In this paper we study the relationship between tensor algebraic curvature tensor, and General conharmonic curvature tensor of Nearly Kahler manifold, i.e. it has a classical symmetry properties of the Riemannian curvature tensor. Relpening generalized Riemannian structure of certain classes of almost Hermitian manifold allows an additional symmetry properties of this tensor.

In this paper we investigated the generalized conharmonic curvature tensor of nearly kahler manifold.

2- Preliminaries

let M be a smooth manifold of dimension 2n, C∞(M) is algebra of smooth function on M; X(M) is the module of smooth vector fields on manifold of M; g= < , > is Riemannian metrics, V is Riemannian connection of the metrics g on M; d is the operator of exterior differentiation. In the further all manifold, Tensor field, etc. objects are assumed smooth a class C∞(M).

Definition 1.[6]: Almost Hermiton structure on a manifold (AH) M is the pair (J, g), where J is an almost complex structure (J2=id) on M, g= < , > is a(pseudo)Riemannian metric on M. In this case JX,JY=<X,Y>, ∀ X,Y∈X(M).

Lemma 1.[7]: Every almost complex manifold there exist an almost Hermiton (AH)-structure. The endomorphism J is called a structural endomorphism, Manifold which is fixed on it an almost Hermiton manifold is called an almost Hermiton manifold (AH) -manifold and denoted by {M, J, g= ,< , >} or simply M.

Definition2.[2]: Almost Hermiton structure (J, g) on manifold M is called;
1 -A Kahler (K) –structure, if
\[ \nabla_J Y=0; X,Y \in X(M). \]
2 -A nearly Kahler (NK)-structure, if M satisfies
\[ \nabla_J Y+\nabla_Y J(X)=0; X,Y \in X(M). \]
Lemma 2. [8]: An almost Hermitian manifold is Kahler if and only if the following identities are satisfied:
1) $J^*V(JY)+V(J)JY=0$
2) $(V(J)Y,Z)+(Y,V(J)Z)=0$

Theorem 3. [8]: An almost Hermitian structure $(J,g)$ on an almost Hermitian manifold is nearly Kahler if and only if the following identities are satisfied:
1) $B(X,Y)=0$
2) $C(X,Y)+C(Y,X)=0$
This structure is called Kahler iff $B=C=0$.

Definition 3. [8]: The set of tensors of type $(r,s)$ in the tensor $T$ are called the elements of the tensor $T$, and the tensors themselves are called elements of the tensor.

Definition 4: Suppose $(M,J,g)$ - NK manifold. We retard alert, that curvature tensor $g$ of the Hermitian metric is introduced by Ishii (1957) [9] as a tensor of type $(4,0)$ on an $n$-dimensional Riemannian manifold, and the formula by the definition of

$$K_{ij} = \{g_{ik} r(JH, JY) + g_{jk} r(JH, JZ) + g_{ki} r(JH, JZ) + g_{kj} r(JH, JY)\}_{ij}$$

(1)

Where $(H)$ is the general Riemann curvature tensor, $r$ is general Ricci tensor. This tensor is invariant under general conformal transformation of space keeping a harmony of functions.

Consider properties tensor general conharmonic curvature $K_{(HR)}$:

1. $K_{(HR)}(X,Y,Z,W) = (HR)(X,Y,Z,W) - 1/2(n-1) \{g(xr) r(HR)(Y,W) + g(yr) r(HR)(X,Z) - g(yr) r(HR)(Y,Z) - g(xr) r(HR)(X,W)\}$(HR)(X,Y,Z,W)

The next ownerships are similarly proved
2. $K_{(HR)}(X,Y,Z,W) = - K_{(HR)}(X,Y,W,Z)$

So general conharmonic curvature tensor $K_{(HR)}$ satisfies all the properties of algebraic curvature tensor $K_{(HR)}$:

2. $K_{(HR)}(X,Y,Z,W) = - K_{(HR)}(X,Y,W,Z)$

(2)

Tensor generalized conharmonic curvature $K_{(HR)}$ looks like:

$$K_{(HR)}(X,Y,Z,W) = (HR)(X,Y,Z,W) - 1/2(n-1) \{r(HR)(Y,Z) + r(HR)(X,W) - g(Y,Z) r(HR)(X,W) - g(X,W) r(HR)(Y,Z)\}$$

(3)

Where $g(xr) = r(HR)(X,Y)$. By definition of the components of the form $K_{(HR)}(X,Y,Z,W)$ are characterized by identity $K_{(HR)}(X,Y,Z,W) = K_{(HR)}(X,Y,W,Z)$.

Theorem 2: All $K_{(HR)}(X,Y,Z,W)$ are characterized by identity $K_{(HR)}(X,Y,Z,W) = K_{(HR)}(X,Y,W,Z)$.

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Tikrit Journal of Pure Science 23 (8) 2018

106
The components of the generalized Riemannian curvature tensor (HR) of NK-manifold are given by the next theorem.

**Theorem 4.6:** The components of the generalized Riemannian curvature by the next formulation:

1) \((HR)_{abcd} = -A_{abcd}^{ac}\)
2) \((HR)_{abcd} = A_{abcd}^{ac}\)
And the others are conjugate to the above components or equal to zero.

**Definition 7.11:** A tensor of type \((2,0)\) which is defined as \(r(HR)_{ij} = (HR)_{ij}^{abc}\) is called generalized Ricci tensor.

**Theorem 5.12:** The components of generalized Ricci tensor of NK-manifold in the adjoint G-structure space are given as the following formation: \(R_{abcd} = -A_{abcd}^{ac}\)
And the others are conjugate to the above component or equal to zero.

**Definition 8.12:** A generalized scalar curvature tensor is denoted by \(K(HR)\) and defined as:

\[ K(HR) = g_{ij} r(HR)_{ij} \]

**Theorem 6.12:** The component of generalized scalar curvature tensor of NK-manifold in the adjoint G-structure space is given as the following form:

\[ K(HR)_{ijkl} = R(HR)_{ijkl} - \frac{1}{2(n-1)} g_{ik} r(HR)_{jk} + g_{jk} r(HR)_{ik} - g_{il} r(HR)_{kj} - g_{jl} r(HR)_{ki} + \frac{1}{2} \delta_{ik}^{a} r(HR)_{ab} + \delta_{ik}^{b} r(HR)_{ac} + \delta_{ik}^{c} r(HR)_{bc} \]

And other components Generalized conharmonic curvature tensor for NK are equal to zero.

**Proof:**
1- Let i=a, j=b, k=c and l=d
- \(K(HR)_{abcd} = (HR)_{abcd} - \frac{1}{2(n-1)} g_{ac} r(HR)_{bd} + g_{bd} r(HR)_{ac} - g_{ad} r(HR)_{bc} - g_{bc} r(HR)_{ad} \)
- \(k_{abcd} = 0 \quad 0 \quad 0 \quad 0 \)
- \(K(HR)_{abcd} = 0 \quad 0 \quad 0 \quad 0 \)

**Definition 6.10:** Ageneralized Riemannian curvature tensor on AH-manifold M is a tensor of type \((4,0)\) which is defined as following formulation:


The components of the generalized Riemannian curvature tensor (HR) of NK-manifold are given by the next theorem.

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And the others are conjugate to the above components or equal to zero.

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- \(K(HR)_{abcd} = (HR)_{abcd} - \frac{1}{2(n-1)} g_{ac} r(HR)_{bd} + g_{bd} r(HR)_{ac} - g_{ad} r(HR)_{bc} - g_{bc} r(HR)_{ad} \)
- \(k_{abcd} = 0 \quad 0 \quad 0 \quad 0 \)
- \(K(HR)_{abcd} = 0 \quad 0 \quad 0 \quad 0 \)
Let $i=\bar{a}$, $j=\bar{b}$, $k=c$ and $l=d$

$$K(\mathcal{H})_{abcd} = (\mathcal{H})_{abcd} - \frac{1}{2(n-1)} g_{dc} r(\mathcal{H})_{bd} + g_{bd} r(\mathcal{H})_{dc}$$

$$r(\mathcal{H})_{bd} - g_{bd} r(\mathcal{H})_{bc} = g_{bd} r(\mathcal{H})_{bc}$$

$$k(\mathcal{H})_{abcd} = 0 - \frac{1}{2(n-1)} \left( \delta_{c}^{a}(0) - (0)(-A_{ak}^{c}) - \delta_{a}^{c}(0) - (0)(-A_{dk}^{a}) \right)$$

$$k(\mathcal{H})_{abcd} = 0$$

3. Let $i=a$, $j=b$, $k=c$ and $l=d$

$$K(\mathcal{H})_{abcd} = (\mathcal{H})_{abcd} - \frac{1}{2(n-1)} g_{ac} r(\mathcal{H})_{bd} + g_{bc} r(\mathcal{H})_{ad}$$

$$r(\mathcal{H})_{bd} - g_{bd} r(\mathcal{H})_{bc} = g_{bd} r(\mathcal{H})_{bc}$$

$$k(\mathcal{H})_{abcd} = 0 - \frac{1}{2(n-1)} \left( \delta_{a}^{c}(0) + (0)(-A_{ak}^{c}) - \delta_{a}^{c}(0) - (0)(-A_{dk}^{a}) \right)$$

$$k(\mathcal{H})_{abcd} = 0$$

4. Let $i=a$, $j=\bar{b}$, $k=c$ and $l=d$

$$K(\mathcal{H})_{abcd} = (\mathcal{H})_{abcd} - \frac{1}{2(n-1)} g_{ac} r(\mathcal{H})_{bd} + g_{bc} r(\mathcal{H})_{ad}$$

$$r(\mathcal{H})_{bd} - g_{bd} r(\mathcal{H})_{bc} = g_{bd} r(\mathcal{H})_{bc}$$

$$k(\mathcal{H})_{abcd} = 0 - \frac{1}{2(n-1)} \left( \delta_{a}^{c}(0) + (0)(-A_{ak}^{c}) - \delta_{a}^{c}(0) - (0)(-A_{dk}^{a}) \right)$$

$$k(\mathcal{H})_{abcd} = 0$$

5. Let $i=a$, $j=\bar{b}$, $k=c$ and $l=d$

$$K(\mathcal{H})_{abcd} = (\mathcal{H})_{abcd} - \frac{1}{2(n-1)} g_{ac} r(\mathcal{H})_{bd} + g_{bc} r(\mathcal{H})_{ad}$$

$$r(\mathcal{H})_{bd} - g_{bd} r(\mathcal{H})_{bc} = g_{bd} r(\mathcal{H})_{bc}$$

$$k(\mathcal{H})_{abcd} = 0 - \frac{1}{2(n-1)} \left( \delta_{c}^{a}(0) + (0)(-A_{ak}^{c}) - \delta_{c}^{a}(0) - (0)(-A_{dk}^{a}) \right)$$

$$k(\mathcal{H})_{abcd} = 0$$

References


الملخص

في هذه البحث تم دراسة العلاقة بين تنزر الانحناء الجبري، وتنزر الانحناء الكونورماني المعمم لمنطوي كوهلر التجريبي، حيث أن هذا التنزر يمتلك خصائص التناظر الكلاسيكي لتنزر الانحناء الريماني، مع احتساب مركبات تنزر الانحناء الكونورماني المعمم في بعض أصناف المنطوي الهرموني التقريبي إضافة إلى دراسة خصائص التناظر لهذا التنزر.