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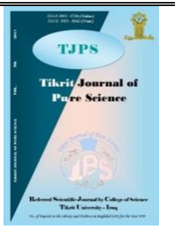

Calculation of the Mott Scattering cross section for Iron and Silver in the energy range (50keV-10MeV)

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Abstract

In the present research work, we tableted the ratio of Mott to Rutherford cross section $(\frac{\sigma_{Mott}}{\sigma_{Rutherford}})$, for Iron and Silver atoms by employing Mckinley - Fashbach equation , in the range of incident electron energy (50KeV-10MeV), by adopting the equations used in the presen calculations by using Visual Basic 2010. The results shows that the ratio $(\frac{\sigma_{Mott}}{\sigma_{Rutherford}})$ is mostly influenced with two parameters :incident electron energy and atomic number of the target material on the Mott cross section. The obtained results were compared with previous results, and they shows a good agreement, and his confirms the validity of the equations used in the employed energy range.

1- Introduction

The scattering of electrons by a nucleus , providing us the most exactly information about nuclear size and charge distribution. The electron may considered as a better nuclear probe than the alpha particles of Rutherford scattering because it is a point particle and can penetrate the nucleus. Mott scattering is considered as an elastic scattering because the incident electron does not change in velocity or mass due to the mass of the nucleus is very large in comparison to the mass of the incident electron, so the elastic scattering, where the lost energy of the particle incident in the coulomb collision with the nucleus . Mott cross-section is a fundamental method for calculating the cross-section of the elastic scattering [1].

The scattering of the electron with the nucleus was developed in phases , and the electron was separated relative to the nucleus by Coulombic field for the point nucleus was derived by Rutherford and then treated by Mott as a relative electron where Dirac used the electron [2-3], the Mott cross-section is very complex so both Mckinley - Fashbach put equation to calculate the Mott cross section by taking the electron spin correction into consideration This was built in by using Born Approximation. Replacing electrons instead of alpha particles (Rutherford scattering) gives us information on nuclear relativity[4], the Mott cross-section is described as the effect of the atomic potential on the scattering factor, and the Mott cross section gives us a description of the scattering of electron radiation, Mott's corrections remove some constraints [5]. The effect of the beams and the relative corrections have their effect on the values of the cross section, and the cross section gives us evidence of the nature of the reactions and their differences from the rest of the reactions and evidence about the shape and size of the nucleus of the atom [6] .Many researchers studied the Mott cross section like Doggett J.A and Spencer L.V[7], Curr R.M. [8] , Yadav H.M[9] , Sherman N [10] , Boschini M.J et al. [11] .

2 – Calculation

2-1 Mott Scattering Equation :

Mott scattering, also known as spin-coupling elastic Coulomb scattering, is the separation of the two spin cases of an electron by scattering the beam off the Coulomb field of heavy atoms like Iron and Silver. It is used to calculate the spin polarization of an electron beam [12]. The spin effect become important, However, if the speed of the electron is high, the Mott cross-section decreases with increasing angle , faster than the Rutherford cross-section . The expression of the cross-section for the electron scattering in the coulomb potential , the recoil from the target can be neglected at low energies .The effect of the electron magnetic moment introduces to the Rutherford relationship for electron scattering , the correction factor as [13] .

$$f_{spin} = 1 - \beta^2 \sin^2 \frac{\theta}{2} \ldots . (1)$$

Where $\beta = v/c$, for relativistic electron . it can be written in another formula , when $\beta = 1$.

$$f_{spin} = 1 - \sin^2 \frac{\theta}{2} = \cos^2 \frac{\theta}{2} = \frac{1+\cos\theta}{2} \ldots . . (2)$$

The spin correction factor does not depend on the kinetic energy K of the incident electron but depends on the scattering angle θ only as shown fig.(1) , If the small scattering angle θ , then the electron spin effect are negligible [14] .

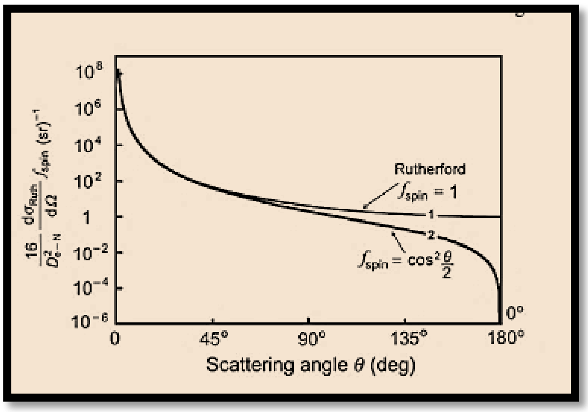

Figure (1) : spin correction factor : The curve(1) represents there is no spin correction factor, but Curved (2) represents there is spin correction factor

Mott used the Dirac equation for electrons and derived the formula of Coulomb's electrons by the point nucleus, since this formula is highly complex, being the sum of two conditions, The McKinley - Faschbach gave an approximation of the Mott formula [15] .

$$\frac{\sigma_{Mott}}{\sigma_{Rutherford}} = 1 - \beta^2 \sin^2\left(\frac{\theta}{2}\right) + \pi Z\beta\alpha \left(\sin(\frac{\theta}{2}) - \sin^2(\frac{\theta}{2})\right) \ldots . (3)$$

Where α = 1/137 is the exact structure constant, $(\frac{\sigma_{Mott}}{\sigma_{Rutherford}})$ the dimensionless quantity, The most important thing addressed by this equation is the development of the original Mott formula, including the correction factor, and was used extensively to correct the errors that existed in the cross section, in addition to the correction of the Born approximation , which was used mainly in the original derivation of the Mott formula.

3 - Results and Discussion

Tables (1 , 2) are the values of the ratio of ($\frac{\sigma_{Mott}}{\sigma_{Rutherford}}$) rd cross-section for Fe and Ag elements , by using the equation (3) within the energy range of (0.005 to 10) MeV , by using Mckinley-Fashbach Equation. The results can be explained as the following [16] .

When the incident electron collides with an electronic target atom , its gradually loss its kinetic energy and deflected continuously causing an ionization and excitation processes continuously, which may leads to several changes such as variations in crystal lattice and material conductivity .But when a swift electron (of high energy) approaches the nucleus, the probability of interaction subjects to [16] .

$$interaction - probability = \frac{1}{\sin^4(\theta)E^4} \ldots (4)$$

As the energy of the incident electron increases , the probability of interaction decreases and in the same way with angle of electron incident. For example , High energy E=10MeV and angle of 165^0, the ratio $\frac{\sigma_{Mott}}{\sigma_{Rutherford}}$, equals to Fe 0.012074 and 0.065891 for Ag and for low energy and incident angle like 30^0 and E=1Mev, the ratio equals to 1.061128 for Fe and for Ag 1.221703 respectively .While the results shows the inverse behavior when compared with respect to atomic numbers Z. this occurs due the interaction of the incident electron with proton inside the nucleus through a series of Columbic interactions .Our present data when compared to the reference data[16] shows an agreement, with small ratio of errors in these data , which may be arises from the mathematical approximations used in derivation the Mickenly –Feshbach equation , and According to figures (2) and (3) .

4 – conclusions

The Mickenly–Feshbach equation is a simple equation suitable for determination the dimensionless ration ($\frac{\sigma_{Mott}}{\sigma_{Rutherford}}$) which is a spin correction factor illustrates a fundamental role in its impact on cross-section values .The incident electron energy and the atomic number of material are the most important parameters that controls the values of the Mott cross-section values in the used energy range .

Table (1) : The results of comparison ratio $\frac{\sigma_{Mott}}{\sigma_{Rutherford}}$ cross-section for Iron [16] .

Θ (deg.)	Electron energy E(MeV)			Z = 26	
	10 MeV Present study Ref[16]	5 MeV Present study Ref[16]	2.5 MeV Present study Ref[16]	1 MeV Present study Ref[16]	0.5 MeV Present study Ref[16]
15	1.056013 1.05868	1.052221 1.05858	1.046714 1.05811	1.041726 1.05617	1.032015 1.05256
30	1.058113 1.07386	1.060798 1.07387	1.062939 1.07390	1.061128 1.07384	1.049533 1.07305
45	1.036809 1.03822	1.039954 1.03863	1.041045 1.03995	1.060273 1.04536	1.067304 1.05345
60	0.949131 0.95469	0.963323 0.95574	0.981740 0.95909	0.990155 0.97313	0.992311 0.99541
75	0.819884 0.83164	0.832479 0.83350	0.859216 0.83948	0.870003 0.86465	0.885243 0.90537
90	0.671314 0.68118	0.704925 0.68396	0.726529 0.69295	0.756224 0.73091	0.790021 0.79285
105	0.509717 0.51772	0.538928 0.52147	0.563671 0.53363	0.621395 0.58500	0.658814 0.66927
120	0.337642 0.35664	0.386052 0.36134	0.407915 0.37654	0.483180 0.44088	0.533965 0.54674
135	0.198826 0.21291	0.236798 0.21843	0.259078 0.23632	0.351721 0.31209	0.438727 0.43703
150	0.078827 0.09971	0.127081 0.10588	0.141045 0.12588	0.244416 0.21059	0.328060 0.35045
165	0.012074 0.02740	0.054697 0.03398	0.073995 0.05531	0.176379 0.14572	0.262947 0.29508
180	0.000872 0.00258	0.020324 0.00927	0.047119 0.03106	0.134945 0.12342	0.247355 0.27603
Θ (deg.)	Electron energy E(MeV)		Z = 26		
	0.25 MeV Present study Ref[16]	0.1 MeV Present study Ref[16]	0.05 MeV Present study Ref[16]	0.025 MeV Present study Ref[16]	0,005 MeV Present study Ref[16]
15	1.009802 1.04623	1.007253 1.03433	1.001513 1.02384	1.000986 1.01351	1.000692 1.00121
30	1.034877 1.06996	1.014279 1.05986	1.007812 1.04778	1.003914 1.03317	1.001471 1.00249
45	1.058168 1.06232	1.030999 1.06643	1.009039 1.06077	1.006028 1.04877	1.004124 1.01043
60	0.998203 1.02390	1.000531 1.05292	1.000877 1.06033	1.001270 1.05624	1.008823 1.02111
75	0.987325 0.95959	0.999274 1.02205	0.999983 1.04781	1.000711 1.05570	1.013512 1.03169
90	0.890073 0.87689	0.997881 0.97858	0.999524 1.02636	0.999976 1.04913	1.017582 1.04190
105	0.811356 0.78478	0.991224 0.92825	0.999075 0.99981	0.999927 1.03903	1.036624 1.05223
120	0.722759 0.69279	0.931118 0.87702	0.993679 0.97206	0.999828 1.02777	1.042562 1.06275
135	0.619666 0.61007	0.863860 0.83052	0.981854 0.94658	0.999449 1.01726	1.048664 1.07280
150	0.525282 0.54435	0.802125 0.79357	0.972030 0.92626	0.999187 1.00888	1.051951 1.08125
165	0.485811 0.50277	0.780378 0.76986	0.961342 0.91320	0.998803 1.00353	1.055212 1.08692
180	0.452989 0.48835	0.779935 0.75169	0.945577 0.90871	0.996258 1.00170	1.059201 1.08891

Table (2) : The results of comparison ratio $\frac{\sigma_{Mott}}{\sigma_{Rutherford}}$ cross-section for Silver [16] .

Θ (deg.)	Electron energy E(MeV)		Z = 47		
	10 MeV Present study Ref[16]	5 MeV Present study Ref[16]	2.5 MeV Present study Ref[16]	1 MeV Present study Ref[16]	0.5 MeV Present study Ref[16]
15	1.170581 1.11557	1.168203 1.11514	1.156931 1.11374	1.139040 1.10750	1.113208 1.09602
30	1.269512 1.21466	1.267791 1.21414	1.254869 1.21243	1.221703 1.20469	1.205513 1.18989
45	1.290715 1.25631	1.289666 1.25605	1.281071 1.25518	1.271259 1.25094	1.263180 1.24149
60	1.251290 1.22547	1.243510 1.22582	1.247624 1.22692	1.253074 1.23101	1.239603 1.23525
75	1.182106 1.12221	1.159204 1.12349	1.161805 1.12760	1.163205 1.14446	1.213532 1.16997
90	0.970021 0.95805	0.987538 0.96051	0.994201 0.96845	1.018226 1.00167	1.106675 1.05458
105	0.762840 0.75305	0.773719 0.75684	0.790811 0.76913	0.887617 0.82090	0.958641 0.90508
120	0.551718 0.53262	0.561442 0.53779	0.583612 0.55453	0.673309 0.62534	0.798013 0.74168
135	0.348645 0.32417	0.351027 0.33060	0.395414 0.35145	0.461558 0.43983	0.621604 0.58590
150	0.181365 0.15357	0.190554 0.16102	0.226551 0.18518	0.312006 0.28776	0.482098 0.45790
165	0.065891 0.04196	0.081148 0.05007	0.097788 0.07639	0.221930 0.18819	0.403562 0.37400
180	0.006873 0.00316	0.042027 0.01150	0.063208 0.03856	0.197021 0.15357	0.381172 0.34481

Θ (deg.)	Electron energy E(MeV)		Z = 47		
	0.25 MeV Present study Ref[16]	0.1 MeV Present study Ref[16]	0.05 MeV Present study Ref[16]	0.025 MeV Present study Ref[16]	0.005 MeV Present study Ref[16]
15	1.088601 1.07678	1.058392 1.04285	1.022743 1.01840	1.006277 1.00537	1.003518 1.00121
30	1.175823 1.16343	1.121797 1.11050	1.071431 1.06306	1.029345 1.02118	1.009524 1.00638
45	1.233617 1.22118	1.195228 1.17120	1.134325 1.11764	1.071468 1.05936	1.000527 0.99636
60	1.245118 1.23363	1.231500 1.20620	1.195002 1.16161	1.129951 1.10211	1.000154 0.99086
75	1.205345 1.19840	1.249845 1.21077	1.213795 1.18772	1.150146 1.13957	1.004553 1.00217
90	1.135431 1.12229	1.216681 1.18856	1.229855 1.19730	1.194552 1.17030	1.053130 1.03381
105	1.021333 1.01798	1.180447 1.14757	1.205114 1.19535	1.215320 1.19682	1.098748 1.08620
120	0.937014 0.90134	1.122169 1.09742	1.199351 1.18737	1.239430 1.22119	1.185002 1.15388
135	0.815020 0.78900	1.089896 1.04748	1.192035 1.17778	1.257519 1.24353	1.285950 1.22582
150	0.719299 0.69625	1.012094 1.00575	1.187001 1.16950	1.273113 1.26221	1.342213 1.28896
165	0.658141 0.63533	1.003383 0.97825	1.183779 1.16409	1.279714 1.27479	1.385991 1.33186
180	0.639710 0.61412	0.993017 0.96867	1.180015 1.16223	1.300051 1.27925	1.417405 1.34702

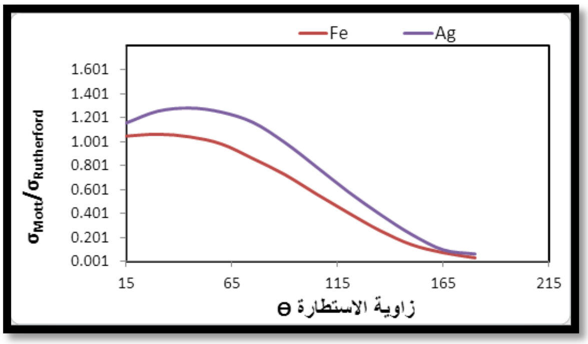

Figure (2) : Mott cross section at 2.5 MeV for Fe and Ag .

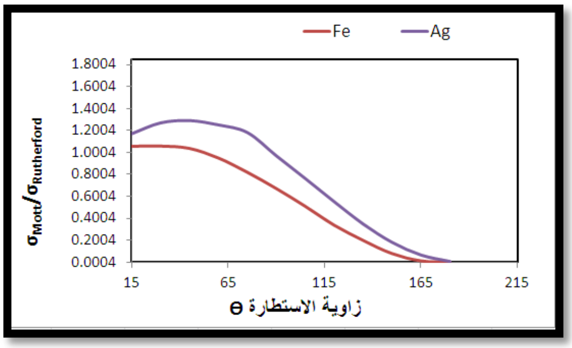

Figure (3) : Mott cross section at 10 MeV for Fe and Ag .

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حساب مساحة المقطع العرضي لاستطارة مووت (Mott) لعنصري الحديد والفضة لمدى الطاقة (50keV-10MeV)

منير رمضان بداح ، صباح محمود امان الله
قسم الفيزياء ، كلية العلوم ، جامعة تكريت ، تكريت ، العراق

الملخص

في البحث الحالي، قمنا بجدولة قيم مساحة المقطع العرضي التفاضلي ($\frac{\sigma_{Mott}}{\sigma_{Rutherford}}$) لعنصري الحديد والفضة باستخدام معادلة Mckinley–Fashbach ضمن مدى الطاقة (50KeV-10MeV) من خلال برمجة المعادلات المستخدمة ببرنامج Visual Basic 2010 . اظهرت النتائج اعتمادية النسبة المتحصلة ($\frac{\sigma_{Mott}}{\sigma_{Rutherford}}$) على عاملين هما: طاقة الكترون الساقط والعدد الذري لمادة الهدف ولقد تم مقارنة النتائج الحالية مع نتائج سابقة، اظهر تطابق جيد مما يشير الى دقة المعادلة المستخدمة في الحسابات الحالية ضمن مدى الطاقة الحالية.

الكلمات المفتاحية: مساحة المقطع العرضي لمووت, معادلة ماكينلي– فيشباخ, عامل تصحيح البرم.