# On Semi- regular $\mathrm{T}_{1}$ and Semi- regular $\mathrm{T}_{\mathbf{2}}$ in Intuitionistic Fuzzy Topological Spaces 

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#### Abstract

The purpose of this paper is to give the new definitions of semi-regular $T_{1}$ and semi-regular $T_{2}$ separation axioms in intuitionistic fuzzy topological spaces. Study the basic properties, characterizations and relationships of these new concepts in intuitionistic fuzzy topological spaces.


Key words: fuzzy set, intuitionistic fuzzy topology, semi-regular $\mathrm{T}_{1}$, semi-regular $\mathrm{T}_{2}$.

## 1. Introduction

After the introduction of fuzzy sets by Zadeh [1], Atanassov in 1983 [2,3] introduced the notion of "intuitionistic fuzzy set " (IFS for short). Using intuitionistic fuzzy sets, Coker [5] introduced the notion of "intuitionistic fuzzy topological spaces. In this paper, we introduce new notions of semi-regular $\mathrm{T}_{1}$ and semi-regular $\mathrm{T}_{2}$ separation axioms in intuitionistic fuzzy topological spaces.

## 2. Preliminaries

The concept of " intuitionistic fuzzy set " (IFS for short) was introduced by Atanassov as an object of the form $\left.A=<x, A_{1}, A_{2}\right\rangle$, where $A_{1}$ and $A_{2}$ are subset of a nonempty fixed set $X$, satisfying the following $\mathrm{A}_{1} \cap \mathrm{~A}_{2}=\varnothing$. Every subset of a nonempty set of IFS having the form $\left\langle\mathrm{x}, \mathrm{A}, \mathrm{A}^{\mathrm{c}}\right\rangle$. Some Boolean algebra operations on IFS is defined by Coker [5] as follows:Let A, B be IF'S where $A=\left\langle x, A_{1}, A_{2}>, B=<x\right.$, $B_{1}, B_{2}>$ belong to a non-empty set $X$ and $\{A i ́: i \in J\}$ be an arbitrary family of IFS in X where $A \hat{i}=<x, \breve{A}_{1}$, $\breve{A}_{2}>$, then :-
$\mathrm{A} \subseteq \mathrm{B} \stackrel{\mathrm{A}_{1} \subseteq \mathrm{~B}_{1} \Lambda \mathrm{~A}_{2} \supseteq \mathrm{~B}_{2} ; ~}{\text {; }}$
$\mathrm{A}=\mathrm{B} \Leftrightarrow \mathrm{A} \subseteq \mathrm{B} \quad \Lambda \mathrm{B} \subseteq \mathrm{A}$;
$\mathrm{A}^{\mathrm{c}}=\left\langle\mathrm{x}, \mathrm{A}_{2}, \mathrm{~A}_{1}\right\rangle$
UAí $\left.=<x, \bigcup_{A_{1}}, \cap \breve{A}_{2}\right\rangle$,
$\cap A i ́=\left\langle x, \cap \check{A}_{1}, \bigcup \breve{A}_{2}\right\rangle$.
$\widetilde{\varnothing}=\langle x, \varnothing, X\rangle, \tilde{X}_{=}=\langle x, X, \varnothing\rangle$.
The an intuitionistic fuzzy topology (IFT for short) on a nonempty set X is a family $\mathcal{T}$ of IF's in X containing $\widetilde{\mathscr{D}}, \tilde{X}$ and closed under finite intersection and arbitrary union, in this case the pair ( $\mathrm{X}, \mathcal{T}$ ) is called an intuitionistic fuzzy topological space (IFTS for short).
Now let A be any IF'S in (X, $\mathcal{T}$ ), then A said to be intuitionistic fuzzy regular (semi) open set ((IFROS), IFSOS for short) if $\mathrm{A}=\operatorname{Int}(\mathrm{ClA})(\mathrm{A} \subseteq \mathrm{CL}(\operatorname{Int} \mathrm{A}))$ and called intuitionistic fuzzy regular (semi)closed set (IFRCS), IFSCS for short) if $\mathrm{A}=\mathrm{Cl}(\operatorname{Int} \mathrm{A})(\mathrm{A} \subseteq \mathrm{CL}$ $(\operatorname{IntA})$ ), when the interior and closure of an IFS A are defined by ;

$$
\begin{aligned}
& \text { Int } \mathrm{A}=\cup\{\mathrm{G}: \mathrm{G} \in \mathcal{T}, \mathrm{G} \subseteq \mathrm{~A}\} \\
& \mathrm{Cl} \mathrm{~A}=\cap\{\mathrm{K}: 1-\mathrm{K} \in \mathcal{T}, \mathrm{~A} \subseteq \mathrm{~K}\}
\end{aligned}
$$

Any IF'S in $\mathcal{T}$ is known an intuitionistic fuzzy open set (IFOS for short) in X. The IF'S $\bar{p}=\left\langle\mathrm{x}, \mathrm{p},\{\mathrm{P}\}^{\mathrm{c}}\right\rangle^{\rangle}$is
called intuitionistic fuzzy point in X. The IF'S $\bar{p}$ is said to be contained in A if $\left(\mathrm{P} \in \mathrm{A}_{1}\right.$ and $\mathrm{P} \notin \mathrm{A}_{2}$, and the set $\overline{\bar{p}}_{=}\left\langle\mathrm{x}, \emptyset,\{\mathrm{P}\}^{\mathrm{c}}>\right.$ is called vanishing Intuitionistic point in X (VIP for short).
2. Some Forms of Semi-regular $T_{1}$ Separation axioms:
In this section, we introduce some new form of the separation axioms namely semi-regular $\mathrm{T}_{1}\left(\mathrm{SRT}_{1}\right.$ for short) in IFTS, we give a definition of semi-regular and semi-regular $T_{1}$ and some of it's properties and relations with each other.
Definition 2.1: Let $(\mathrm{X}, \mathcal{T})$ be an IFTS, A subset A of X is said to be semi-regular if A is both semi open and semi closed [5].
The set of all semi-regular sets of $X$ is denoted by $\operatorname{SR}(X)$, the intersection of all semi-regular sets of $X$ containing A is called the semi-regular closure of A and denoted by $\operatorname{SRCL}(\mathrm{A})$ and the union of all semiregular sets of X contained in A is called the semiregular interior of A and denoted by $\operatorname{SRI}(\mathrm{A})$.
Definition 2.2: Let $(\mathrm{X}, \mathcal{T})$ be an IFTS, than $(\mathrm{X}, \mathcal{T})$ is said to be :-

1. $\mathbf{S R T}_{1}(\mathbf{i} \mathbf{)}$ if for each $x, y \in X, x \neq y, \exists U, V \in$ $\mathrm{SR}(\mathrm{X})$ s.t $\bar{x} \in \mathrm{U}, \bar{y} \notin \mathrm{U}$ and $\bar{y} \in \mathrm{~V}, \bar{x} \notin \mathrm{~V}$.
2. SRT $_{1}$ (íí) if for each $x, y \in X, x \neq y, \exists U, V \in$ $\operatorname{SR}(X)$ s.t $\overline{\bar{x}} \in U,{ }^{\bar{y}} \notin \mathrm{U}$ and ${ }^{\overline{\bar{y}}} \in \mathrm{~V}, \overline{\bar{x}} \notin \bar{x} \in \mathrm{~V}$.
3. $\mathbf{S R T}_{1}$ (íií) if for each $x, y \in X, x \neq y, \exists U, V \in$ $\operatorname{SR}(\mathrm{X})$ s.t $\bar{X} \in \mathrm{U} \subseteq \bar{Y}^{\mathrm{c}}$ and $\bar{y} \in \mathrm{~V} \subseteq \tilde{X}^{\mathrm{c}}$.
4. $\mathbf{S R T}_{1}$ (iv) if for each $x, y \in X, x \neq y, \exists U, V \in$ $\operatorname{SR}(\mathrm{X})$ s.t $\overline{\bar{x}} \in \mathrm{U} \subseteq \overline{\bar{Y}}^{\mathrm{c}}$ and ${ }^{\overline{\bar{y}}} \in \mathrm{~V} \subseteq \overline{\bar{X}}^{\mathrm{c}}$.
5. $\mathbf{S R T}_{\mathbf{1}}(V)$ if for each $x, y \in X, x \neq y, \exists U, V \in$ $\mathrm{SR}(\mathrm{X})$ s.t y $\notin \mathrm{V}$ and $\bar{x} \notin \mathrm{~V}$.
6.SRT $\mathbf{1}_{\mathbf{1}}(\mathrm{Vi})$ if for each $\mathrm{x}, \mathrm{y} \in \mathrm{X}, \quad \mathrm{x} \neq \mathrm{y}, \exists \mathrm{U}, \mathrm{V}$ $\in \operatorname{SR}(X)$ s.t $\bar{y}^{\mathrm{c}} \notin \mathrm{U}$ and $\overline{\bar{x}} \notin \mathrm{~V}$.
The following theorem appears in [4] for IFOS without proof, we generalize it for SR sets and give it here with proof.

Theorem 2.3 : Let ( $\mathrm{X}, \mathcal{T}$ ) be an IFTS, then the following implication are valid.

$\mathrm{SRT}_{1}$ (iií) $\mathrm{SRT}_{1}$ (ív)
Proof: To prove $\mathrm{SRT}_{1}(\mathrm{ví}) \rightarrow \mathrm{SRT}_{1}(\mathrm{v}):-$
Let $x, y \in X, x \neq y, \quad$ since $\operatorname{SRT}_{1}(v i ́)$ hold so there exists $\mathrm{U}, \mathrm{V} \in \mathrm{SR}(\mathrm{X})$ s.t $\overline{\bar{y}} \notin \mathrm{U}$ and $\overline{\bar{x}} \notin \mathrm{~V}$, this implies that $y \in u_{2}$ and $x \in V_{2}$, Since $u_{1} \cap u_{2}=\emptyset$ and $v_{1} \cap v_{2}=$ $\emptyset$, we get $y \notin u_{1}$ and $x \notin V_{1}$, therefore
$\bar{x} \notin \mathrm{~V}$ and $\bar{y} \notin \mathrm{U}$ so $\mathrm{SRT}_{1}(\mathrm{v})$ holds.
To prove $\mathrm{SRT}_{1}(\mathrm{i}) \rightarrow \mathrm{SRT}_{1}(\mathrm{v}):-$
Let $x, y \in X$. Since $\operatorname{SRT}_{1}(\hat{i})$ hold, so there exists $U$, $\mathrm{V} \in \mathrm{SR}(\mathrm{X})$ s.t $\bar{x} \in \mathrm{U}, \bar{y} \notin \mathrm{U}$ and $\mathrm{y} \in \mathrm{V}$, $\mathrm{x} \notin \mathrm{U}$, this implies that $\bar{x} \notin \mathrm{U}$ and $\bar{y} \in \mathrm{~V}, \bar{x} \notin \mathrm{~V}, \mathrm{x} \notin \mathrm{V}$ and $\bar{y}$ $\notin \mathrm{U}$, therefore $\mathrm{SRT}_{1}(\mathrm{v})$ hold.
In order to prove $\operatorname{SRT}_{1}($ ií $) \rightarrow \operatorname{SRT}_{1}(v i ́)$, take $x, y \in$ $X, x \neq y$. Since $\operatorname{SRT}_{1}($ ií $)$ hold, so there exists $U, V \in$ $\mathrm{SR}(\mathrm{X})$ s.t $\overline{\bar{x}} \in \mathrm{U}, \overline{\bar{y}}_{\notin \mathrm{U} \text { and }}{ }^{\overline{\bar{y}}} \in \mathrm{~V}, \overline{\bar{x}} \notin \bar{x}, \in \mathrm{~V}$. From this we have $\bar{X} \notin \mathrm{~V}$ and ${ }^{\bar{y}} \notin \mathrm{U}$, therefore $\mathrm{SRT}_{1}(\mathrm{ví})$ hold.
$\mathrm{SRT}_{1}(\mathrm{i})+\mathrm{SRT}_{1}(\mathrm{i} \mathbf{i}) \longrightarrow \mathrm{SRT}_{1}(\mathrm{i})$ and
$\mathrm{SRT}_{1}(\mathrm{i})+\mathrm{SRT}_{1}(\mathrm{ii}) \longrightarrow \mathrm{SRT}_{1}(\mathrm{ii})$ is direct.
To prove $\mathrm{SRT}_{1}(\mathrm{i})+\mathrm{SRT}_{1}(\mathrm{i} i ́) \rightarrow \mathrm{SRT}_{1}($ iií $):-$
Let $x, y \in X, x \neq y$. Since $\operatorname{SRT}_{1}(i) \quad \& \operatorname{SRT}_{1}(i i)$ hold so $\exists \mathrm{U}, \mathrm{V} \in \mathrm{SR}(\mathrm{X})$ s.t $\bar{x} \in \mathrm{U}, \bar{y} \in \mathrm{~V}, \bar{x} \notin \mathrm{~V}$ and $\bar{y} \notin \mathrm{U}$, so $\overline{\bar{x}} \in \mathrm{U},{ }^{\bar{y}} \notin \mathrm{U}$ and ${ }^{\overline{\bar{y}}} \in \mathrm{~V}, \overline{\bar{x}} \notin \bar{x} \in \mathrm{~V}$.

First we har to prove :-
$\bar{x} \in \mathrm{U} \subseteq \bar{Y}^{\mathrm{c}}$ and $\bar{y} \in \mathrm{~V} \subseteq \bar{X}^{\mathrm{c}}$, we have from assumption $\bar{x} \in \mathrm{U}$ and $\bar{y} \in \mathrm{~V}$.

To prove $\mathrm{U} \subseteq \bar{Y}^{\mathrm{c}}$, let $\mathrm{U}=\left\langle\mathrm{x}, \mathrm{u}_{1}, \mathrm{u}_{2}\right\rangle$ and $\bar{Y}^{\mathrm{c}}=\langle\mathrm{y}$, $\{y\}^{c},\{y\}>$, since $\bar{y} \notin U$, so $y \in u_{1}$, therefore $u_{1} \subseteq\{y\}^{c}$ and $\{\mathrm{y}\} \subseteq \mathrm{u}_{2}$, this implies that $\mathrm{U} \subseteq \bar{Y}^{\mathrm{c}}$. In a similar way, we can prove $\mathrm{V} \subseteq \bar{X}^{\mathrm{c}}$. Hence $\mathrm{SRT}_{1}($ (iíi) holds.
In order to prove $\mathrm{SRT}_{1}($ iíí $) \rightarrow \mathrm{SRT}_{1}(\mathrm{i})+\mathrm{SRT}_{1}(\mathrm{ii}):-$
First we have to prove $\mathrm{SRT}_{1}($ iií $) \rightarrow \mathrm{SRT}_{1}(\mathrm{i})$

Let $x, y \in X, x \neq y$. Since $R_{1}$ (iií) hold, so $\exists U, V \in$ $\mathrm{SR}(\mathrm{X})$ s.t $\bar{x} \in \mathrm{U} \subseteq \subseteq \bar{Y}^{\mathrm{c}}$ and $\bar{y} \in \mathrm{~V} \subseteq \bar{X}^{\mathrm{c}}$, we have to prove $\bar{x} \in \mathrm{U}, \bar{y} \notin \mathrm{U}$ and $\bar{y} \approx \mathrm{~V}, \bar{x} \notin \mathrm{~V}$ this implies that $\bar{x} \in \mathrm{U}$ and $\mathrm{Y} \subseteq \mathrm{U}$ so $\bar{x} \in \mathrm{U}, \bar{y} \notin \mathrm{U}$ and since $\bar{y}$ $\in \mathrm{V} \subseteq \bar{X}^{\mathrm{c}}$, so we get that $\bar{y} \in \mathrm{~V}, \bar{x} \notin \mathrm{~V}$, therefore $\mathrm{SRT}_{1}(\mathrm{i})$ holds.
Similarly, we can prove that $\mathrm{SRT}_{1}($ (ií $) \rightarrow \mathrm{SRT}_{1}($ (ií $)$.
The following implication all proved by transitivity :-
$\mathrm{SRT}_{1}(\mathrm{ii})+\mathrm{SRT}_{1}(\mathrm{i}) \rightarrow \mathrm{SRT}_{1}(\mathrm{ví})$,
$\mathrm{SRT}_{1}(\mathrm{ii})+\mathrm{SRT}_{1}(\mathrm{i}) \rightarrow \mathrm{SRT}_{1}(\mathrm{v})$
Remark 2.4: The converse of the last theorem are not true in general. The following counter example shows the cases.

## Example 2.5 :

1. Let $\mathrm{X}=\{1,2,3\}$ and define $\mathcal{T}=\{\tilde{\Phi}, \tilde{X}, \mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D}$, E, F \} where $\mathrm{A}=\langle\mathrm{x},\{1\},\{2,3\}\rangle, \mathrm{B}=\langle\mathrm{x},\{2\}$, $\{1,3\}>, C=\langle x,\{1,2\},\{3\}\rangle, D=\langle x,\{1,3\},\{2\}\rangle$, $\mathrm{E}=\langle\mathrm{x},\{2,3\}, \emptyset\rangle, \mathrm{F}=\langle\mathrm{x},\{1,3\}, \emptyset\rangle, \operatorname{so} \operatorname{SR}(\mathrm{X})=$ $\left\{\widetilde{\Phi}, \tilde{X}^{\tilde{X}}, \mathrm{~B}, \mathrm{D}\right\}$, then $(\mathrm{X}, \mathcal{T})$ is $\mathrm{SRT}_{1}(\mathrm{i})$, but not $\mathrm{SRT}_{1}(\mathrm{ii})$. 2. Let $\mathrm{X}=\{1,2\}$ and $\mathcal{T}=\{\widetilde{\Phi}, \tilde{X}, \mathrm{~A}, \mathrm{~B}\}$, where $\mathrm{A}=<$ $\mathrm{x}, \emptyset,\{1\}>, \mathrm{B}=\langle\mathrm{x}, \emptyset,\{2\}>$ and $\operatorname{SR}(\mathrm{X})=\{\widetilde{\Phi}, \widetilde{\bar{X}}, \mathrm{~A}$, $\mathrm{C}, \mathrm{D}\}$ where $\mathrm{C}=\langle\mathrm{x}, \emptyset,\{1\}\rangle$ and $\mathrm{D}=\langle\mathrm{x},\{2\}, \emptyset\rangle$, then $(\mathrm{X}, \mathcal{T})$ is $\mathrm{SRT}_{1}(\mathrm{vi})$, but not $\mathrm{SRT}_{1}(\mathrm{i})$.
2. Let $\mathrm{X}=\{1,2,3\}$ and define $\mathcal{T}=\{\widetilde{\varnothing}, \tilde{X}, \mathrm{~A}, \mathrm{~B}, \mathrm{C}$, $\mathrm{D}, \mathrm{E}, \mathrm{F}\}$ on X where $\mathrm{A}=\langle\mathrm{x}, \emptyset,\{1,2\}\rangle, \mathrm{B}=\langle\mathrm{x}$, $\{3\},\{1,2\}>, C=<x, \emptyset,\{2,3\}>, D=\langle x,\{3\},\{2\}>$, $\mathrm{E}=\langle\mathrm{x},\{1,3\},\{2\}\rangle, \mathrm{F}=\langle\mathrm{x}, \emptyset,\{2\}\rangle$, then $(\mathrm{X}, \mathcal{T})$ is $\operatorname{SRT}_{1}(\mathrm{ví})$, but not $\mathrm{SRT}_{1}(\mathrm{i} i ́)$ and not $\mathrm{SRT}_{1}(\mathrm{iii})$.
3. Let $\mathrm{X}=\{1,2,3\}$ and $\mathcal{T}=\{\widetilde{\Phi}, \widetilde{X}, \mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}$, $\mathrm{G}, \mathrm{H}, \mathrm{K}\}$ where $\mathrm{A}=\langle\mathrm{x},\{1\},\{3\}\rangle, \mathrm{B}=\langle\mathrm{x},\{2\}$, $\{1\}>, C=\langle x,\{1\},\{2,3\}\rangle, D=\langle x, \emptyset,\{2\}\rangle, E=<$ $\mathrm{x},\{1,2\}, \emptyset>, \mathrm{F}=<\mathrm{x}, \emptyset,\{1,3\}>, \mathrm{G}=<\mathrm{x}, \emptyset,\{2,3\}>$, $\mathrm{K}=\langle\mathrm{x},\{1\}, \emptyset\rangle \operatorname{So}(\mathrm{X}, \mathcal{T})$ is $\operatorname{SRT}_{1}(\mathrm{i})$ but not $\mathrm{SRT}_{1}$ (iií).

## 3. Semi- regular $T_{2}$ in intuitionistic Fuzzy Topological Spaces :

The aim of this part is to introducing some new form of $T_{2}$ separation axioms namely semi-regular
$\mathrm{T}_{2}$ in IFTS and study properties and it's relations of each other.
Definition 3.1: Let (X, $\mathcal{T}$ ) be an IFTS. (X, $\mathcal{T})$ is said to be :-

1. $\operatorname{SRT}_{2}(\hat{i})$ if for all $x, y \in X, x \neq y, \exists U, V \in \operatorname{SR}(X)$ such that $\bar{x} \in \mathrm{U}, \bar{y} \in \mathrm{~V}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\varnothing}$.
2. $\operatorname{SRT}_{2}($ ií $)$ if for all $x, y \in X, x \neq y, \exists U, V \in \operatorname{SR}(X)$ such that $\overline{\bar{x}} \in \mathrm{U},{ }^{\bar{y}} \in \mathrm{~V}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\varnothing}$.
3. $\operatorname{SRT}_{2}($ (iíí) if for all $x, y \in X, x \neq y, \exists U, V \in \operatorname{SR}(X)$ such that $\bar{x} \in \mathrm{U}, \bar{y} \in \mathrm{~V}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\varnothing}$.
4. $\operatorname{SRT}_{2}($ ív $)$ if for all $x, y \in X, x \neq y, \exists U, V \in \operatorname{SR}(X)$ such that $\overline{\bar{X}} \in \mathrm{~V}$ and $\mathrm{U} \subseteq \mathrm{V}$.
5. $\operatorname{SRT}_{2}(v)$ if for all $x, y \in X, x \neq y, \exists U, V \in S R(X)$ such that $\bar{x} \in U \subseteq \bar{Y}^{\mathrm{c}}, \bar{y} \in \mathrm{~V} \subseteq \bar{X}^{\mathrm{c}}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\Phi}$.
6. $\operatorname{SRT}_{2}(\mathrm{ví})$ if for all $x, y \in X, x \neq y, \exists U, V \in \operatorname{SR}(X)$ such that $\overline{\bar{x}} \in \mathrm{U} \subseteq \bar{Y}^{\mathrm{c}}, \bar{y}^{\mathrm{c}} \in \mathrm{V} \subseteq \bar{X}^{\mathrm{c}}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\varnothing}$.

Theorem 3.2 : Let $(\mathrm{X}, \mathcal{T})$ be an IFTS, then the following implications are valid.


## Proof :-

1. Let $(\mathrm{X}, \mathcal{T})$ be IFTS satisfy $\mathrm{SRT}_{2}(\mathrm{~V})$, to prove that $(\mathrm{X}, \mathcal{T})$ is satisfy $\mathrm{SRT}_{2}\left(\mathrm{v} \mathrm{v}^{\prime}\right)$. Let $\mathrm{x}, \mathrm{y} \in \mathrm{X}, \mathrm{x} \neq \mathrm{y}$. Since $\operatorname{SRT}_{2}(\mathrm{v})$ holes. Then $\exists \mathrm{U}, \mathrm{V} \in \operatorname{SR}(\mathrm{X})$ such that $\bar{x} \in$ $\mathrm{U} \subseteq \bar{Y}^{\mathrm{c}}, \bar{y} \in \mathrm{~V} \subseteq \bar{X}^{\mathrm{c}} \quad$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\varnothing}$, Since $\bar{x} \in \mathrm{U}$ and $\bar{y} \in \mathrm{~V}$ then we can get easily that $\overline{\bar{x}} \in \mathrm{U}$ and $\overline{\bar{y}} \in \mathrm{~V}$, therefore $\overline{\bar{x}} \in \mathrm{U}, \overline{\bar{y}} \in \mathrm{~V}, \mathrm{U} \subseteq \bar{Y}^{\mathrm{c}}, \mathrm{V} \subseteq \bar{X}^{\mathrm{c}}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\Phi}$ from hypotheses, so we get that $(\mathrm{X}, \mathcal{T})$ is satisfies $\mathrm{SRT}_{2}$ (ví).
2. To prove $\mathrm{SRT}_{2}(\mathrm{i}) \rightarrow \mathrm{SRT}_{2}(\mathrm{i} \hat{i})$, let $(\mathrm{X}, \mathcal{T})$ be IFTS satisfy $\operatorname{SRT}_{2}(i)$ and $x, y \in X, x \neq y$, so $\exists U, V \in \operatorname{SR}(X)$ such that $\bar{x} \in \mathrm{U}, \bar{y} \in \mathrm{~V}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\varnothing}$. Then we can get easily that $\overline{\bar{x}} \in U$ and $\overline{\bar{y}} \in \mathrm{~V}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\bar{\Phi}}$, therefore $\mathrm{SRT}_{2}$ (ií) holds.
3. Let $(X, \mathcal{T})$ be IFTS $x, y \in X, x \neq y$ and $\operatorname{SRT}_{2}(i)$ holds, to prove $\mathrm{SRT}_{2}\left(\right.$ (iíi) is satisfy, since $\mathrm{SRT}_{2}(\mathrm{i})$ holds so $\exists \mathrm{U}, \mathrm{V} \in \mathrm{SR}(\mathrm{X})$ such that $\bar{x} \in \mathrm{U}, \bar{y} \in \mathrm{~V}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\Phi}$, since $\bar{x} \in \mathrm{U}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\Phi}$ this implies that $\bar{x} \notin \mathrm{~V}$, so $\bar{x} \in \mathrm{~V}^{\mathrm{c}}$, this prove that for every $\mathrm{x} \in \mathrm{X}$ , if $\bar{x} \in \mathrm{U}$, then $\bar{x} \in \mathrm{~V}^{\mathrm{c}}, \bar{y} \in \mathrm{~V}$, i.e. $\mathrm{U} \subseteq \mathrm{V}$, therefore $\mathrm{SRT}_{2}$ (iíí) holds.
4. Suppose that $\mathrm{SRT}_{2}$ (íi) holds, to prove $\mathrm{SRT}_{2}$ (iv), let $\mathrm{x}, \mathrm{y} \in \mathrm{X}, \mathrm{x} \neq \mathrm{y}$, since $\mathrm{SRT}_{2}($ ií $)$ is hold so
$\exists \mathrm{U}, \mathrm{V} \in \operatorname{SR}(\mathrm{X})$ such that $\overline{\bar{x}} \in \bar{U}^{\mathrm{c}} \subseteq \overline{\bar{y}}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\varnothing}$, since $\overline{\bar{X}} \in U$, then $\bar{X} \notin \mathrm{~V}^{\mathrm{c}}=\emptyset$, so
$\overline{\bar{x}} \in \mathrm{~V}$, therefore $\mathrm{u} \in \mathrm{V}$, that is mean $\mathrm{SRT}_{2}(\mathrm{iv})$ holds.
5. In order to prove $\mathrm{SRT}_{2}$ (ií) satisfy when $\mathrm{SRT}_{2}$ (ví) holds. Let $x, y \in X, x \neq y$, so $\exists U, V \in S R(X)$ such that $\overline{\bar{x}} \in \bar{U}^{\mathrm{c}} \subseteq{ }_{\overline{\bar{Y}}}, \overline{\bar{y}} \in \bar{V}^{\mathrm{c}} \subseteq \mathrm{X}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\bar{\Phi}}$, from this we get directly that $\exists \mathrm{U}, \mathrm{V} \in \operatorname{SR}(\mathrm{X})$ such that $\overline{\bar{x}} \in$ $\mathrm{U}, \overline{\bar{y}} \in \mathrm{~V}$ and $\mathrm{U} \cap \mathrm{V}=\widetilde{\bar{\Phi}}$, therefore $\mathrm{SRT}_{2}(\mathrm{ii})$ holds.
6. $\mathrm{SRT}_{2}(\mathrm{iv}) \rightarrow \mathrm{SRT}_{2}(\mathrm{i})$ is clear.
7. To prove $\mathrm{SRT}_{2}(\mathrm{i} v)$ satisfy when $\mathrm{SRT}_{2}$ (iíí) holds, suppose that $x, y \in X, x \neq y$ so $\exists U, V \in \operatorname{SR}(X)$ such that $\bar{x} \in \mathrm{U}, \bar{y} \in \mathrm{~V}$ and $\mathrm{U} \subseteq \mathrm{V}^{\mathrm{c}}$, so we get directly that $\overline{\bar{x}} \in \mathrm{U}, \overline{\bar{y}} \in \mathrm{~V}$ and
$\mathrm{U} \cap \mathrm{V}=\widetilde{\Phi}$, therefore $\mathrm{SRT}_{2}$ (iv) holds.
Remark 3.3: In general the converse of the diagram appears in the theorem is not true in general. The following counter example shows the cases.

## Example 3.4 :

(i) Let $\mathrm{X}=\{1,2,3\}$ and define $\mathcal{T}=\{\widetilde{\widetilde{\Phi}}, \tilde{X}, \mathrm{~A}, \mathrm{~B}, \mathrm{C}\}$ on $X$ where $A=\langle x,\{1\},\{2,3\}>, B=\langle x,\{2\},\{1,3\}>, C$ $=\langle x,\{1,2\},\{3\}\rangle$, then $\operatorname{SR}(X)=\{\widetilde{\Phi}, X, D, E\}$ where $D$ $=\langle x,\{1\},\{2\}\rangle, E=\langle x,\{2\},\{1\}\rangle$, so the IFTS (X, $\mathcal{T})$ is $\mathrm{SRT}_{2}(\mathrm{ii})$ but not $\mathrm{SRT}_{2}(\mathrm{i})$.
(ií) Let $\mathrm{X}=\{1,2\}$ and define $\mathcal{T}=\{\widetilde{\boldsymbol{\Phi}}, \tilde{X}, \mathrm{~A}, \mathrm{~B}\}$ on X where $\mathrm{A}=\langle\mathrm{x}, \emptyset,\{2\}>, \mathrm{B}=\langle\mathrm{x}, \emptyset\{1\}>$, then the $\operatorname{IFTS}(\mathrm{X}, \mathcal{T})$ is $\mathrm{SRT}_{2}\left(\mathrm{i} \mathrm{i}^{\prime}\right)$, but not $\mathrm{SRT}_{2}(\mathrm{i})$.
(iií) Let $\mathrm{X}=\{1,2,3\}$ and define $\mathcal{T}=\{\widetilde{\mathscr{D}}, \tilde{X}, \mathrm{~A}, \mathrm{~B}\}$ on X where $\mathrm{A}=<\mathrm{x}, \emptyset,\{2,3\}>, \mathrm{B}=<\mathrm{x}, \emptyset,\{1,3\}>$, then the $\operatorname{IFTS}(\mathrm{X}, \mathcal{T})$ is $\mathrm{SRT}_{2}(\mathrm{v} \mathrm{i})$, but not $\mathrm{SRT}_{2}(\mathrm{v})$.
Since every $T_{2}$ separation axiom is $T_{1}$ separation axiom in general topology, then we have the following corollary :-
Corollary 3.5: Let (X, $\mathcal{T}$ ) be IFTS, then if (X, $\mathcal{T}$ ) is satisfies $\operatorname{SRT}_{2}(\mathrm{n})$, then it satisfies $\mathrm{SRT}_{1}(\mathrm{n})$, where
 corollary is not true in general and the following examples show the cases :-

## Example 3.6 :

1. Let $\mathrm{X}=\{1,2,3\}$ and define $\mathcal{T}=\left\{\widetilde{\Phi}^{\widetilde{X}}, \mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D}\right.$, E, F $\}$ where $\mathrm{A}=\langle\mathrm{x}, \emptyset,\{1,2\}\rangle, \mathrm{B}=\langle\mathrm{x}, \emptyset,\{2,3\}>$, $C=\langle x,\{3\},\{1,2\}\rangle, \quad D=\langle x,\{3\},\{2\}\rangle, \quad E=\langle x$, $\{1,3\},\{2\}\rangle, F=\langle x, \emptyset,\{2\}\rangle$, so $\operatorname{SR}(X)=\{\widetilde{\Phi}, \tilde{X}, M$, $H\}$ where $M=<x,\{3\}, \emptyset>, H=<x, \emptyset,\{3\}>$, so $(X$, $\mathcal{T}$ ) is SRT1(vi), but not SRT2(vi).
2. In the example 3.4(1) we see $(\mathrm{X}, \mathcal{T})$ is $\mathrm{SRT}_{1}(\mathrm{i})$ but not $\mathrm{SRT}_{2}(\mathrm{i})$ and in the (iii) of the example 3.4 we see ( $\mathrm{X}, \mathcal{J}$ ) is $\mathrm{SRT}_{1}(\mathrm{v})$, but not $\mathrm{SRT}_{2}(\mathrm{v})$.
3. Let $\mathrm{X}=\{1,2\}$ and define $\mathcal{T}=\{\widetilde{\widetilde{\phi}}, \tilde{X}, \mathrm{~A}, \mathrm{~B}\}$ where A $=\langle\mathrm{x}, \emptyset,\{1\}\rangle, \mathrm{B}=\langle\mathrm{x}, \emptyset,\{2\}\rangle$,so $\operatorname{SR}(\mathrm{X})=\mathcal{T}$.

Hence, $(\mathrm{X}, \mathcal{T})$ is $\mathrm{SRT}_{1}(\mathrm{ii})$, but not $\mathrm{SRT}_{2}(\mathrm{ii})$.
4. Take $\mathrm{X}=\{1,2,3\}$ and define $\mathcal{T}=\{\widetilde{\Phi}, \widetilde{X}, \mathrm{~A}, \mathrm{~B}, \mathrm{C}$,

D, E, F, G $\}$, where $A=\langle x,\{1\},\{2,3\}>, B=\langle x,\{2\}$, $\{1,3\}>, C=\langle x,\{1,2\},\{3\}>, D=\langle x,\{3\},\{1,2\}\rangle, E$ $=\langle x\{1,3\},\{2\}\rangle, F=\langle x,\{2,3\},\{1\}\rangle$, so $\operatorname{SR}(X)=\mathcal{T}$, then $(\mathrm{X}, \mathcal{T})$ is $\mathrm{SRT}_{1}(\mathrm{i} i \mathrm{i})$, but not $\mathrm{SRT}_{2}(\mathrm{i} i \mathrm{i})$.

## References

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Let $\mathrm{X}=\{1,2,3\}$ and define $\mathcal{T}=\{\widetilde{\Phi}, \tilde{X}, \mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D}, \mathrm{E}$, F, G, H, K $\}$ where $A=\langle x,\{1\},\{3\}\rangle, B=\langle x,\{2\}$, $\emptyset>, C=\langle x,\{3\}, \emptyset\rangle, D=\langle x,\{1,2\}, \emptyset\rangle, E=\langle$ $\mathrm{x},\{1,3\}, \emptyset\rangle, \mathrm{F}=\langle\mathrm{x},\{2,3\}, \emptyset\rangle, \mathrm{G}=\langle\mathrm{x}, \emptyset,\{3\}\rangle$, $\mathrm{H}=\langle\mathrm{x}, \emptyset, \emptyset\rangle, \mathrm{K}=\langle\mathrm{x}\{1\}, \emptyset\rangle$, then the IFS (X, $\mathcal{T}$ ) on X is $\mathrm{SRT}_{1}$ (iv), but not $\mathrm{SRT}_{2}$ (iv).
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## حول الشبهه المنتظم T1 والثببه المننظم $\mathbf{T}_{2}$ في الفضاءات الثبولوجية الحدسية <br> فاطمة محمود محمد

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[^0]:     الثبه المنتظم T2 ودراسة بعض صفاتهما وتعميمها مع بعض التفاصيل والعلاقات التي تربط بينهما .

