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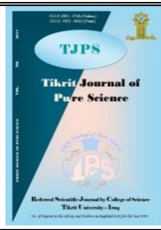

Some New Types of open Functions Via (l_0, m_1)-Fuzzy alpha^m- Closed Sets

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Abstract

In this paper, we derive more on $alpha^m$-continuous functions and α-irresolute functions with $alpha^m$-open functions and $alpha^m$-closed functions in double fuzzy topological spaces via (l_0, m_1)-fuzzy alpha^m-closed sets. Also, we reached some relationships between these new types of functions and compare them with their opposite with illustrative examples in the same space.

1. Introduction

After the conference paper of Zadeh [1], the search of intuitionistic fuzzy sets started by Atanassove [2], [3] where he added another membership function to Zade's function and called it non-membership function, but the definition of the topology in Chang's sense gave by Coker [4] Later that, Samanta and Mondal [5] introduced the notion of intuitionistic gradation of openness of fuzzy sets. The expression "intuitionistic" evaporate used in literature until 2005, when Gutierrez Garcia and Rodabaugh [6] concluded that the most suited work under the name "double ".

In this paper we discuss $alpha^m$-continuous function and α-irresolute with $alpha^m$-open function and $alpha^m$-closed function. Also, discuss some characterization of the new concepts.

2. Preliminaries

Throughout this paper, $(X, \tau_{X1}, \tau_{X1}^*)$ and $(Y, \tau_{Y2}, \tau_{Y2}^*)$ represent to double fuzzy topological spaces. Suppose X be any non-empty set and $I_{l_0} = (0,1]$, $I_{m1} = [0,1)$ which are subset of closed interval $I = [0,1]$. For any fuzzy set λ in $(X, \tau_{X1}, \tau_{X1}^*)$. 1- λ is denote the complement of λ in X.

Definition 2.1 [6]. Let X be a non-empty set and a double fuzzy topology (τ_X, τ_X^*) is a pair of functions $\tau_X, \tau_X^*: I^x \to I$, which satisfies the following properties:

$(O1)$ $\tau_X(\lambda_1) \leq 1 - \tau_X^*(\lambda_1)$ for each $\lambda_1 \in I^x$.

$(O2)$ $\tau_X(\lambda_1 \wedge \lambda_2) \geq \tau_X(\lambda_1) \wedge \tau_X(\lambda_2)$ and $\tau_X^*(\lambda_1 \wedge \lambda_2) \leq \tau_X^*(\lambda_1) \vee \tau_X^*(\lambda_2)$ for each $\lambda_1, \lambda_2 \in I^x$.

$(O3)$ $\tau_X(\vee_{i\in r} \lambda_i) \geq \wedge_{i\in r} \tau_X(\lambda_i)$ and $\tau_X^*(\vee_{i\in r}\lambda_i) \leq \vee_{i\in r}\tau_X^*(\lambda_i)$ for each, $\lambda_i \in I^x, i \in r$.

The triplex (X, τ_X, τ_X^*) is called a double fuzzy topological spaces (dfts, for short).

Definition 2.2 [6]. If (X, τ_X, τ_X^*) be a dfts. Then double fuzzy closure operator and double fuzzy interior operator of $\lambda_1 \in I^X$ are defined by:

$C_{\tau,\tau^*}(\lambda_1, l_0, m_1) = \wedge \{\beta \in I^x, \lambda_1 \leq \beta, \tau(1-\beta) \geq l_0, \tau^*(1-\beta) \leq m_1\}$,

$I_{\tau,\tau^*}(\lambda_1, l_0, m_1) = \vee \{\beta \in I^x, \beta \leq \lambda_1, \tau(\beta) \geq l_0, \tau^*(\beta) \leq m_1\}$.

Where $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$ with $l_0 + m_1 \leq 1$.

Definition 2.3[11] Let (X, τ_X, τ_X^*) be a dfts. A fuzzy point is defined by

$$x_t(y) = \begin{cases} t, & if\ y = x \\ , & \\ 0, & if\ y \neq x \end{cases} \quad \text{for } x \in X \text{ and } t \in I_0.$$

Definition 2.4 [11] Let (X, τ_X, τ_X^*) be a dfts. $\lambda \in I^X$, $x_t \in p_t(x)$, $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$. A fuzzy set λ is called (l_0, m_1)-fuzzy neighborhood of x_t if $\tau(\lambda) \geq l_0$, $\tau^*(\lambda) \leq m_1$ and $x_t q \lambda$.

Now, we introduce the following definitions :-

Definition 2.5 Let λ be a subset of a dfts (X, τ_X, τ_X^*) $l_0 \in I_{l0}$, $m_1 \in I_{m1}$ and $\lambda \in I^x$ is called:

1. An (l_0, m_1)-fuzzy α-open set[7] if $\lambda \leq I_{\tau,\tau^*}(C_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda, l_0, m_1), l_0, m_1), l_0, m_1)$. And an (l_0, m_1)-fuzzy α-closed set if $C_{\tau,\tau^*}(I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, l_0, m_1), l_0, m_1), l_0, m_1) \leq \lambda$.
2. An (l_0, m_1)-fuzzy $alpha^m$-closed set [8] (briefly, (l_0, m_1)-f α^m-closed) if $I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, l_0, m_1), l_0, m_1) \leq \beta$, whenever $\lambda \leq \beta$ and β is (l_0, m_1)-fα-open.
3. An (l_0, m_1)-fuzzy $alpha^m$-open set[8] (briefly, (l_0, m_1)-fα^m-open) iff 1-λ is (l_0, m_1)-fuzzy α^m- closed.

Definition 2.6 [8] If (X, τ, τ^*) be a dfts. So, for each $\lambda, \mu \in I^x$, $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$.

We have the double fuzzy α^m-Closure and double fuzzy α^m-Interior of λ is defined as:

$\alpha^m\, C_{\tau,\tau^*}(\lambda, l_0, m_1) = \wedge\{\beta \in I^X : \lambda < \beta, \beta \text{ is } (l_0, m_1)\text{-f}\alpha^m\text{-closed set}\}$.

$\alpha^m I_{\tau,\tau^*}(\lambda, l_0, m_1) = \vee\{\beta \in I^X : \beta < \lambda, \beta \text{ is } (l_0, m_1)\text{-f}\alpha^m\text{-open set}\}$.

Definition 2.7 Let $(X, \tau_{X1}, \tau_{X1}^*)$ and $(Y, \tau_{Y2}, \tau_{Y2}^*)$ be two dfts's. A function f: $(X, \tau_{X1}, \tau_{X1}^*) \rightarrow (Y, \tau_{Y2}, \tau_{Y2}^*)$ is called**:**

1. Double fuzzy irresolute [9] (briefly, df-irr) if $f^{-1}(\beta)$ is (l_0, m_1)-fs-open set for each $\beta \in I^Y$, $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$.
2. Double fuzzy continuous [10] if $(f^{-1}(\gamma)) \geq \tau_{Y2}(\gamma)$ and $\tau_{X1}^*(f^{-1}(\gamma)) \leq \tau_{Y2}^*(\gamma)$ for each $\gamma \in I^Y$.
3. Double fuzzy open function[2] (briefly, df-open) if $\tau_{Y2}(f(\lambda)) \geq \tau_{X1}(\lambda)$ and $\tau_{Y2}^*(f(\lambda)) \leq \tau_{X1}^*(\lambda)$ for each $\lambda \in I^X$, $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$.
4. Double fuzzy closed[10] (briefly, df-closed) if $\tau_{Y2}(f(1-\lambda)) \geq \tau_{X1}(1-\lambda)$ and $\tau_{Y2}^*(f(1-\lambda)) \leq \tau_{X1}^*(1-\lambda)$ for each $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$.

3. Double fuzzy α^m-open functions

In this part of the research, we generalized the definitions (2.5) but in the function, and used to find new results in dfts.

Definition 3.1 Let $(X, \tau_{X1}, \tau_{X1}^*)$ and $(Y, \tau_{Y2}, \tau_{Y2}^*)$ be two dfts's. $\lambda \in I^X$, $\beta \in I^Y$, $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$. A function f: $(X, \tau_{X1}, \tau_{X1}^*) \rightarrow (Y, \tau_{Y2}, \tau_{Y2}^*)$ is called:

1. Double fuzzy $alpha^m$-open function (briefly, dfα^m-open) if for each (l_0, m_1)-f α^m-open set λ, f(λ) is (l_0, m_1)-f α^m open set.
2. Double fuzzy $alpha^m$-closed function (briefly, dfα^m-closed) if for each (l_0, m_1)-f α^m-closed set λ, f(λ) is (l_0, m_1)-f α^m-closed set.
3. Double fuzzy α-irresolute (briefly, dfα-irr) if $f^{-1}(\gamma)$ is (l_0, m_1)-fuzzy α- closed in X for each (l_0, m_1)-fuzzy α -closed set γ in Y.
4. Double fuzzy $alpha^m$-continuous (briefly, dfα^m-c) if $f^{-1}(\beta)$ is (l_0, m_1)-fα^m-open such that $\tau_{Y2}(\beta) \geq l_0$ and $\tau_{Y2}^*(\beta) \leq m_1$.

Now, we introduce the following theorem (3.2) expensive the relation between dfα^m-closed function and dfα^m-open set under condition.

Theorem 3.2 Let $(X, \tau_{X1}, \tau_{X1}^*)$ and $(Y, \tau_{Y2}, \tau_{Y2}^*)$ be two dfts's. Then a function f: $(X, \tau_{X1}, \tau_{X1}^*) \rightarrow (Y, \tau_{Y2}, \tau_{Y2}^*)$ is dfα^m-closed function iff for each $\mu_Y \leq Y$ and $f^{-1}(\mu_Y) \leq \beta$, there is an (l_0, m_1)-dfα^m-open set γ in Y such that $\mu_Y \leq \gamma$ and $f^{-1}(\gamma) \leq \beta$.

Proof. Let f be df α^m-closed function and suppose that $\mu_Y \leq Y$, $\tau_{X1}(\beta) \geq l_0$ and $\tau_{X1}^*(\beta) \leq m_1$ whenever, $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$ such that $f^{-1}(\mu_Y) \leq \beta$.

Then, $\gamma = 1 - f(1-\beta)$ is an (l_0, m_1)-fα^m-open set containing μ_Y such that $f^{-1}(\gamma) \leq \beta$.

Conversely, Let 1-β_x be an (l_0, m_1)-fuzzy closed set in X then, $f^{-1}(f(1-(1-\beta_x)) \leq \beta_x$, $\tau_{X1}(\beta_x) \geq l_0$ and $\tau_{X1}^*(\beta_x) \leq m_1$ by hypothesis, there exist $\gamma \in I^Y$ is an (l_0, m_1)-fα^m-open set such that f $(\beta_x) \leq \gamma$.

Since, $\beta_x \leq f^{-1}(f(\beta_x))$ then, $\beta_x \leq f^{-1}(\gamma)$ So, 1-$\beta_x \leq 1-(f^{-1}(\gamma))$ hence,

1- $\gamma \leq f(1-\beta_x) \leq f(1-f^{-1}(\gamma) \leq 1-\gamma \Rightarrow$ f(1-β_x) =1-γ.

Since 1-γ is (l_0, m_1)-f α^m-closed set then, f(1-β_x) is (l_0, m_1)-fα^m-closed. Therefore, f is df α^m-closed function.

Proposition 3.3 Let $(X, \tau_{X1}, \tau_{X1}^*)$ and $(Y, \tau_{Y2}, \tau_{Y2}^*)$ be two dfts's. and f: $(X, \tau_{X1}, \tau_{X1}^*) \rightarrow (Y, \tau_{Y2}, \tau_{Y2}^*)$ is dfα-irr and λ is an (l_0, m_1)-f α^m-closed in X . Then f (λ) is (l_0, m_1)-f α^m-closed in Y.

Proof. Suppose that β is an (l_0, m_1)-fα^m-open set in Y such that f(λ) $\leq \beta$ whenever, λ is (l_0, m_1)-fα^m-closed in X, $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$.

Since f is dfα-irr function $\lambda \leq f^{-1}(\beta)$ and $f^{-1}(\beta)$ is an (l_0, m_1)-fα^m-open set. Hence $C_{\tau,\tau^*}(\lambda, l_0, m_1) \leq f^{-1}(\beta)$ hence, λ is (l_0, m_1)-fα^m-closed. But, f($C_{\tau,\tau^*}(\lambda, l_0, m_1)$) is an (l_0, m_1)-fα^m-closed contained in the (l_0, m_1)-f α-open set β, this implies that $C_{\tau,\tau^*}(f(C_{\tau,\tau^*}(\lambda, l_0, m_1), l_0, m_1)) \leq \beta$ and $C_{\tau,\tau^*}(f(\lambda), l_0, m_1) \leq \beta$.

$\therefore f(\lambda)$ is an (l_0, m_1)-f α^m -closed set in Y.

Corollary 3.4 Let $(X, \tau_{X1}, \tau_{X1}^*)$, $(Y, \tau_{Y2}, \tau_{Y2}^*)$ and $(Z, \tau_{z3}, \tau_{z3}^*)$ be dfts's and the function f: $(X, \tau_{X1}, \tau_{X1}^*) \rightarrow (Y, \tau_{Y2}, \tau_{Y2}^*)$ is df α^m-closed and g: $(Y, \tau_{Y2}, \tau_{Y2}^*) \rightarrow (Z, \tau_{z3}, \tau_{z3}^*)$ is dfα^m-closed and df α-irr function. Then, g o f: $(X, \tau_{X1}, \tau_{X1}^*) \rightarrow (Z, \tau_{z3}, \tau_{z3}^*)$ is dfα^m-closed function.

Proof. Suppose λ is (l_0, m_1)-f closed set in X, whenever, $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$ by hypothesis f (λ) is an (l_0, m_1)-f α^m-closed set in Y. Since g is dfα^m-closed and dfα-irr by using Proposition 3.2, we get g(f(λ)) = (g of)(λ) is df α^m-closed in Z.

Then, we have g o f is dfα^m-closed function.

Theorem 3.5 Let $(X, \tau_{X1}, \tau_{X1}^*)$, $(Y, \tau_{Y2}, \tau_{Y2}^*)$ and $(Z, \tau_{z3}, \tau_{z3}^*)$ be dfts's. f: $(X, \tau_{X1}, \tau_{X1}^*) \rightarrow (Y, \tau_{Y2}, \tau_{Y2}^*)$ and g: $(Y, \tau_{Y2}, \tau_{Y2}^*) \rightarrow (Z, \tau_{z3}, \tau_{z3}^*)$ be two functions, where f is continuous and surjective and g o f: $(X, \tau_{X1}, \tau_{X1}^*) \rightarrow (Z, \tau_{z3}, \tau_{z3}^*)$ is dfα^m -closed function. Then, g is dfα^m-closed function.

Proof. Suppose that $\tau_{Y2}(1-\lambda) \geq l_0$ and $\tau_{Y2}^*(1-\lambda) \leq m_1$ whenever, $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$. Since f is continuous so, $\tau_{X1}(1- f^{-1}(\lambda)) \geq l_0$ and $\tau_{X1}{}^{*}(1 - f^{-1}(\lambda)) \leq m_1$. Since g o f is df$\alpha^m$-closed, (g o f)($f^{-1}(\lambda)$) is (l_0, m_1)-fα^m-closed in Z. That is, g(λ) is (l_0, m_1)-fα^m-closed in Z, but f is surjective. So, we get g is a dfα^m-closed function.

Proposition 3.6 Let $(X, \tau_{X1}, \tau_{X1}{}^{*})$, $(Y, \tau_{Y2}, \tau_{Y2}{}^{*})$ and $(Z, \tau_{z3}, \tau_{z3}{}^{*})$ be dfts's. And f: $(X, \tau_{X1}, \tau_{X1}{}^{*}) \to (Y, \tau_{Y2}, \tau_{Y2}{}^{*})$ is df-closed function and g: $(Y, \tau_{Y2}, \tau_{Y2}{}^{*}) \to (Z, \tau_{z3}, \tau_{z3}{}^{*})$ dfα^m-closed function. Then, g o f: $(X, \tau_{X1}, \tau_{X1}{}^{*}) \to (Z, \tau_{z3}, \tau_{z3}{}^{*})$ is dfα^m -closed function.

Proof. Suppose $\tau_{X1}(1-\lambda) \geq l_0$ and $\tau_{X1}{}^{*}(1-\lambda) \leq m_1$, whenever $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$.

Then, $\tau_{Y1}(1- f^{-1}(\lambda)) \geq l_0$ and and $\tau_{Y1}{}^{*}(1 - f^{-1}(\lambda)) \leq m_1$ by hypotheses, g(f(λ)) is dfα^m-closed in Z, so (g o f)(λ) is df α^m-closed in Z. That is, g o f is df α^m- closed function.

Remark 3.7 Every dfα^m-open function is df α^m-continuous function, but the convers is not true and we can show that by the following example.

Example 3.8 Let $X = \{a_1, b_1, c_1\}$ and $Y = \{x, y, z\}$. define fuzzy sets $\lambda_1, \lambda_2, \beta_1$ as follows:

$\lambda_1(a_1) = 0.4$, $\lambda_1(b_1) = 0.3$, $\lambda_1(c_1) = 0.2$,

$\lambda_2(a_1) = \beta_1(x) = 0.6$, $(b_1) = \beta_1(y) = 0.7$, $\lambda_2(c_1) = \beta_1(z) = 0.8$.

And the dfts's $(X, \tau_{X1}, \tau_{X1}{}^{*})$ and $(Y, \tau_{Y2}, \tau_{Y2}{}^{*})$ are define as follows:

$$\tau_1(\lambda) = \begin{cases} 1, & \text{if } \lambda = 0 \text{ or } 1 \\ \frac{1}{2}, & \text{if } \lambda = \lambda_1 \\ 0, & \text{otherwise} \end{cases}$$

$$, \quad \tau_1{}^{*}(\lambda) = \begin{cases} 0, & \text{if } \lambda = 0 \text{ or } 1 \\ \frac{1}{2}, & \text{if } \lambda = \lambda_1 \\ 1, & \text{otherwise} \end{cases}$$

$$\tau_2(\beta) = \begin{cases} 1, & \text{if } \beta = 0 \text{ or } 1 \\ \frac{1}{2}, & \text{if } \beta = \beta_1 \\ 0, & \text{otherwise} \end{cases} ,$$

$$\tau_2{}^{*}(\beta) = \begin{cases} 0, & \text{if } \beta = 0 \text{ or } 1 \\ \frac{1}{2}, & \text{if } \beta = \beta_1 \\ 1, & \text{otherwise} \end{cases}$$

The function f: $(X, \tau_{X1}, \tau_{X1}{}^{*}) \to (Y, \tau_{Y2}, \tau_{Y2}{}^{*})$ define as:

f (a_1) = x, f(b_1) = y, f(c_1) = z.

So, since β_1 is an $(\frac{1}{2}, \frac{1}{2})$-f open set and $f^{-1}(\beta_1) = \lambda_2$ is an $(\frac{1}{2}, \frac{1}{2})$-f$\alpha^m$ -c-open set then, $C_{\tau,\tau^*}(I_{\tau,\tau^*}(C_{\tau,\tau^*}(\beta, l_0, m_1), l_0, m_1), l_0, m_1) \leq \beta$ $\Rightarrow C_{\tau,\tau^*}(I_{\tau,\tau^*}\left(\left(\lambda_1{}^{c}, \frac{1}{2}, \frac{1}{2}\right), \frac{1}{2}, \frac{1}{2}\right)$

$C_{\tau,\tau^*}(\beta, \frac{1}{2}, \frac{1}{2}) = \lambda_1{}^{c} \leq \beta \Rightarrow \beta$ is an $\left(\frac{1}{2}, \frac{1}{2}\right)$-f α-closed and $\lambda \leq \lambda^c \Rightarrow \beta$ is an $(\frac{1}{2}, \frac{1}{2})$-f α^m-open.

Since $f^{-1}(\beta_1) = \lambda_2$ and $\lambda_2 \notin (\tau_{X1}, \tau_{X1}{}^{*})$

$\therefore f^{-1}(\beta_1) = \lambda_2$ is an $(\frac{1}{2}, \frac{1}{2})$-f α^m-open

So, β is an $(\frac{1}{2}, \frac{1}{2})$-f α^m-continuous But, f(β) is not α^m-open function.

Now, we introduce new concept $alpha^m$-neighborhood and theorem illustrates the important properties of dfα^m-open function .

Definition 3.9 Let $(X, \tau_X, \tau_X{}^{*})$ be a dfts and $\lambda \in I^X$. A subset δ of X is called fuzzy $alpha^m$-neighborhood of λ (f α^m-nbhd, for short) if there exist an (l_0, m_1)-f α^m-open set ρ_0 such that $\lambda \in \rho_0 \leq \delta$

Theorem 3.10 Let $(X, \tau_X, \tau_X{}^{*})$ be adfts's. λ is an (l_0, m_1)-fα^m-closed in X .Then $x \in \alpha^m C_{\tau,\tau^*}(\lambda, l_0, m_1)$ iff any fuzzy α^m -nbhd δ of x in X, $\delta \wedge \lambda \neq 0$.

Proof. Suppose that the fuzzy α^m-nbhd δ of $x \in I^X$ such that $\delta \wedge \lambda = 0$. So, there exist $\rho_0 \in I^X$ is an (l_0, m_1)-fuzzy α^m-open set such that $x \in \rho_0 \leq \delta$, whenever, $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$. So, we have $\rho_0 \wedge \lambda = 0$ and $x \in 1 - \rho_0$, then $\alpha^m C_{\tau,\tau^*}(\lambda, l_0, m_1) \in 1 - \rho_0$.

Therefore, $x \notin \alpha^m C_{\tau,\tau^*}(\lambda, l_0, m_1)$ which is contradiction to hypothesis $x \in \alpha^m C_{\tau,\tau^*}(\lambda, l_0, m_1)$ then, $\delta \wedge \lambda \neq 0$.

Conversely, Let $x \notin \alpha^m C_{\tau,\tau^*}(\lambda, l_0, m_1)$ then, there exist $\rho_0 \in I^X$ be an (l_0, m_1)-f α^m-closed set such that $\lambda \leq \rho_0$ and $x \notin \rho_0$. Then, $x \in 1-\rho_0$ and $1-\rho_0$ is (l_0, m_1)-fα^m-open set in X and hence $1-\rho_0$ is fuzzy α^m-nbhd of $x \in I^X$. But $\lambda \wedge (1 - \rho_0) = 0$. which is a contradiction. Then, $x \in \alpha^m C_{\tau,\tau^*}(\lambda, l_0, m_1)$.

Theorem 3.11 Let $(X, \tau_{X1}, \tau_{X1}{}^{*})$ and $(Y, \tau_{Y2}, \tau_{Y2}{}^{*})$ be two dfts's and let f: $(X, \tau_{X1}, \tau_{X1}{}^{*}) \to (Y, \tau_{Y2}, \tau_{Y2}{}^{*})$ be function. Then, the following statements are equivalent:

(1) f is dfα^m-open function

(2) λ subset of X, f $(I_{\tau,\tau^*}(\lambda, l_0, m_1)) \leq \alpha^m I_{\tau,\tau^*}(f(\lambda), l_0, m_1))$.

(3) $\forall x \in I^X$ and for each neighborhood β of x in X there exist, fuzzyα^m -nbhd δ of f(x) in Y such that $\delta \leq f(\beta)$.

Proof. (1) $\Rightarrow$ (2) Suppose f is dfα^m-open function, whenever $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$ then, $I_{\tau,\tau^*}(\lambda, l_0, m_1)$ is open in X and so, f $(I_{\tau,\tau^*}(\lambda, l_0, m_1))$ is (l_0, m_1)-f α^m-open in Y. We have, $f(I_{\tau,\tau^*}(\lambda, l_0, m_1)) \leq f(\lambda)$

Then we get, $f(I_{\tau,\tau^*}(\lambda, l_0, m_1)) \leq \alpha^m I_{\tau,\tau^*}(f(\lambda), l_0, m_1))$

(2) $\Rightarrow$ (3) Assume (2) holds and let $x \in I^X$ and β be an neighborhood of x in X. Then, there exist an open set ρ_0 such that $x \in \rho_0 \leq \beta$.

By hypothesis, $f(\rho_0) = f(I_{\tau,\tau^*}(\rho_0, l_0, m_1)) \leq \alpha^m I_{\tau,\tau^*}(f(\rho_0), l_0, m_1))$

$\Rightarrow f(\rho_0) = \alpha^m I_{\tau,\tau^*}(f(\rho_0), l_0, m_1))$

we have, $f(\rho_0)$ is an (l_0, m_1)-f α^m-open in Y. $f(x) \in f(\rho_0) \leq f(\beta)$ and so, (3) is holds by taking $\delta = f(\rho_0)$.

(3)$\Rightarrow$ (1) Assume (3) is hold and let $\tau_{X1}(\beta) \geq l_0$ and $\tau_{X1}{}^{*}(\beta) \leq m_1$, $x \in \beta$ and

f(x) = y . Then, $y \in f(\beta)$ and by hypothesis there exist a fuzzy α^m-nbhd δ_y of y in Y such that $\delta_y \leq f(\beta)$. Since δ_y is fuzzy α^m-nbhd of y, so there exist γ_y is an (l_0, m_1)- f α^m–open set in Y such that

$y \in \gamma_y \leq \delta_y$ then, $f(\beta) = \{\gamma_y : y \in f(\beta)\}$ is an (l_0, m_1)-f α^m-open set in Y, this implies that f is df α^m-open function.

Corollary 3.12 A function f: (X, τ_{X1}, τ_{X1}^*) → (Y, τ_{Y2}, τ_{Y2}^*) form a dfts (X, τ_{X1}, τ_{X1}^*) into the dfts (Y, τ_{Y2}, τ_{Y2}^*). Is a dfα^m-open function iff $f^{-1}(\alpha^m C_{\tau,\tau^*}(\beta, l_0, m_1)) \leq C_{\tau,\tau^*}(f^{-1}(\beta), l_0, m_1)$, for each $\beta \in I^Y$.

Proof. Let f be dfα^m-open function then, for any $\beta \leq Y$ whenever, $l_0 \in I_{l0}$ and $m_1 \in I_{m1}$, $f^{-1}(\beta) \leq C_{\tau,\tau^*}(f^{-1}(\beta), l_0, m_1)$, there exist $\gamma \in I^Y$is (l_0, m_1)-f α^m-closed set such that $\beta \leq \gamma$and $f^{-1}(\gamma) \leq C_{\tau,\tau^*}(f^{-1}(\beta), l_0, m_1)$.

Therefore, $f^{-1}(\alpha^m C_{\tau,\tau^*}(\beta, l_0, m_1)) \leq f^{-1}(\gamma) \leq C_{\tau,\tau^*}(f^{-1}(\beta), l_0, m_1)$,
since γ is (l_0, m_1)-f α^m -closed set in Y.

Conversely; Suppose μ_Y is any subset of Y and 1-ρ_0 is any closed set containing $f^{-1}(\mu_Y)$. Put $\gamma = \alpha^m C_{\tau,\tau^*}(\mu_Y, l_0, m_1)$ then, γ is an (l_0, m_1)-fα^m-closed set and $\mu_Y \leq \gamma$ so, by hypothesis

$$f^{-1}(\gamma) = f^{-1}\left(\alpha^m C_{\tau,\tau^*}(\mu_Y, l_0, m_1)\right) \leq C_{\tau,\tau^*}(f^{-1}(\mu_Y), l_0, m_1) \leq 1 - \rho_0$$

Then, f is df α^m-open function.

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بعض انواع الدوال المفتوحة بالنسبة للمجموعة (l_0, m_1)-fuzzy alpha^m المغلقة

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الملخص

في هذ البحث تطرقنا الى دراسة العديد من الدوال المستمرة من نوع -$alpha^m$, والدوال المحيرة α مع الدوال المفتوحة و المغلقة. $alpha^m$ والمغلقة من نوع$alpha^m$ في الفضاءات التبولوجية المزدوجة بالنسبة للمجموعات (l_0, m_1)-fuzzy alpha^m ايضا توصلنا الى بعض العلاقات بين هذه الانواع الجديدة من الدوال ومن ثم قمنا بأجراء بعض المقارنات بين الدوال المختلفة في نفس الفضاء.