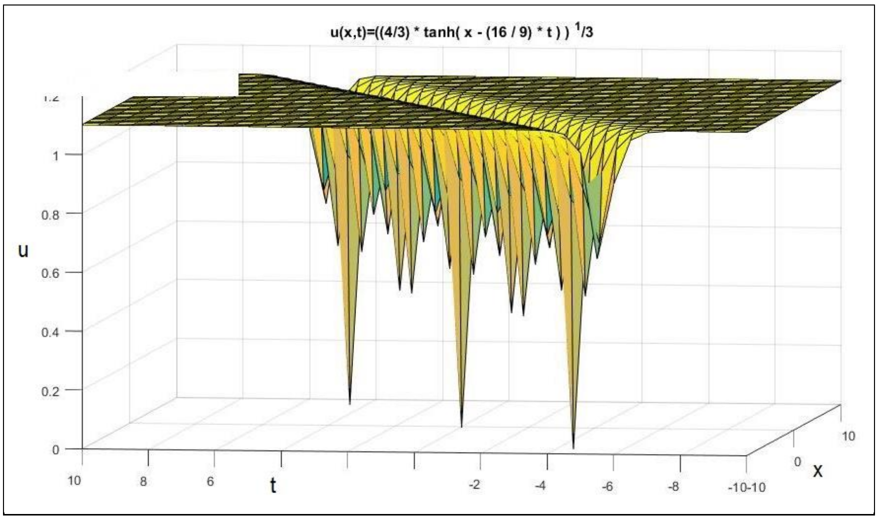

Figure 1 The soliton solution of $u_1(x,t)$ for $-10 \leq x \leq 10$, and $-10 \leq t \leq 10$.

Finally, we would like to obtain the traveling wave solution of equation (2), when $m = 5$ and $n = 9$

$$u_t = (u^5)_{xx} + (u^9)_x \quad (16)$$

Using the wave transformation

$$u(x,t) = u(\zeta),\ \zeta = (x - ct) \quad (17)$$

Substituting equation (17) into equation (16), we obtain the following ordinary differential equations (ODE):

$$cu' = 20u^3 (u')^2 + 5u^4u'' + 9u^8u' \quad (18)$$

Considering the positive integer m can be determined by making a balance between the linear terms of highest order derivative and non-linear terms of u' and u^4u'' in equation (18), gives $m = \frac{1}{4}$

Now, we use the transformation $u(\zeta) = w^{\frac{1}{4}}(\zeta)$ in to equation (18)

We get the ODE in the following form.

$$cww' - 9w^3w' - 5w^2w'' - \frac{5}{4}ww'^2 = 0 \quad (19)$$

According to homogeneous balance between ww' and w'' in equation (19) we get $m = 1$

Therefore, the solution of (19) can be written in the form

$$u(\zeta) = a_0 + a_1 y \quad (20)$$

Where a_0,a_1 and c are constants which are unknown.

Substituting (20) into (19), collecting all terms with the same powers of y^i and setting each coefficient to zero, we obtain a system of algebraic equations for a_0,a_1 and c as follows.

y^1: $9a_1^3 - \frac{45}{4}a_1^2 = 0$,

y^2: $18a_0a_1^2 - 10a_0a_1 = 0$,

y^3: $-ca_1 + 9a_0^2a_1 - 9a_1^3 + \frac{25}{2}a_1^2 = 0$,

y^4: $-18a_0a_1^2 + 10a_0a_1 = 0$,

y^5: $= ca_1 - 9a_0^2a_1 - \frac{5}{4}a_1^2 = 0$.

Solving this system by Maple gives

Case 1:

$a_o = a_o$, $a_1 = 0$, $c = c$

Case 2:

$a_o = 0$, $a_1 = \frac{5}{4}$, $c = \frac{25}{16}$

The solution formulae of equation (18) can be written as following:

$$u_2(x,t) = (\frac{5}{4} \tanh\left(x - \frac{26}{16}t\right))^{\frac{1}{4}} \quad (21)$$

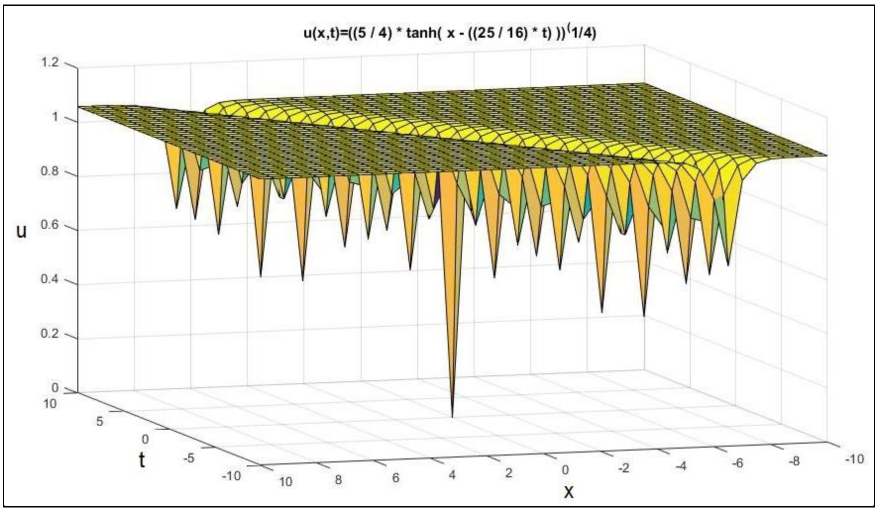

Figure 2 The soliton solution of $u_2(x,t)$ for $-10 \leq x \leq 10$, and $-10 \leq t \leq 10$.

4. Conclusion

In this paper, standard tanh method has been successfully used to obtain traveling wave solutions of the nonlinear diffusion–convection equation, when $m = 4$ and $n = 7$, and $m = 5$ and $n = 9$. Also, comparison was made between the solution of the standard tanh method and F-Expansion method under special conditions. We observe that our results represented by equations (15) and (21) are similar to the results obtained by F-Expansion method if the condition h=1, C=-1, and B=2 in equation (46) [15]. The closed–form solution obtained via this method is in good agreement with the F-Expansion method. The solution of standard tanh method gives many-soliton solutions for nonlinear partial differential equations. In addition, the solutions contain free parameters. These solutions will be very useful in various physical situations. In the end, we provided numerical simulations to complete the study.

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حلول جديده للموجات المتنقلة لمعادلة الحمل بالانتشار الغير خطية باستخدام طريقة الظل الزائدي

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الملخص

تم الحصول على حلول جديدة للموجات المتنقلة بواسطة طريقة الظل الزائدي القياسية لمعادلة الحمل بالانتشار الغير خطية. طريقة الظل الزائدي طبقت لاول مرة لمعادلة الحمل بالانتشار الغير خطية عندما (n, m) اعداد صحيحة حيث كانت رتبة $n = 7, m = 4$ وكذلك رتبة $n = 9, m = 5$ وحصلنا على الحلول الصريحة للمعادلة بشكل متعددة الحلول $\tanh x$ وتم مقارنة النتائج مع اسلوب التوسيع F

الكلمات المفتاحية: طريقة الظل الزائدي القياسية، الموجات المتنقلة، معادلة الحمل بالانتشار.