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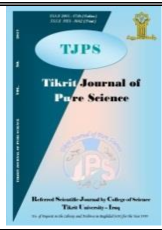

SOME TYPES OF CONTIONUOUS FUNCTION VIA (r_0, s_1)-FUZZY α^m-CLOSED SETS

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Abstract

The purpose of this paper is to introduce and study the notions of some types of continuous functions via (r_0, s_1)-fuzzy α^m-closed sets in double fuzzy topological space. Also, we reached some relationships among these new types of functions and compare them with their opposite with illustrative examples in the same space.

1. Introduction

The term of fuzzy sets was studied originally by Zadeh in his paper [1]. Then, Chang [2], introduced the concept of fuzzy topological space. Later, as an extension of Zadeh's study of fuzzy sets, Coker [3] defined the topology of intuitionistic fuzzy sets. The concept intuitionistic fuzzy sets was introduced by Atanassov [4]. The expression "intuitionistic" evaporate used in literature until 2005, when Gutierrez Garcia and Rodabaugh [5], they suggested that the double fuzzy set is a more appropriate name than intuitionistic and completed that their research project under the name " double" rather than intuitionistic.

The goal of this present is to continue and to the allocation study of Fatimah et al. [6,7]. Also, we will give new definitions of double fuzzy α^m-continuous function, double fuzzy α^m-open function and double fuzzy-α^m generalized-continuous function. We study them with various examples.

2. Preliminaries

Throughout this present paper, spaces X and Y always means non empty sets and I is the closed interval [0,1], I_{r0}=(0,1] and I_{s1}= [0,1). The class of all fuzzy sets in X and Y are denoted by I^X and I^Y respectively. By $\overline{0}$ and $\overline{1}$, we denote the smallest and the greatest fuzzy sets on X. For a fuzzy set $\lambda_1 \in I^X$. For two fuzzy sets ρ_1 and δ_1 in X where $\rho_1 = \{(x, \mu_{\rho1}(x)\text{: } x \in X\}$ and $\delta_1 = \{(x, \mu_{\delta1}(x)\text{: } x \in X\}$, then their union $\rho_1 \vee \delta_1$, intersection $\rho_1 \wedge \delta_1$ and complement $\rho_1^{\,c} = \overline{1}- \rho_1$ and the subset $\rho_2 \le \delta_2$ if and only if $\mu_{\rho2}(x) \le \mu_{\delta2}(x)$ and $\gamma_{\rho2}(x) \ge \gamma_{\delta2}(x)$ for all $x \in X$, where $\rho_2=\{<x, \mu_\rho(x), \gamma_\rho(x)>\text{: } x \in X\}$, $\delta_2 = \{<x, \mu_\delta(x), \gamma_\delta(x)>\text{: } x \in X\}$. All other notations are standard notations of fuzzy set theory.[1]

We recall the following definitions used in this paper.

Definition 2.1 [5] A double fuzzy topology (τ_X, τ_X^*) on a non-empty set X is a pair of functions $\tau_X, \tau_X^*\text{: } I^X \to I$, which satisfies the following properties:

$(O1)$ $\tau_X(\lambda_1) \le \overline{1} - \tau_X^*(\lambda_1)$ for each $\lambda_1 \in I^X$.

$(O2)$ $\tau_X(\lambda_1 \wedge \lambda_2) \ge \tau_X(\lambda_1) \wedge \tau_X(\lambda_2)$ and $\tau_X^*(\lambda_1 \wedge \lambda_2) \le \tau_X^*(\lambda_1) \vee \tau_X^*(\lambda_2)$ for each $\lambda_1, \lambda_2 \in I^X$.

$(O3)$ $\tau_X(\vee_{i\in r} \lambda_i) \ge \wedge_{i\in r} \tau_X(\lambda_i)$ and $\tau_X^*(\vee_{i\in r}\lambda_i) \le \vee_{i\in r}\tau_X^*(\lambda_i)$ for each $\lambda_i \in I^X, i \in r$.

The triplex (X, τ_X, τ_X^*) is called a double fuzzy topological spaces (dfts, for short), and denoted by X.

Definition 2.2 [5, 6] If X is a dfts. Then a double fuzzy closure operator and double fuzzy interior operator of $\lambda_1 \in I^X$ are defined by:

$C_{\tau_x,\tau_x^*}(\lambda_1, r_0, s_1) = \wedge \{\mu_1 \in I^X, \lambda_1 \le \mu_1, \tau_X(\overline{1} - \mu_1) \ge r_0, \tau_X^*(\overline{1} - \mu_1) \le s_1\}$,

$I_{\tau_x,\tau_x^*}(\lambda_1, r_0, s_1) = \vee \{\mu_1 \in I^X, \mu_1 \le \lambda_1, \tau_X(\mu_1) \ge r_0, \tau_X^*(\mu_1) \le s_1\}$.

where $r_0 \in I_{r0}$ and $s_1 \in I_{s1}$ with $r_0 + s_1 \leq \bar{1}$.

Definition 2.3 Let X be a dfts $\lambda_1, \mu_1 \in I^X$, $r_0 \in I_{r0}$ and $s_1 \in I_{s1}$. A fuzzy set λ_1 is called:

1. An (r_0, s_1)-fuzzy open set (r_0, s_1)-fo, for short) [6] if $\tau_X(\lambda_1) \geq r_0$ and $\tau_X^*(\lambda_1) \leq s_1$, whenever $r_0 \in I_{ro}$ and $s_1 \in I_{s1}$. A fuzzy set λ_1 is called an (r_0, s_1)-fuzzy closed set ((r_0, s_1)-fc, for short), whenever $\tau_X(\bar{1}-\lambda_1) \geq r_0$ and $\tau_X^*(\bar{1}-\lambda_1) \leq s_1$.
2. An (r_0, s_1)-fuzzy α-open set ((r_0, s_1)-fα-open, for short) [8], if $\lambda_1 \leq I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda_1, r_0, s_1), r_0, s_1))$ and an (r_0, s_1)-fuzzy α-closed set ((r_0, s_1)-fα-closed, for short), if $C_{\tau,\tau^*}(I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda_1, r_0, s_1), r_0, s_1)) \leq \lambda_1$.
3. An (r_0, s_1)-generalized fuzzy closed ((r_0, s_1) -gf closed, for short) [9], if $C_{\tau x,\tau x^*}(\lambda_1, r_0, s_1) \leq \mu_1$ whenever $\lambda_1 \leq \mu_1$, $\tau_X(\mu_1) \geq r_0$ and $\tau_X^*(\mu_1) \leq s_1$. λ_1 is called (r_0, s_1)-generalized fuzzy open ((r_0, s_1)-gf open, for short) if $(\bar{1}-\lambda_1)$ is an (r_0, s_1)-gfc set.

Definition 2.4 [6] Let X and Y be two dfts's. A function f : X → Y is said to be a double fuzzy continuous function iff $\tau_x(f^{-1}(\nu)) \geq \tau_Y(\nu)$ and $\tau_X^*(f^{-1}(\nu)) \leq \tau_Y^*(\nu)$ for each $\nu \in I^Y$.

Definition 2.5 [7] A subset λ_1 in a double fuzzy topological space (X, τ_X, τ_X^*) is called (r_0, s_1)-fuzzy α^m- closed sets ((r_0,s_1)-f α^m-closed, for short) iff $I_{\tau x,\tau x^*}(C_{\tau x,\tau x^*}(\lambda_1,r_0, s_1),r_0, s_1) \leq \mu_1$, whenever $\lambda_1 \leq \mu_1$ and μ_1 is an (r_0, s_1)-α-open for each $\mu_1 \in I^X$, $r_0 \in I_{r0}$ and $s_1 \in I_{s1}$. λ_1 is called (r_0, s_1)-fα^m-open iff $\bar{1}-\lambda_1$ an (r_0, s_1)-fα^m-closed.

Definition 2.6 [7] If X is a dfts, for each $\lambda_1, \mu_1 \in I^X$, $r_0 \in I_{r0}$ and $s_1 \in I_{s1}$ then, the α^m-Closure and α^m-Interior operator of λ_1 is defined as:

$\alpha^m C_{\tau_X,\tau_X^*}(\lambda_1, r_0, s_1) = \wedge\{\mu_1 \in I^X: \lambda_1 \leq \mu_1, \mu_1$ is (r_0, s_1)-fα^m-closed$\}$.

$\alpha^m I_{\tau_X,\tau_X^*}(\lambda_1, r_0, s_1) = \vee\{\mu_1 \in I^X: \lambda_1 \geq \mu_1, \mu_1$ is (r_0, s_1)-fα^m-open$\}$.

3. Continuous Functions Via (r_0, s_1)- Fuzzy α^m-Closed Sets

In this section, we introduce new continuous functions via (r_0, s_1)-fuzzy α^m-closed sets called them double fuzzy α^m-continuous functions, double fuzzy α^m-open functions and double fuzzy- α^m generalized -continuous functions. After that, we get some propositions, theorems to show the relationships between different functions.

Proposition 3.1 Let (X, τ_X, τ_X^*) be dfts . λ_1 is (r_0, s_1)-fα^m-open in X iff μ_1 is (r_0, s_1)-fα-closed set such that $\mu_1 \leq \lambda_1$ and $\mu_1 \leq C_{\tau x,\tau x^*}(I_{\tau x,\tau x^*}(\lambda_1, r_0, s_1), r_0, s_1)$ whenever, $r_0 \in I_{r0}$ and $s_1 \in I_{s1}$.

Proof. λ_1 is (r_0, s_1)-fα^m-open then, $\bar{1}-\lambda_1$ is (r_0, s_1)-fα^m-closed. So, $\bar{1}-\lambda_1 \leq U$, where U is (r_0, s_1)-fα-open set then, $I_{\tau x,\tau x^*}(C_{\tau,\tau^*}(\bar{1}-\lambda_1, r_0, s_1), r_0, s_1) \leq U$. Put $\bar{1}-\lambda_1 = \mu_1$ and $\bar{1}- C_{\tau x,\tau x^*}(I_{\tau x,\tau x^*}(\lambda_1, r_0, s_1), r_0, s_1) \leq U$, for each $\mu_1 \leq \lambda_1$ and $\mu_1 \leq C_{\tau x,\tau x^*}(I_{\tau x,\tau x^*}(\lambda_1, r_0, s_1))$.

⇐ To prove $\bar{1}-\lambda_1$ is (r_0, s_1)-fα^m-closed set. We take, λ_1 be (r_0, s_1)-fα^m-open. So, for each μ_1 is (r_0, s_1)-fα-closed set. Put $\bar{1}-\mu_1 = \nu$.

Then, $\bar{1}-\mu_1 \geq \bar{1}- C_{\tau x,\tau x^*}(I_{\tau x,\tau x^*}(\lambda_1, r_0, s_1), r_0, s_1)$ therefor $\bar{1}-\mu_1 \geq I_{\tau x,\tau x^*}(C_{\tau x,\tau x^*}(\bar{1}-\lambda_1, r_0, s_1), r_0, s_1)$ for each $\lambda_1 \leq \mu_1$ so, $(\bar{1}-\lambda_1)$ is (r_0, s_1)-fα^m-closed.

Definition 3.2 Let (X, τ_X, τ_X^*) be a dfts $\lambda_1, \mu_1 \in I^X$, $r_0 \in I_{r0}$, $s_1 \in I_{s1}$, λ_1 is called an (r_0, s_1)-α^m-generalized fuzzy closed set (for short, (r_0, s_1)-α^m-gf-closed set) if $\alpha^m C_{\tau x,\tau x^*}(\lambda_1, r_0, s_1) \leq \mu_1$ such that $\lambda_1 \leq \mu_1$ and μ_1 is an (r_0, s_1)-fα^m-open set. λ_1 is called an (r_0, s_1)-α^m-generalized fuzzy open (for short, (r_0, s_1)-α^m-gf-open set) if $\bar{1}-\lambda_1$ is an (r_0, s_1)-α^m-gf- closed set.

Definition 3.3 Let X and Y are two dfts's for each $\lambda_1 \in I^X$, $\mu_1 \in I^Y$, $r_0 \in I_{r0}$ and $s_1 \in I_{s1}$. Then a function f: X → Y is called:

(1) A double fuzzy α^m-continuous functions (df-α^m-c, for short) if $f^{-1}(\mu_1)$ is an (r_0, s_1)-fα^m-open such that $\tau_Y(\mu_1) \geq r_0$ and $\tau_Y^*(\mu_1) \leq s_1$.

(2) A double fuzzy α^m-open functions (dfα^m-open, for short) if $f(\lambda_1)$ is an (r_0, s_1)-fα^m-open in Y for each $\tau_X(\lambda_1) \geq r_0$ and $\tau_X^*(\lambda) \leq s_1$.

(3) A double fuzzy-α^m-closed (df-α^m-closed, for short) if $f(\lambda_1)$ is an (r_0, s_1)-fα^m-closed in Y for each $\tau_X(\bar{1}-\lambda_1) \geq r_0$ and $\tau_X^*(\bar{1}-\lambda_1) \leq s_1$.

(4) A double fuzzy α^m generalized-continuous function (df-α^mg-c, for short) if the $f^{-1}(\mu_1)$ is an (r_0,s_1)-α^m-gf-closed set in X for each $\tau_Y(\bar{1}-\mu_1) \geq r_0$ and $\tau_Y^*(\bar{1}-\mu_1) \leq s_1$.

Remark 3.4

1- Every (r_0, s_1)-fuzzy closed set is an (r_0, s_1)-fuzzy-α^m-closed set.

2- Every (r_0, s_1)- fuzzy α^m-closed set is an (r_0, s_1)-α^m gf-closed set.

Theorem 3.4 Let (X, τ_X, τ_X^*) and (Y, τ_Y, τ_Y^*) be a dfts's. If f:(X, τ_X, τ_X^*) → (Y, τ_Y, τ_Y^*) is a double fuzzy continuous function, then f is a double fuzzy -α^m-continuous function.

Proof. Suppose that X and Y be a dfts's, f: X→ Y, $\tau_Y(\bar{1}-\lambda_1) \geq r_0$, $\tau_Y^*(\bar{1}-\lambda_1) \leq s_1$. Then, $f^{-1}(\bar{1}-\lambda_1)$ is (r_0, s_1)-fuzzy closed set in X. Since every (r_0, s_1)-fuzzy closed set is (r_0, s_1)-fuzzy-α^m-closed set so, $f^{-1}(\bar{1}-\lambda_1)$ is (r_0, s_1)- fuzzy -α^m-closed set in X. Therefore, f is double fuzzy-α^m-continuous function.

Theorem 3.5 Let f: X → Y be a function between dfts's X and Y, f is df-α^m-c function iff $f^{-1}(\lambda_1)$ is (r_0,s_1)-fα^m-open set in X, such that $\tau_Y(\lambda_1) \geq r_0$, $\tau_Y^*(\lambda_1) \leq s_1$, whenever $\lambda_1 \in I^X$, $r_0 \in I_{r0}$ and $s_1 \in I_{s1}$.

Proof. Suppose that f: X → Y is dfα^m-c function, $\tau_Y(\lambda_1) \geq r_0$, $\tau_Y^*(\lambda_1) \leq s_1$, then $\tau_Y(\bar{1}-\lambda_1) \geq r_0$ and $\tau_Y^*(\bar{1}-\lambda_1) \leq s_1$.

But $f^{-1}(\bar{1}-\lambda_1) = \bar{1}- f^{-1}(\lambda_1)$ is an (r_0, s_1)-fα^m-closed set in X. So $f^{-1}(\lambda_1)$ is an (r_0,s_1)-fα^m-open set in X.

← Suppose that $f^{-1}(\lambda_1)$ is an (r_0, s_1)-fα^m -open set in X, put $\mu_1 = \bar{1}-\lambda_1$.

So, $\tau_Y(\bar{1}-(\bar{1}-\mu_1)) \geq r_0$ and $\tau_Y^*(\bar{1}-(\bar{1}-\mu_1)) \leq s_1$.

Since $f^{-1}(\bar{1}-\mu_1) = \bar{1}- f^{-1}(\mu_1)$ is an (r_0, s_1)-fα^m- open set in X, so $f^{-1}(\mu_1)$ is an (r_0, s_1)-fα^m- closed set in X. Therefore f is df α^m-c function.

Proposition 3.6 Let X and Y be dfts's. f: X → Y is a double fuzzy-continuous function, then f is a double fuzzy-α^m generalized-continuous function.

Proof. Let $\tau_Y(\mu_1) \geq r_0$ and $\tau_Y^*(\mu_1) \leq s_1$,since f is df-c, then

$\tau_X(f^{-1}(\mu_1)) \geq r_0$ and $\tau_X^*(f^{-1}(\mu_1)) \leq s_1$

Since, every an(r_0,s_1)-fuzzy open set is an (r_0,s_1)-α^mgf-open set.
That is for each $\tau_Y(\mu_1) \geq r_0$ and $\tau_Y^*(\mu_1) \leq s_1$, $f^{-1}(\mu_1)$ is an (r_0,s_1)- α^mgf-open set in X.
Therefore, f is df-α^mg-c function.

Proposition 3.7 Let X and Y be a dfts's. If f: X → Y is double fuzzy-α^m-continuous, then f is double fuzzy-α^m generalized-continuous.

Proof . Suppose that X and Y are dfts's and f: X → Y, $\tau_Y(\bar{1}-\lambda_1) \geq r_0$, $\tau_Y^*(\bar{1}-\lambda_1) \leq s_1$. Since f is df-$\alpha^m$-continuous, then
$f^{-1}(\bar{1}-\lambda_1)$ is (r_0, s_1)-fα^m-closed set in X
Since every (r_0, s_1)-fα^m-closed set is an (r_0, s_1)-α^m-gf closed set. Therefore $f^{-1}(\bar{1}-\lambda_1)$ is (r_0, s_1)-α^m-gf closed set in X. that is f is df-α^mg-c.

Definition 3.8 Let X be a dfts and $\lambda_1 \in I^X$. The α^m-generalized closure of the set λ_1 denoted by $\alpha^m GC_{\tau x,\tau x^*}(\lambda_1, r_0, s_1)$ is the intersection of all (r_0,s_1)- α^m-gf closed set of X such that $\lambda_1 \leq \alpha^m GC_{\tau x,\tau x^*}(\lambda_1, r_0, s_1)$.

Remark 3.9 It is clear that $\lambda_1 \leq \alpha^m GC_{\tau x,\tau x^*}(\lambda_1, r_0, s_1) \leq C_{\tau x,\tau x^*}(\lambda_1, r_0, s_1)$ for each $\lambda_1 \in I^X$.

Theorem 3.10 Let X and Y be a dfts's. If f: X → Y is df-α^mg-c function then,
$f(\alpha^m GC_{\tau x,\tau x^*}(\lambda_1, r_0, s_1)) \leq C_{\tau y,\tau y^*}(f(\lambda_1, r_0, s_1)$,
for each $\lambda_1 \in I^X$.

Proof. Let $\lambda_1 \in I^X$ and $C_{\tau y,\tau y^*}(f(\lambda_1), r_0, s_1)$ be an (r_0, s_1)-f closed set in Y.
Since, f is df- α^mg-c function , $f^{-1}(C_{\tau y,\tau y^*}(f(\lambda_1), r_0, s_1))$ is an (r_0, s_1)-α^m gf-closed set in X.
And, $\lambda_1 \leq f^{-1}(f(\lambda_1))$.
Then, $\lambda_1 \leq f^{-1}(C_{\tau y,\tau y^*}(f(\lambda_1), r_0, s_1))$.

Therefore by Remark 3.9, $\alpha^m GC_{\tau x,\tau x^*}(\lambda_1, r_0, s_1) \leq f^{-1}(C_{\tau y,\tau y^*}(f(\lambda_1), r_0, s_1))$.
Hence, $f(\alpha^m GC_{\tau x,\tau y^*}(\lambda_1, r_0, s_1)) \leq C_{\tau y,\tau y^*}(f(\lambda_1), r_0, s_1)$.

Definition 3.11 A dfts X is called double fuzzy $\alpha^m(\tau_X, \tau_X^*)_{\frac{1}{2}}$ space (dfα^m-$(\tau_X, \tau_X^*)_{\frac{1}{2}}$, for short) if each (r_0,s_1)-α^mgf-closed set in X is an (r_0, s_1)-fα^m-closed set in X.

Theorem 3.12 Let f: X → Y be a df-α^mg-c function and g: Y → Z is a df-c function, then gof: X→ Z is a df-α^mg-c function.

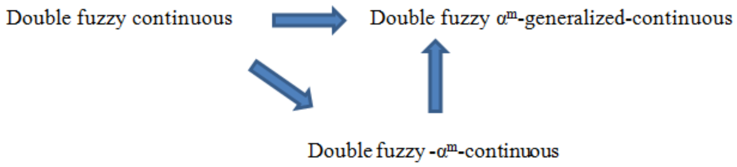
Proof. Let $\tau_Z(\bar{1}-\lambda_1) \geq r_0$ and $\tau_Z^*(\bar{1}-\lambda_1) \leq s_1$, since g is df-c function and
$\tau_Y(g^{-1}(\bar{1}-\lambda_1)) \geq r_0$ and $\tau_Y^*(g^{-1}(\bar{1}-\lambda_1)) \leq s_1$
Since, $f^{-1}(g^{-1}(\bar{1}-\lambda_1))$ is an (r_0,s_1)-α^mgf-closed set, so $(gof)^{-1}(\bar{1}-\lambda_1) = f^{-1}(g^{-1}(\bar{1}-\lambda_1))$ is an (r_0,s_1)-α^mgf-closed set in X. That is gof is df-α^mg-c function.

Theorem 3.13 Let X, Y and Z be adfts's. If f: X → Y and g: Y→ Z are two df-α^mg-c such that Y is df-$\alpha^m(\tau_Y, \tau_Y^*)_{\frac{1}{2}}$ space, then gof: X → Z is df- α^mg-c function.

Proof. Let $\tau_Z(\bar{1}-\lambda_1) \geq r_0$ and $\tau_Z^*(\bar{1}-\lambda_1) \leq s_1$, since g is df-α^mg-c function and $g^{-1}(\bar{1}-\lambda_1)$ is an (r_0, s_1)- α^mgf -closed set in Y.
$f^{-1}(g^{-1}(\bar{1}-\lambda_1))$ is an (r_0, s_1)-α^mgf-closed set in X, because f is df-α^mg-c function.
$(gof)^{-1}(\bar{1}-\lambda_1) = f^{-1}(g^{-1}(\bar{1}-\lambda_1)$ is an (r_0,s_1) -α^mgf-closed set. That is gof is df-α^mg-c function.

4. Interrelations

The following implication explain the relationship between different functions:



Remark 4.1 The following example explain the convers of above relationship is not true.

Example 4.2

1. Let X={p, q} , Y= {m, n} and δ_1, δ_2 are fuzzy sets, we define $(\tau_X(\delta), \tau_{X^*}(\delta))$ on X by:

$$\tau_X(\delta) = \begin{cases} \bar{1}, & if \quad \delta \in \{\bar{0}, \bar{1}\}, \\ \frac{1}{2}, & \delta(x) = \delta_1 \\ \frac{1}{4}, & \delta(x) = \delta_2 \\ \bar{0}, & otherwise \end{cases},$$

$$\tau_X^*(\delta) = \begin{cases} \bar{0}, & if \quad \delta \in \{\bar{0}, \bar{1}\}, \\ \frac{1}{2}, & \delta(x) = \delta_1 \\ \frac{3}{4}, & \delta(x) = \delta_2 \\ \bar{1}, & otherwis \end{cases}$$

Such that, $\delta_1(p) = 0.4$, $\delta_1(q) = 0.4$,
And, $\delta_2(p) = 0.6$, $\delta_2(q) = 0.7$.
Also, we define $(\tau_Y(\Psi), \tau_{Y^*}(\Psi))$ on Y by:

$$\tau_Y(\Psi) = \begin{cases} \bar{1}, & if \quad \Psi \in \{\bar{0}, \bar{1}\}, \\ \frac{1}{2}, & \Psi(y) = \Psi_1 \\ \bar{0}, & otherwise \end{cases},$$

$$\tau_Y^*(\Psi) = \begin{cases} \bar{0}, & if \quad \Psi \in \{\bar{0}, \bar{1}\}, \\ \frac{1}{2}, & \Psi(y) = \Psi_1 \\ \bar{1}, & otherwise \end{cases}$$

Such that, $\Psi_1(m) = 0.6$, $\Psi_1(n) = 0.2 \rightarrow \Psi_1^c(m) = 0.4$, $\Psi_1^c(n) = 0.8$.
When, the function f between two dfts's $(\tau_X(\delta), \tau_X^*(\delta))$ and $(\tau_Y(\Psi), \tau_Y^*(\Psi))$ is defined by:
f: $(X, \tau_X, \tau_X^*) \rightarrow (Y, \tau_Y, \tau_Y^*)$ as, f(p) = m, f(q) = n.
So, $I_{\tau x,\tau x^*}(C_{\tau x,\tau x^*}(\Psi_1^c, \frac{1}{2}, \frac{1}{2})\frac{1}{2}, \frac{1}{2}) = I_{\tau x,\tau x^*}(\delta_1^c, \frac{1}{2}, \frac{1}{2}) = \delta_2$ and, $f^{-1}(\Psi_1^c)$ is an $(\frac{1}{2}, \frac{1}{2})$-f$\alpha^m$- closed set → $f^{-1}(\Psi_1)$ is an $(\frac{1}{2}, \frac{1}{2})$-f$\alpha^m$-open set.
That is, f is df- α^m-c function, but $f^{-1}(\Psi_1) \notin \tau_X$ → f is not df-c function.

2. Let X={p, q} and Y= {m, n}, and take $(\tau_X(\delta), \tau_X^*(\delta))$ and $(\tau_Y(\Psi), \tau_Y^*(\Psi))$ on X and Y respectively, by as follow as (1), such that:
$\delta_1(p) = 0.3$, $\delta_1(q) = 0.4$,
$\delta_2(p) = 0.7$, $\delta_2(q) = 0.6$,

$\Psi_1(m) = 0.7$, $\Psi_1(n) = 0.8$,
And, $\Psi_2(m) = 0.3$, $\Psi_2(n) = 0.2$.
When, the function f between two dfts $(\tau_X(\delta), \tau_X^*(\delta))$ and $(\tau_Y(\Psi), \tau_Y^*(\Psi))$ is defined by:
$f: (X, \tau_X, \tau_X^*) \rightarrow (Y, \tau_Y, \tau_Y^*)$ as, $f(p) = m$, $f(q) = n$.
So, $f^{-1}(\Psi_1) = (p_{0.7}, q_{0.8})$, $f^{-1}(\Psi_1) \leq \delta_2$, δ_2 is an $(\frac{1}{2}, \frac{1}{2})$-$f\alpha^m$-closed set.
$\alpha^m C_{\tau x,\tau x^*}(f^{-1}(\Psi_1), \frac{1}{2}, \frac{1}{2}) = \wedge\{\delta_2 \in I^X, f^{-1}(\Psi_1) \leq \delta_2$, then $I_{\tau x,\tau x^*}(C_{\tau x,\tau x^*}(\delta_2, \frac{1}{2}, \frac{1}{2}) \frac{1}{2}, \frac{1}{2}) = \delta_2 \leq \delta_2$

But, $f^{-1}(\Psi_1)$ is an $(\frac{1}{2}, \frac{1}{2})$-$\alpha^m$gf-closed set $\rightarrow$ f is df-α^mg-c function.
Since $f^{-1}(\Psi_1) \notin \tau_X \rightarrow$ f is not df- c function.
And $f^{-1}(\Psi_1) \not\leq C_{\tau x,\tau x^*}(, I_{\tau x,\tau x^*}(f^{-1}(\Psi_1), \frac{1}{2}, \frac{1}{2}) \frac{1}{2}, \frac{1}{2})$, since
$C_{\tau x,\tau x^*}(, I_{\tau x,\tau x^*}(f^{-1}(\Psi_1), \frac{1}{2}, \frac{1}{2}) \frac{1}{2}, \frac{1}{2}) = \bar{0}$, $f^{-1}(\Psi_1) \not\leq \bar{0}$, so $f^{-1}(\Psi_1)$ is not $(\frac{1}{2}, \frac{1}{2})$- $f\alpha^m$- open set. That is f is not df- α^m-c function.

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بعض انواع الدوال المستمرة عن طريق المجموعات المغلقة– (r_0, s_1)-fuzzy α^m

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الملخص

الغرض من هذا البحث هو تقديم ودراسة مصطلحات بعض انواع الدوال المستمرة عن طريق المجموعات. في الفضاءات التبولوجية المضببة المزدوجة. ايضا توصلنا الى بعض العلاقات بين (r_0, s_1)-fuzzyα^m المغلقة. انواع الجديدة من الدوال ومقارنتها مع الاتجاه المقابل لها مع الامثلة التوضيحية في نفس الفضاء.