



APPROXIMATE SOLUTION FOR BURGER'S-FISHER EQUATION BY VARIATIONAL ITERATION TRANSFORM METHOD

Ali Al-FAYADH

Department of Mathematics & Computer Applications, College of Science, Al-Nahrain University, Baghdad, Iraq

ARTICLE INFO.

Article history:

-Received: 29 / 10 / 2017

-Accepted: 15 / 11 / 2017

-Available online: / / 2018

Keywords: Laplace Transform, Variational iteration transform method, Generalized Burger's-Fisher equation.

Corresponding Author:

Name: Ali Al-FAYADH

E-mail: aalfayadh@yahoo.com

Tel:

Abstract

Nonlinear partial differential equations represent the most important phenomena occurring in the world and are encountered in various fields of science. Generalized Burger's-Fisher equation is very important for describing different mechanisms. Burger's-Fisher equation arises in the field of applied mathematics and physics applications. This equation shows a prototypical model for describing the interaction between the reaction mechanisms, convection effect, and diffusion transport. In this paper, variational iteration transform method that combines Laplace transform and the variational iteration method, which used to obtain approximate analytical solutions of Burger's - Fisher equation. Comparison of the results obtained by the present method with the exact solution and other existing methods reveals the accuracy and fast convergence of the proposed method.

1. Introduction

Most of the problems in engineering and science are modeled by differential equations. Differential equations describe exchange of energy, information, matter, or any other quantities; often as they vary in time and space. These equations arise in various scientific fields such as the gas dynamics, traffic flow, and in applied mathematics and physics applications.

A compelling strategy is required to break down the scientific model which gives arrangements fitting in with physical reality. The nonlinear models of genuine issues are still exceptionally hard to unravel either numerically or hypothetically, more presumptions must be made pointlessly to make nonlinear models feasible. As of late, much consideration dedicated to the look for better and more productive arrangement strategies for deciding an answer, rough or correct, systematic or numerical, to nonlinear models [1–5]. Finding precise or estimated arrangements of these nonlinear conditions is intriguing and critical.

Many methods have been developed for providing approximate solutions of NPDEs. Some of these methods include pseudo spectral method [6], spectral collocation method [7], Adomian decomposition method (ADM) [8–10], homotopy perturbation method (HPM) [11–13], the differential transform

method (DTM) [14–16] and variational iteration method (VIM) [17–18]. One of these approximate methods which have received a great deal of attention is the (VIM). The VIM is first proposed by his paper [19] in which the technique implemented on several nonlinear ordinary and partial differential equations [20–23].

The variational iteration transform method (VITM) [24] that combines Laplace transform and Obtaining approximate and exact solutions of linear and nonlinear differential equations by using the variational iteration method. Applying the VITM in this paper is mainly used to get the approximate solutions of the generalized nonlinear Burger's-Fisher equation that put the interaction between reaction mechanisms, convection effects and diffusion transports [23]. The numerical results are compared with the exact solutions and that obtained previously by the VIM [25].

2. The Generalized Burger's-Fisher Equations

Consider the generalized Burger's-Fisher equation:

$$\frac{\partial u}{\partial t} + \alpha u^\sigma \frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = \beta u(1 - u^\sigma), 0 \leq x \leq 1, t \geq 0 \quad (1)$$

With initial condition

$$u(x, 0) = \left(\frac{1}{2} + \frac{1}{2} \tanh\left(\frac{-\alpha\sigma}{2(\sigma+1)}\right)\right)^{\frac{1}{\sigma}}, \quad (2)$$

and boundary conditions

$$u(0, t) = \left(\frac{1}{2} + \frac{1}{2} \tanh \left[\frac{-\alpha\sigma}{2(\sigma+1)} \left(\left(\frac{\alpha}{\sigma+1} + \frac{\beta(\sigma+1)}{\alpha} \right) t \right) \right] \right)^{\frac{1}{\sigma}}, t \geq 0 \quad (3)$$

$$u(1, t) = \left(\frac{1}{2} + \frac{1}{2} \tanh \left[\frac{-\alpha\sigma}{2(\sigma+1)} \left(1 - \left(\frac{\alpha}{\sigma+1} + \frac{\beta(\sigma+1)}{\alpha} \right) t \right) \right] \right)^{\frac{1}{\sigma}}, t \geq 0 \quad (4)$$

The exact solution of Eq. (1) is

$$u(x, t) =$$

$$\left(\frac{1}{2} + \frac{1}{2} \tanh \left[\frac{-\alpha\sigma}{2(\sigma+1)} \left(x - \left(\frac{\alpha}{\sigma+1} + \frac{\beta(\sigma+1)}{\alpha} \right) t \right) \right] \right)^{\frac{1}{\sigma}} \quad (5)$$

where α, β and σ are constants, Generalized Burger's equation will be obtained when $\beta = 0$.

3. Variational Iteration Transform Method (VITM):

The basic idea of this method can be illustrated by considering the equation of a general nonlinear non-homogeneous partial differential with initial conditions as follow

$$\mathfrak{D}u(x, t) + \mathcal{R}u(x, t) + \mathfrak{N}u(x, t) = g(x, t) \quad (6)$$

$$u(x, 0) = h(x), u_t(x, 0) = f(x)$$

where \mathfrak{D} is the second order linear differential operator $\mathfrak{D} = \frac{\partial^2}{\partial t^2}$, \mathcal{R} is linear differential operator of less order than \mathfrak{D} , \mathfrak{N} represent the general nonlinear differential operator and $g(x,t)$ is the source term.

Taking Laplace Transform on both sides of Eq.(6)

$$L[\mathfrak{D}u(x, t)] + L[\mathcal{R}u(x, t)] + L[\mathfrak{N}u(x, t)] = L[g(x, t)] \quad (7)$$

$$s^2 L[u(x, t)] - su(x, 0) - u_t(x, 0) + L[\mathcal{R}u(x, t)] + L[\mathfrak{N}u(x, t)] = L[g(x, t)] \quad (8)$$

Taking Inverse Laplace transform

$$u(x, t) = f(x) + th(x) + \frac{1}{s^2} L^{-1} [L(g(x, t))] - \frac{1}{s^2} L^{-1} [L(\mathcal{R}(x, t))] - \frac{1}{s^2} L^{-1} [L\mathfrak{N}u(x, t)] \quad (9)$$

Derivative by $\frac{\partial}{\partial t}$ both sides of Eq.(9)

$$u_t(x, t) = h(x) + \frac{\partial}{\partial t} L^{-1} \left(\frac{1}{s^2} L \{g(x, t)\} \right) - \frac{\partial}{\partial t} L^{-1} \left(\frac{1}{s^2} L \{ \mathcal{R}u(x, t) \} \right) - \frac{\partial}{\partial t} L^{-1} \left(\frac{1}{s^2} L \{ \mathfrak{N}u(x, t) \} \right) \quad (10)$$

By the correction function of the irrational method

$$u_{n+1}(x, t) = u_n(x, t) - \int_0^t ((u_n)_\xi(x, \xi) + \frac{\partial}{\partial \xi} (L^{-1} \left(\frac{1}{s^2} L \{ \mathcal{R}u_n(x, \xi) \} + L^{-1} \left(\frac{1}{s^2} L \{ \mathfrak{N}u_n(x, \xi) \} \right) - L^{-1} \left(\frac{1}{s^2} L \{g(x, \xi)\} \right) - h(x) \right) d\xi \quad (11)$$

Finally, the solution $u(x,t)$ is given by

$$u(x, t) = \lim_{n \rightarrow \infty} u_n(x, t)$$

4. Numerical Results

In this section we illustrate the ability and simplicity of the presented method for solving the nonlinear Burger's-Fisher equation. Two cases will be studied: the first case is when $\alpha = \beta = 0.01$ and $\sigma = 1$ in Eq. (1), whereas the second case is when $\alpha = \beta = 0.001$ and $\sigma = 1$, for various values of x and t .

Example :

Suppose the equation of generalized Burger's-Fisher:

$$\frac{\partial u}{\partial t} + \alpha u^\sigma \frac{\partial u}{\partial x} - \frac{\partial^2 y}{\partial x^2} = \beta u(1 - u^\sigma), 0 \leq x \leq 1, t \geq 0$$

With initial condition

$$u(x, 0) = \left(\frac{1}{2} + \frac{1}{2} \tanh \left(\frac{-\alpha\sigma}{2(\sigma+1)} \right) \right)^{\frac{1}{\sigma}}$$

Case 1: when $\alpha = \beta = 0.01$ and $\sigma = 1$.

$$u_t - u_{xx} + 0.01uu_x + 0.01u(u - 1) = 0 \quad (12)$$

with initial condition

$$u(x, 0) = \frac{1}{2} - \frac{1}{2} \tanh \left(\frac{0.01}{4} x \right) \quad (13)$$

Taking Laplace transform on both sides on Eq.12

$$L[u_t] - L[u_{xx}] + 0.01L[uu_x] + 0.01L[u(u - 1)] = 0 \quad (14)$$

This can be written as

$$sLu(x, t) - u(x, 0) - L[u_{xx}] + 0.01L[uu_x] + 0.01L[u(u - 1)] = 0 \quad (15)$$

Substituting the initial condition in Eq.(15) we get

$$sLu(x, t) - \frac{1}{2} - \frac{1}{2} \tanh(0.0025x) - L[u_{xx}] + 0.01L[uu_x] + 0.01L[u(u - 1)] = 0 \quad (16)$$

$$L[u(x, t)] = \frac{1}{2s} + \frac{1}{2s} \tanh(0.0025x) + \frac{1}{s} L[u_{xx}] - \frac{0.01}{s} L[uu_x] - \frac{0.01}{s} L[(u - 1)] \quad (17)$$

Take Inverse Laplace to Eq.(17) obtain:

$$u(x, t) = L^{-1} \left[\frac{1}{2s} \right] + L^{-1} \left[\frac{1}{2s} \tanh(0.0025x) \right] + L^{-1} \left(\frac{1}{s} L[u_{xx}] \right) - L^{-1} \left(\frac{0.01}{s} L[uu_x] \right) - L^{-1} \left(\frac{0.01}{s} L[(u - 1)] \right) \quad (18)$$

$$= \frac{1}{2} + \frac{1}{2} \tanh(0.0025x) + L^{-1} \left(\frac{1}{s} L[u_{xx}] \right) - L^{-1} \left(\frac{0.01}{s} L[uu_x] \right) - L^{-1} \left(\frac{0.01}{s} L[u(u - 1)] \right) \quad (19)$$

Derivative by $\frac{\partial}{\partial t}$ both sides of Eq.(19) we get

$$u_t(x, t) = \frac{\partial}{\partial t} \left[\left(\frac{1}{s} L[u_{xx}] \right) \right] - L^{-1} \left(\frac{0.01}{s} L[u_{xx}] \right) - L^{-1} \left(\frac{0.01}{s} L[u(u - 1)] \right) \quad (20)$$

$$u_t(x, t) - \frac{\partial}{\partial t} \left[L^{-1} \left(\frac{1}{s} L[u_{xx}] \right) \right] - 0.01L^{-1} \left(\frac{1}{s} L[u_{xx}] \right) - 0.01L^{-1} \left(\frac{1}{s} L[u(u - 1)] \right) = 0 \quad (21)$$

Making the correction function yields

$$u_{n+1}(x, t) = u_n(x, t) - \int_0^t \left((u_n)_\xi(x, \xi) - \frac{\partial}{\partial \xi} \left[\left(\frac{1}{s} L[u_n]_{xx} \right) - 0.01L^{-1} \left(\frac{1}{s} L[u_n(u_n)_x] \right) - 0.01L^{-1} \left(\frac{1}{s} L[u_n(u_n - 1)] \right) \right] \right) d\xi \quad (22)$$

Now we use the initial condition

$$u_0(x, t) = u(x, 0) = \frac{1}{2} (1 - \tanh(0.0025x)) \quad (23)$$

$$u_1(x, t) = u_0(x, t) - \int_0^t \left((u_0)_\xi(x, \xi) - \frac{\partial}{\partial \xi} \left[L^{-1} \left(\frac{1}{s} L[(u_0)_{xx}] \right) - 0.01L^{-1} \left(\frac{1}{s} L[u_0(u_0)_x] \right) - 0.01L^{-1} \left(\frac{1}{s} L[u_0(u_0 - 1)] \right) \right] \right) d\xi \quad (24)$$

$$u_1(x, t) = \frac{1}{2} - \frac{1}{2} \tanh(0.0025x) + \int_0^t \left(\frac{\partial}{\partial \xi} \left[L^{-1} \left(\frac{1}{s} \left[\left(\frac{1}{2} - \tanh(0.0025x) \right)_{xx} \right] \right) - 0.01L^{-1} \left(\frac{1}{s} \left[\left(\frac{1}{2} - \tanh(0.0025x) \right) \left(\frac{1}{2} - \tanh(0.0025x) \right) \right] \right) - 0.01L^{-1} \left(\frac{1}{s} \left[\left(\frac{1}{2} - \frac{1}{2} \tanh(0.0025x) \right) \right] \right) \right] \right) d\xi \quad (25)$$

$$u_1(x, t) = \frac{1}{2} - \frac{1}{2} \tanh(0.0025x) + \int_0^t \left[\frac{\partial}{\partial \xi} \left[L^{-1} \left(\frac{(0.0025)^2}{s} \left(\left[\frac{s^2 - (0.0025)^2}{s} \right]^2 \left(\frac{0.0025}{s^2 - (0.0025)^2} \times \frac{s^2 - (0.0025)^2}{s} \right) - 0.01 L^{-1} \left[\left(\frac{-0.0025}{4s} \right) \times \left(\frac{s^2 - (0.0025)^2}{s} \right)^2 + \frac{0.0025}{4s} \left[\left(\frac{0.0025}{s^2 - (0.0025)^2} \right) \times \left(\frac{s^2 - (0.0025)^2}{s} \right) \right] \right) \right] \right] - 0.01 L^{-1} \left(\frac{1}{4s^2} + \frac{4}{s} \left(\frac{0.0025}{s^2 - (0.0025)^2} \times \frac{s^2 - (0.0025)^2}{s} \right) \right) \right] d\xi \quad (26)$$

$$= \frac{1}{2} - \frac{1}{2} \tanh(0.0025x) + \int_0^t \left[\frac{\partial}{\partial \xi} \left[\left(-2(0.0025)^5 + \frac{(0.0025)^7}{2} \xi^2 \right) - \frac{0.01(0.0025)}{4} \left[2(0.0025)^2 - \frac{(0.0025)^4}{2} \xi^2 + \right. \right. \right.$$

$$\left. \left. 2(0.0025)^3 \xi - \frac{(0.0025)^5}{3!} \xi^3 \right] - 0.01 \left(\frac{\xi}{4} + 2(0.0025)^2 \xi^2 \right) \right] d\xi \quad (27)$$

$$u_1(x, t) = \frac{1}{2} - \frac{1}{2} \tanh(0.0025x) + \frac{0.01(0.0025)^6}{24} t^3 + \left(\frac{(0.0025)^7}{2} + \frac{0.01(0.0025)^5}{8} + 2(0.01(0.0025)^2) \right) t^2 - 0.01 \left(\frac{(0.0025)^4}{2} + \frac{1}{4} + 4(0.0025)^2 \right) t \quad (28)$$

Therefore, the approximate solution:

$$u(x, t) \approx u_1(x, t) = \frac{1}{2} - \frac{1}{2} \tanh(0.0025x) + \frac{0.01(0.0025)^6}{24} t^3 + \left(\frac{(0.0025)^7}{2} + \frac{0.01(0.0025)^5}{8} + 2(0.01(0.0025)^2) \right) t^2 - 0.01 \left(\frac{(0.0025)^4}{2} + \frac{1}{4} + 4(0.0025)^2 \right) t \quad (29)$$

Table (1) shows the approximate solution using the proposed VITM method against the exact solution for various values of x and t .

Table (1): The Approximation Solution against the Exact Solution When $\alpha = \beta = 0.01$ and $\sigma = 1$.

x	t	Exact sol.	App. Sol.	Error
0.01	0.02	0.500043749999888	0.500037500312506	0.000006249687382
0.02		0.500031249999959	0.500025000312525	0.000006249687434
0.03		0.500018749999991	0.500012500312574	0.000006249687417
0.04		0.500006250000000	0.500000000312671	0.000006249687329
0.05		0.499993750000000	0.499987500312829	0.000006249687171
0.01	0.03	0.500068749999567	0.500062500703133	0.000006249296434
0.02		0.500056249999763	0.500050000703152	0.000006249296611
0.03		0.500043749999888	0.500037500703201	0.000006249296687
0.04		0.500031249999959	0.500025000703297	0.000006249296662
0.05		0.500018749999991	0.500012500703456	0.000006249296535
0.01	0.04	0.500093749998901	0.500087501250010	0.000006248748891
0.02		0.500081249999285	0.500075001250029	0.000006248749256
0.03		0.500068749999567	0.500062501250078	0.000006248749489
0.04		0.500056249999763	0.500050001250175	0.000006248749588
0.05		0.500043749999888	0.500037501250333	0.000006248749555
0.01	0.05	0.500118749997767	0.500112501953137	0.000006248044630
0.02		0.500106249998401	0.500100001953156	0.000006248045245
0.03		0.500093749998901	0.500087501953205	0.000006248045696
0.04		0.500081249999285	0.500075001953301	0.000006248045984
0.05		0.500068749999567	0.500062501953460	0.000006248046107
0.01	0.06	0.500143749996039	0.500137502812514	0.000006247183525
0.02		0.500131249996985	0.500125002812533	0.000006247184452
0.03		0.500118749997767	0.500112502812582	0.000006247185185
0.04		0.500106249998401	0.500100002812678	0.000006247185723
0.05		0.500093749998901	0.500087502812837	0.000006247186064
0.01	0.07	0.500168749993593	0.500162503828141	0.000006246165452
0.02		0.500156249994914	0.500150003828159	0.000006246166755
0.03		0.500143749996039	0.500137503828209	0.000006246167830
0.04		0.500131249996985	0.500125003828305	0.000006246168680
0.05		0.500118749997767	0.500112503828464	0.000006246169303
0.01	0.08	0.500193749990302	0.500187505000018	0.000006244990284
0.02		0.500181249992061	0.500175005000036	0.000006244992025
0.03		0.500168749993593	0.500162505000086	0.000006244993507
0.04		0.500156249994914	0.500150005000182	0.000006244994732
0.05		0.500143749996039	0.500137505000341	0.000006244995698

Case 2: when $\alpha = \beta = 0.001$ and $\sigma = 1$. Table (2) shows the approximate solution using the proposed VITM method against the exact solution for various

values of x and t . In addition, the absolute errors resulted by using the proposed VITM and existing VIM [25] is presented.

Table (2): The Exact Solution and the Approximation Solution When $\alpha = \beta = 0.001$ and $\sigma = 1$.

x	T	Exact sol.	App. Sol.	Error VITM	Error VIM
0.01	0.02	0.500003812500000	0.500003750003125	0.000000062496875	0.0025031102
	0.04	0.500008812499999	0.500008750012500	0.000000062487499	0.0025081138
	0.06	0.500013812499996	0.500013750028125	0.000000062471871	0.0025131170
	0.08	0.500018812499991	0.500018750050000	0.000000062449991	0.0025181206
0.04	0.02	0.500000625000000	0.500000000003125	0.0000000624996875	0.0099961959
	0.04	0.500005062500000	0.500005000012500	0.000000062487500	0.0100011899
	0.06	0.500010062499999	0.500010000028125	0.000000062471874	0.0100061907
	0.08	0.500015062499995	0.500015000050000	0.000000062449995	0.0100111915

5. Conclusion

The generalized Burger's-Fisher has been analyzed using the variational iteration transform method. The proposed method provides dependable results in the form of analytical approximation converging very rapidly, this method can give very good

6. References

- [1] Wang, X.Y. (1988). Exact and explicit solitary wave solutions for the generalized Fisher equation. *Phys Lett A*, **131(4/5)**:277–9.
- [2] Jeffrey, A. and Mohamad, M.N.B. (1991). Exact solutions to the KdV–Burgers equation. *Wave Motion*, **14**:369–75.
- [3] Wadati, M. (1972). The exact solution of the modified Korteweg–de Vries equation. *J Phys Soc Jpn*, **32**:1681–7.
- [4] Kivshar, Y.S. and Pelinovsky, D.E. (2000). Self-focusing and transverse instabilities of solitary waves. *Phys Rep*, **331**:117–95.
- [5] Hereman, W. and Takaoka, M. (1990). Solitary wave solutions of nonlinear evolution and wave equations using a direct method and MACSYMA. *J Phys A*; **23**:4805–22.
- [6] Javidi, M. (1990). A numerical solution of the generalized Burger's–Huxley equation by pseudospectral method and Darvishi's preconditioning. *Appl Math Comput*, **175**:1619–28.
- [7] Javidi, M. (2006). A numerical solution of the generalized Burger's–Huxley equation by spectral collocation method. *Appl Math Comput*, **178(2)**:338–44.
- [8] Adomian, G. (1994). Solving frontier problems of physics: the decomposition method. Boston: Kluwer Academic.
- [9] Hashim; I. Noorani, M.S.M. and Batiha, B. (2006). A note on the Adomian decomposition method for the generalized Huxley equation. *Appl Math Comput*, doi:10.1016/16.amc.03.011, in press.
- [10] Hashim; I. Noorani, M.S.M. and Al-Hadidi, M.R.S. (2006). Solving the generalized Burgers–Huxley equation using the Adomian decomposition method. *Math Comput Model*, **43**:1404–11.
- [11] He, JH. (2005). Application of homotopy perturbation method to nonlinear wave equations. *Chaos, Solitons & Fractals*, **26(3)**: 695–700.
- [12] He, JH. (2005). Homotopy perturbation method for bifurcation of nonlinear problems. *Int J Nonlin Sci Numer Simul*, **6(2)**:207–8.
- [13] He, J.H. (2005). Limit cycle and bifurcation of nonlinear problems. *Chaos, Solitons & Fractals*, **26(3)**:827–33.
- [14] He, J.H. (1997). A new approach to nonlinear partial differential equations. *Communications in nonlinear Science and Numerical Simulation*, **2(4)**:230–235.
- [15] He, J.H. (1997). Variational iteration method for delay differential equations. *Communications in nonlinear Science and Numerical Simulation*, **2(4)**:235–6.
- [16] He, J.H. (1999). Variational iteration method – a kind of non-linear analytical technique: some examples *International Journal of Non-Linear Mechanics*, **34(4)**:699–708.
- [17] He, J.H. (2000). Variational iteration method for autonomous ordinary differential systems. *Applied Mathematics and Computation*, **114(2–3)**:115–23.
- [18] He, J.H. (2003). A simple perturbation approach to Blasius equation. *Applied Mathematics and Computation*, **140(2–3)**:217–22.
- [19] He; J.H. Wan, Y-Q. and Guo, Q. (2004). An iteration formulation for normalized diode characteristics. *International Journal of Circuit Theory and Application*, **32(6)**:629–32.
- [20] Abdou, M.A. and Soliman, A.A. (2005). Variational iteration method for solving Burger's and coupled Burger's equations. *Journal of Computational and Application Mathematics*, **181(2)**:245–251.
- [21] Momani, S. and Abuasad, S. (2006). Application of He's variational iteration method to Helmholtz equation. *Chaos, Solitons & Fractals*, **27(5)**:1119–23.
- [22] Moghimi, M. and Hejazi, F.S.A. (2007). Variational iteration method for solving generalized Burger–Fisher and Burger equations. *Chaos, Solitons & Fractals*, **33(5)**: 1756-1761.
- [23] Satsuma, J. (1987). Topics in soliton theory and exactly solvable nonlinear equations. Singapore: World Scientific.

[24] Toheeb; A. Oladapo, O. and Yusuf, O. (2014). A Combination of the Laplace Transform and the Variational Iteration Method for the Analytical Treatment of Delay Differential Equations. International Journal of Differential Equations and Applications, **13** (3): 164-175.

[25] Kocacoban; D., Koc, A. B. Kurnaz, A. and Keskin, Y. (2011). A Better Approximation to the Solution of Burger-Fisher Equation. Proceedings of the World Congress on Engineering Vol I, WCE 2011, July 6 - 8, 2011, London, U.K.

الحل التقريبي لمعادلة بورجر - فشر باستخدام طريقة التكرار التبايني التحويلي

علي الفياض

قسم الرياضيات وتطبيقات الحاسوب ، كلية العلوم ، جامعة النهريين ، بغداد ، العراق

الملخص

تمثل المعادلات النفاضلية الجزئية غير الخطية اهم الظواهر التي تحصل في العالم وتشارك في حقول العلوم المتنوعة. تعتبر معادلة بورجر - فيشر من المعادلات الاكثر اهمية لوصف مختلف الديناميكيات. تظهر معادلة بورجر - فيشر في حقل الرياضيات التطبيقية وتطبيقات الفيزياء. تمثل هذه المعادلة النموذج الرئيس لوصف الترابط بين ميكانيكية التفاعل, التأثير الحراري والانتشار. في هذا البحث تم استخدام طريقة تحويل التكرار التبايني والتي تتألف من تحويلات لابلاس وطريقة تكرار التبايني لغرض الحصول على الحلول التحليلية التقريبية لمعادلة بورجر - فيشر. ان مقارنة النتائج المستحصلة باستخدام هذه الطريقة مع الحل التام والطرق الموجودة الاخرى تبين الدقة وسرعة التقارب للطريقة المقدمة.