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Further extension of the improved hyperbolic function method with application to general Sawada–Kotera equation of fifth-order, The 3D potential-YTSF equation and (mKdV) equation.

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Abstract

In this study, a new improved hyperbolic function method is applied for the first time. The proposed method used to calculate more general exact solutions for the general Sawada–Kotera equation of fifth-order, 3D potential-YTSF equation and modified Korteweg-de Vries (mKdV) equation. The obtained results show that the improved hyperbolic function method provides and easy and fast solution when used with mathematical software such as Maple. The finding also indicates that the proposed method helps in understanding the physical structures of the problem.

1-Introduction

Finding the exact solutions of nonlinear evolution equation is very important because the nonlinear complex physical phenomena related to the nonlinear evolution equation. Complex physical phenomenas are widely presented in many fields such as fluid mechanics, physics, biology, chemistry and engineering. Therefore, obtaining an exact solution for this kind of nonlinear evolution equation is crucial [1,2].

Based on this fact, a variety of powerful and direct method such as Homotopy Analysis method (HAM), Homotopy Perturbation method, Modified Adomian's method, Variational iteration method (VIM), Finite difference method, Finite volume method, Jacobi elliptic function expansion method, and Application of the (G'/G)-Expansion method were proposed to obtain exact significant solutions and numerical solutions of nonlinear evolution equation [3,4,5,6,7,8,9,10,11].

The best known equation is the general Sawada–Kotera (SK), which is a special case of the fifth-order KdV equation (fKdV)[12].

$$w_t + w_{5x} + \gamma w w_{3x} + \beta w_x w_{2x} + \alpha w^2 w_x = 0 \quad (1.1)$$

where α , β and γ are arbitrary nonzero and real parameters, and $w(x,t) = w(\varphi)$is a differentiable function. The fifth-order KdV equation is an important mathematical model with wide applications in quantum mechanics and nonlinear optics. Moreover the fifth-order KdV equation describes motions of long waves in shallow water under gravity and in a one-dimensional nonlinear lattice. The general form of the Sawada–Kotera equation is characterized by the values of β and α.

$$\beta = \gamma, \quad \alpha = \frac{1}{5}\gamma^2 \quad (1.2)$$

Using equation (1,2), equation (1,1) could be reduced to general SK equation:

$$w_t + w_{5x} + \gamma w w_{3x} + \gamma w_x w_{2x} + \frac{1}{5}\gamma^2 w^2 w_x = 0 \quad (1.3)$$

Recently, Yu, Toda, Sasa, and Fukuyama proposed a new 3D nonlinear evolution equation called the potential- YTSF equation [13].

$$-4w_{xt} + w_{xxxz} + 4w_x w_{xz} + 2w_{xx} w_z + 3w_{yy} = 0 \quad (1.4)$$

Another system is a modified Korteweg-de Vries (mKdV) equation are written in the form:

$$w_t + w_{3x} + 6w^2 w_x = 0 \quad (1.5)$$

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Which is presented in electric circuits and multicomponent plasmas. The modified KdV equation has a pulse travelling solution [14,15]. This paper divided into five parts. Comprehensive introduction is presented in section one. Then in section two we describe the methodology of improved hyperbolic function method for finding exact travelling wave solutions of nonlinear evolution equations. In Section three, we applied the proposed improved hyperbolic function method to obtain the traveling wave solution for the 3D potential-YTSF equation, general Sawada– Kotera equation, and modified Korteweg-de Vries (mKdV) equation. Then the result is discussed and compared in section four. Section five list the advantages of the improved method. Finally, a graphical exemplification is presented in section six.

2. Methodology

Suppose we are given a nonlinear evolution equation in the form of a PDE for a function $w(x,t)$. The procedures for solving this kind of equation using improved hyperbolic function can be presented as follows. First, travelling wave solutions are applied by taking $w(x,t) = w(\varphi)$, where $\varphi = (x - Bt)$. Substitution into the PDE yields an ODE for $w(\varphi)$. If possible, the ODE is integrated term by term one or more times and this introduces one or more constants of integration (This step is obtional). The key step is to introduce the ansatz.

$$w(\varphi) = H_0 + \sum_{i=1}^{N}(H_i T^i + F_i T^{-i}) + T^{-2}(h_0 + \sum_{i=1}^{N}(h_i T^i + f_i T^{-i}) \qquad (2.1)$$

Where, $T = \tanh(\varphi)$

T satisfies the differential equation

$$\frac{dT}{d\varphi} = (1 - T^2) \qquad (2.2)$$

Also,

$$\frac{d}{d\varphi} \equiv (1 - T^2)\ \frac{d}{dT} \qquad (2.3)$$

In (2.1), N is a positive integer (homogeneous balance between the highest order derivative and the nonlinear term). The $(H_0,\ h_0,\ H_i, F_i, and\ f_i\ )$ are real constants $(i = 1, \ldots, N)$. Substitution of (2.1) and (2.3) into the ODE, integrated if possible, introduce an algebraic equation in powers of T. If possible, N is determined; usually this involves balancing the linear term of highest-order in the algebraic equation with the highest-order nonlinear term. With N determined, the coefficients of each power of T are equated to zero in the algebraic equation. This yields a system of algebraic equations involving the $(H_0,\ h_0,\ H_i, F_i, h_i, f_i\ )$ and, B $.(i = 1, \ldots, N)$. If the original evolution equation contains some arbitrary constant coefficients, these will appear in the system of algebraic equations.

Our observations that involving hyperbolic functions can be extended to trigonometric functions as follows when is taken as $\tanh\varphi = i\tan\varphi$

3- Exact analytical solutions

3-1: We can get the solitary wave solution of (1.3), by using the transformations

$$\varphi = x - Bt \qquad (3.1.1)$$

Eq. (1.3) turn into

$$-Bw' + w^{(5)} + \gamma w w''' + \gamma w' w'' + \frac{1}{15}\gamma^2 w^2 w' = 0 \qquad (3.1.2)$$

Integrating (3.1.2) with respect to φ , we have

$$C - Bw + w^{(4)} + \gamma w w'' + \frac{1}{15}\gamma^2 w^3 = 0 \qquad (3.1.3)$$

The solution of Eq. (3.1.3) can be expressed by a polynomial in hyperbolic functions

$$w(\varphi) = H_0 + (H_1 T^1 + F_1 T^{-1}) + T^{-2}(h_0 + h_1 T^1 + f_1 T^{-1}) + (H_2 T^2 + F_2 T^{-2}) + T^{-2}(h_2 T^2 + f_2 T^{-2}) \qquad (3.1.4)$$

Where $(H_0,\ h_0,\ H_1, F_1, h_1, f_1, H_2, F_2, h_2, f_2\ )$ and, B all are constants to be determined.

Using the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (3.1.3) we can obtain N=2.

Substituting Eq. (3.1.4) into Eq. (3.1.3), the left-hand side of this equation is converted into polynomials in T^i and $i = (0,1,2, \ldots, 18)$. Setting each coefficient of these polynomials to zero, we obtain a set of system algebraic equations for $(H_0,\ h_0,\ H_1, F_1, h_1, f_1, H_2, F_2, h_2, f_2\ )$ and, B. When solving this system of algebraic equation by using Maple software, we have the following unknown parameters.

Family1:

$C = -\frac{1}{15}\gamma^2 H_0^3 - \frac{1}{5}\gamma^2 H_0^2 h_2 - \frac{1}{5}\gamma^2 h_2^2 H_0 - \frac{1}{15}\gamma^2 h_2^3 - \gamma H_0 - B\ H_0 - \gamma h_2 - B h_2 = 0, B = B$

$F_1 = -h_1, F_2 = -h_0, H_0 = H_0, H_1 = 0, H_2 = 0, f_1 = 0, f_2 = 0, h_0 = h_0, h_1 = h_1, h_2 = h_2$

Family2:

$C = \frac{5922}{\gamma^2(-\frac{\gamma h_0 \mp 30i\sqrt{2}}{\gamma} + h_0)}, F_1 = -h_1, F_2 = -\frac{\gamma h_0 \mp 30i\sqrt{2}}{\gamma},$

$H_0 = -\frac{\gamma(\gamma h_0 \mp 30i\sqrt{2})h_2 + \gamma^2 h_0 h_2 - 1170}{\gamma^2\left(-\frac{\gamma h_0 \mp 30i\sqrt{2}}{\gamma} + h_0\right)}, H_1 = 0, H_2 = 0, f_1 = 0, f_2 = 0, h_0 = h_0,$

$h_1 = h_1, h_2 = h_2, B = -\gamma + \frac{241}{10}.$

Family3:

$C = \mp\frac{\frac{987}{10}i\sqrt{2}}{\gamma}, F_1 = -h_1, F_2 = -h_0, H_0 = \mp\frac{\frac{39}{2}i\sqrt{2}}{\gamma} - h_2, H_1 = 0, H_2 = \mp\frac{30i\sqrt{2}}{\gamma},$

$f_1 = 0, f_2 = 0, h_0 = h_0, h_1 = h_1, h_2 = h_2, B = -\gamma + \frac{241}{10}.$

Family 4:

$C = \mp\frac{\frac{66987}{10}i\sqrt{2}}{\gamma}, F_1 = -h_1, F_2 = \pm\frac{30i\sqrt{2}}{\gamma} - h_0, H_0 = \mp\frac{\frac{39}{2}i\sqrt{2}}{\gamma} - h_2, H_1 = 0$

$H_2 = \pm\frac{30i\sqrt{2}}{\gamma}, f_1 = 0, f_2 = 0, h_0 = h_0, h_1 = h_1, h_2 = h_2, B = -\gamma + \frac{3841}{10}$.

Substituting (Family1,2,3,4) into Eq. (3.1.4), we obtain the solitary wave solutions of Eq. (1.3):

$$w_{1,2} = -\frac{\gamma(\gamma h_0 \mp 30i\sqrt{2})h_2 + \gamma^2 h_0 h_2 - 1170}{\gamma^2\left(-\frac{\gamma h_0 \mp 30i\sqrt{2}}{\gamma} + h_0\right)} + \frac{h_0}{T^2} - \frac{\gamma h_0 \mp 30i\sqrt{2}}{\gamma} T^{-2} + h_2 = -\frac{\gamma(\gamma h_0 \mp 30i\sqrt{2})h_2 + \gamma^2 h_0 h_2 - 1170}{\gamma^2\left(-\frac{\gamma h_0 \mp 30i\sqrt{2}}{\gamma} + h_0\right)} \frac{30i\sqrt{2}}{\gamma} coth^2\varphi + h_2 \quad (3.1.5)$$

$$w_{3,4} = \mp\frac{\frac{39}{2}i\sqrt{2}}{\gamma} \mp \frac{30i\sqrt{2}}{\gamma} T^2 = \mp\frac{\frac{39}{2}i\sqrt{2}}{\gamma} \mp \frac{30i\sqrt{2}}{\gamma} tanh^2\varphi \quad (3.1.6)$$

$$w_{5,6} = \mp\frac{\frac{39}{2}i\sqrt{2}}{\gamma} + \frac{h_0}{T^2} \pm \frac{30i\sqrt{2}}{\gamma} T^2 \pm \frac{\frac{30i\sqrt{2}}{\gamma} - h_0}{T^2} = \mp\frac{\frac{39}{2}i\sqrt{2}}{\gamma} \pm \frac{30i\sqrt{2}}{\gamma} tanh^2\varphi \pm \frac{30i\sqrt{2}}{\gamma} coth^2\varphi \quad (3.1.7)$$

3-2: Substituting the traveling wave variable $w(x,y,t,z) = w(\varphi)$, potential-YTSF equation (1.4), leads to an ODE:

$$4Bw'' + w'''' + 6w'w'' + 3w'' = 0 \quad (3.2.1)$$

Integrating Eq. (3.2.1) once and letting the integral constant to zero, we obtain

$$4Bw' + w''' + 3(w')^2 + 3w' = 0 \quad (3.2.2)$$

The homogeneous balance ($N = 1$), Therefore, the solution of Eq. (3.2.2) takes the form:

$$w(\varphi) = H_0 + (H_1 T^1 + F_1 T^{-1}) + T^{-2}(h_0 + h_1 T^1 + f_1 T^{-1}) \quad (3.2.3)$$

Substituting (3.2.3) together with (2.2) into (3.2.2), yields a set of simultaneous algebraic equations for (H_0, h_0, H_1, F_1, h_1, f_1) and, B.

Solving these algebraic equations with the help of symbolical computation software, such as, Maple, we obtain the following:

Family1:

$B = B, F_1 = F_1, H_0 = H_0, H_1 = 0, f_1 = 0, h_0 = 0, h_1 = -F_1$

Family2:

$B = -\frac{7}{4}, F_1 = F_1, H_0 = H_0, H_1 = 0, f_1 = 0, h_0 = 0, h_1 = -F_1 + 2$

Family3:

$B = -\frac{7}{4}, F_1 = F_1, H_0 = H_0, H_1 = 2, f_1 = 0, h_0 = 0, h_1 = -F_1$

Family4:

$B = -\frac{19}{4}, F_1 = F_1, H_0 = H_0, H_1 = 2, f_1 = 0, h_0 = 0, h_1 = -F_1 + 2$

Substituting (Family1,2,3,4) into Eq. (3.2.3), we obtain the solitary wave solutions of Eq. (1.4):

$$w_1 = H_0 + \frac{F_1}{T} + \frac{-F_1+2}{T} = H_0 + 2\coth\varphi \quad (3.2.4)$$

$$w_2 = H_0 + 2T + \frac{F_1}{T} - \frac{F_1}{T} = H_0 + 2\tanh\varphi \quad (3.2.5)$$

$$w_3 = H_0 + 2T + \frac{F_1}{T} + \frac{-F_1+2}{T} = H_0 + 2\tanh\varphi + 2\coth\varphi \quad (3.2.6)$$

3-3 Look for the travelling wave solution of Eq. (1.5) in the form $w(x,t) = w(x - Bt)$,

The speed B is to be determined later, Eq. (1.5) is converted into an ODE, by using the travelling wave variable.

$$-Bw' + 6w^2w' + w''' = 0 \quad (3.3.1)$$

After, integrating with respect to φ

$$M - Bw + 2w^3 + w'' = 0 \quad (3.3.2)$$

Where, M is an integration constant that is to be determined later. Suppose that the solution of Eq. (3.3.2) is of the form of Eq. (3.2.3).

By substituting (3.2.3), (2.2) into Eq. (3.3.2), collecting all the terms of powers of T, and setting each coefficient to zero, we get simultaneous equations:

T^1: $2{H_1}^3 + 2H_1 = 0$,

T^2: $6{H_1}^2 H_0 = 0$,

T^3: $6F_1{H_1}^2 + 6{H_0}^2 H_1 + 6{H_1}^2 h_1 - H_1 B - 2H_1 = 0$,

T^4: $12F_1H_1H_0 + 2{H_0}^3 + 12H_1H_0h_1 + 6{H_1}^2 h_0 - 2H_0 + 2h_0 = 0$,

T^5: $6{F_1}^2 H_1 + 6F_1{H_0}^2 + 12F_1H_1h_1 + 6{H_0}^2 h_0 + 12H_0H_1h_0 + 6{H_1}^2 f_1 + 6H_1{h_1}^2 - F_1B - Bh_1 - 2F_1 + 6f_1 - 2h_1 = 0$,

T^6: $6{F_1}^2 H_0 + 12F_1H_0h_0 + 12F_1H_1h_0 + 6{H_0}^2 h_0 + 12H_0H_1f_1 + 6{h_1}^2 H_0 + 12H_1h_0h_1 - Bh_0 - 8h_0 = 0$

T^7: $2{F_1}^3 + 6{F_1}^2 h_1 + 12F_1H_0h_0 + 12F_1H_1h_1 + 6{h_1}^2 F_1 + 6H_0^2 f_1 + 12H_0h_0h_1 + 12H_1f_1h_1 + 6H_1h_0^2 + 2h_1^3 - Bf_1 + 2F_1 - 18f_1 + 2h_1 = 0$

T^8: $6{F_1}^2 h_0 + 12F_1H_0f_1 + 12F_1h_0h_1 + 12H_0f_1h_1 + 6H_0h_0^2 + 12H_1f_1h_0 + 6h_0h_1^2 + 6h_0 = 0$

T^9: $6{F_1}^2 f_1 + 12F_1h_1f_1 + 6F_1{h_0}^2 + 12H_0f_1h_1 + 6H_1f_1^2 + 6f_1h_1^2 + 6{h_0}^2 h_1 + 12f_1 = 0$

T^{10}: $12F_1h_0f_1 + 6H_0f_1^2 + 12f_1h_1h_0 + 2h_0^3 = 0$

T^{11}: $6F_1{f_1}^2 + 6h_1f_1^2 + 6f_1{h_0}^2 = 0$

T^{12}: $6{f_1}^2 h_0 = 0$

T^{13}: $2{f_1}^3 = 0$

Solving these system $\{T^1, T^2, \ldots, T^{13}\}$, we obtain

Family1:

$B = B, F_1 = F_1, H_0 = 0, H_1 = 0, f_1 = 0, h_0 = 0, h_1 = -F_1$

Family2:

$B = 2H_0^2, F_1 = F_1, H_0 = H_0, H_1 = 0, f_1 = 0, h_0 = 0, h_1 = -F_1$

Family3:

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$B = -2, F_1 = F_1,\ H_0 = 0, H_1 = 0, f_1 = 0, h_0 = 0, h_1 = -F_1 \pm i$

Family4:

$B = -2, F_1 = F_1, H_0 = 0, H_1 = \pm i, f_1 = 0, h_0 = 0, h_1 = -F_1$

Family5:

$B = \pm 6iF_1 \pm 6i(-F_1 \pm i) - 2, F_1 = F_1,\ H_0 = 0, H_1 = \pm i, f_1 = 0, h_0 = 0, h_1 = -F_1 \pm i$

Substituting (Family1,2,3,4,5) into Eq. (3.2.3), we obtain the solitary wave solutions of Eq. (1.5):

$$w_1 = H_0 \quad (3.3.3)$$

$$w_{2,3} = \pm iT^{-1} = \pm i \coth\varphi \quad (3.3.4)$$

$$w_{4,5} = \pm iT = \pm i \tanh\varphi \quad (3.3.5)$$

$$w_{6,7} = \pm iT \pm iT^{-1} = \pm i \tanh\varphi \ \pm i \coth\varphi \quad (3.3.5)$$

4. Summary

In this paper, improved hyperbolic function method has been successfully used to obtain travelling wave solution, of the nonlinear evolution equation (general Sawada–Kotera equation of fifth-order, the 3D potential-YTSF equation and modified Korteweg-de Vries (mKdV) equation). We calculated new exact solutions that are not obtained by any hyperbolic function method (3.1.5), (3.1.6) and (3.1.7). We proved The valaidty of our solutions by putting them back into the original Eq. (1.3). Also, a comparison was made between the solutions of the improved method and [13,14], and [15] under special conditions. We observed that our results are in good agreement with [13], [14], and [15] see Tables 1,2.

We obtained further new exact travelling wave solutions of hyperbolic function $w_{6,7}$ in Table 1 and w_3 in Table 2, which have not been reported. The solutions contain many free parameters that can be used in different physical applications where the equations arise. Also, when the free parameters assumed particular values the obtained solutions are reduced to special function.

Table 1 Comparison between Expansion Method [13]	Obtained solutions.
If, $A = 1, C = 0, \Omega = \lambda^2 - 4\mu, d = -\frac{\lambda}{2}, B = \frac{\lambda}{2}, \lambda = 2, \mu = 0, a_0 = 2, E = \frac{1}{2}, \psi = 1$ $v_1 = \frac{1}{A}(2\psi d + B + \sqrt{\Omega}\tanh\{\frac{\sqrt{\Omega}}{2A}\zeta\}) + a_0$	If, $H_0 = 2$ $w_1 = H_0 + 2\coth\{x + y + z + \frac{7}{4}t\}$ $w_2 = H_0 + 2\tanh\{x + y + z + \frac{7}{4}t\}$
$v_2 = \frac{1}{A}(2\psi d + B + \sqrt{\Omega}\coth\{\frac{\sqrt{\Omega}}{2A}\zeta\}) + a_0$ $\zeta = x + y + z - \frac{1}{4A^2}(3A^2 + 4E\psi + B^2)t$	$w_3 = H_0 + 2\tanh\varphi + 2\coth\varphi$ w_3, new exact traveling wave solution

Table 2 Comparison between Expansion Method [14]	Obtained solutions.
If, $\sigma = -1, \lambda = 2, \mu = 0, \delta = 1$ $u_{1,2} = \pm\frac{1}{2}\sqrt{\sigma\delta(\lambda^2 - 4\mu)}\ \tanh\frac{1}{2}\sqrt{(\lambda^2 - 4\mu)}\ \zeta$	$w_{2,3} = \pm i\coth\{x + 2t\}$ $w_{4,5} = \pm i\tanh\{x + 2t\}$ $w_{6,7} = \pm i\tanh\varphi \ \pm i\coth\varphi$
$\zeta = x - \left(2\delta\mu - \frac{1}{2}\delta\lambda^2\right)t = x + 2t$	$w_{6,7}$, new exact traveling wave solution

5-The advantages of the improved hyperbolic function method

1- The improved hyperbolic function method is used to seek solutions of higher-order nonlinear equations which can be reduced to ODEs of the order greater than 3.

2- The results include kink-profile solitary-wave solutions, bell-profile solitary-wave solutions, periodic wave solutions, rational solutions, singular solutions and other new formal solutions.

3- The improved methods yield a general solution with free parameters, so no need to apply the initial and boundary conditions at the outset.

4- The solution procedure can be easily implemented in Mathematica or Maple

5- Finally, we can see, tanh- coth type solitary wave solutions of nonlinear evolution equation are obtained by tanh-coth function method.

6. Graphical exemplification

In this section, Some of the obtained travelLing wave solutions of nonlinear evolution equations of the (3+1)-dimensional potential-YTSF equation, are represented in Figures. 1–3 with the aid of symbolic computation software like Maple.

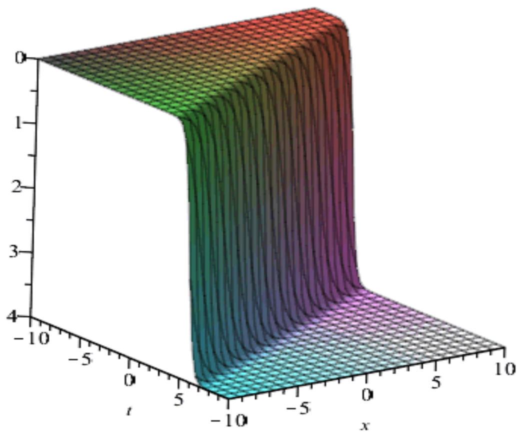

Figure 1 Modulus plot of Kink wave, Shape of (3,2,5) when, $H_0 = 2, y = 0, z = 0$.

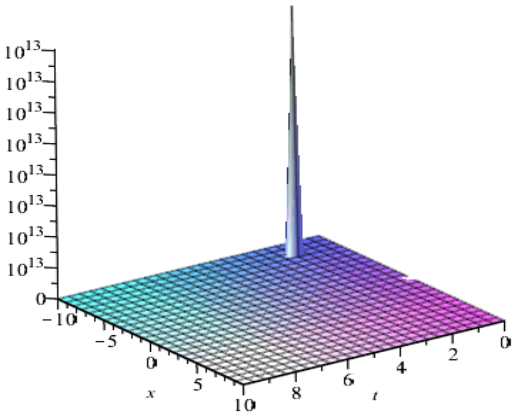

Figure 2 Modulus plot of single soliton wave, Shape of (3,2,6) when, H_0 =0, $y = 0$, $z = 0$.

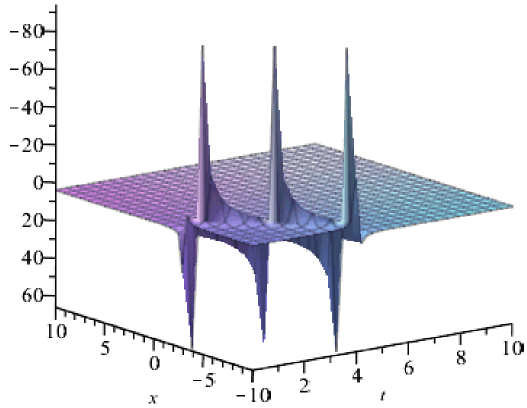

Figure 3 Modulus plot periodic wave, Shape of (3,2,4) when, H_0 =2, $y = 2$, $z = 1$.

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مزيد من التوسيع في تحسين طريقة الدوال الزائدية مع التطبيق للمعادلة العامة من الرتبة الخامسة mKdV ومعادلة TSF- equation والمعادلة ثلاثية الابعادSawada–Kotera

وفاء محي الدين طه

قسم الرياضيات ، كلية التربية للعلوم الصرفة ، جامعة تكريت ، تكريت ، العراق

الملخص

في هذه الدراسة , تحسين جديد في طريقة الدوال الزائدية يطبق الاول مرة . الطريقة المقترحة استخدمت لحساب حلول دقيقة اكثر عمومية للمعادلة العامة من الرتبة الخامسة لي mKdV ومعادلة YTSF- equation والمعادلة ثلاثية الابعادSawada–Kotera

والنتائج التي تم الحصول عليها تبين ان التحسين في طريقة الدوال الزائدية يوفر حل سهل وسريع عند استخدامها مع البرمجيات الرياضية وتشير النتيجة أيضا إلى أن الطريقة المقترحة تساعد في فهم البنى المادية للمشكلة.