# Structures of Pseudo - BG Algebra and Sime pseudo - BG - Algebra 

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## 1 Introduction

BCK-algebras and BCI-algebras were introduced by Imai. and Iseki as two classes of abstract algebras in 1966 [1, 2]. It is known that the class of BCKalgebras is a proper subclass of BCI-algebras. In 1983, BCH-algebras as a wide class of abstract algebras were introduced by Hu and $\mathrm{Li}[3,4]$. In their study, it is given that the class of BCI-algebras are proper subclasses of BCH -algebras. In 1999, the notion of d-algebras that is another useful generalization of BCK-algebras was introduced by Neggers and Kim [5]. In 2001, a new notion called a Q-algebras was introduced by J. Neggers, S. S. Ahn and H. S. Kim [6]. At the same time pseudo-BCKalgebras as an extension of BCK-algebras was introduced by G. Geordscu, and A. Iorgulescu [7] In 2008, pseudo-BCK-algebras as a natural generalization of BCI-algebras and pseudo- BCKalgebras were introduced by W. A. Dudek and Y. B. Jun [8]. These algebras have also connections with other algebras of logics such as pseudo-MV-algebras and pseudo-BL-algebras defined by G. Georgesuc and A. Iorgulescu [9] and [10], respectively. As a generalization of many algebras, these pseudo algebras have been studied by many researchers [11, 12, 13, 14, 15]. Bajalan and Ozbal introduced Some properties and homomorphisms of pseudo-Q algebras [16]. In this paper, we introduced the notion new types of algebras pseudo BG- algebra, pseudo sub BG


#### Abstract

In this paper, we introduced the notion new types of algebras pseudo BG- algebra, pseudo sub BG -algebra, Pseudo Ideal and pseudo strong Ideal of Pseudo-BG-Algebras. We state some Proposition and examples which determine the relationships between these notions and some types of ideal and we introduced the notion semi pseudo BG- algebra, pseudo sub BG -algebra, Pseudo Ideal and pseudo strong Ideal of semi pseudo-BG-Algebras. We investigated a new notion, of algebra called semi pseudo BG- algebra. We state some Proposition and examples which determine the relationships between these notions and some types of ideals defined minimal and homomorphism and kernel.


-algebra, Pseudo Ideal and pseudo strong Ideal of Pseudo-BG-Algebras.

## 2 Preliminaries

2.1 Definition [20]

A $\boldsymbol{B} \boldsymbol{G}$ - algebra is a non-empty set $\boldsymbol{X}$ with a constant $\mathbf{0}$ and a binary operation $" *$ " satisfying the following axioms:
I. $x * x=0$
II. $x * 0=x$
III. $(x * y) *(0 * y)=x$, For all $x, y \in X$
2.2 Definition [1]
2.3 A BH - algebra, we mean an algebra $(X ; *, 0)$ of type ( 2,0 ) satisfying the following conditions:
I. $x * x=0$,
II. $x * 0=x$,
III. $x * y=0$ and $y * x=0$ imply $x=y \forall x, y \in$ $X$.
2.4 Definition [ 2]

A pseudo BH -algebra is a non-empty set X with a constant 0 and two binary operations "*" and " $\stackrel{\text { " }}{ }$ satisfying the following axioms:
(P1) $x * x=x \diamond x=0$;
(P2) $x * 0=x \diamond 0=x$;
(P3) $x * y=y \diamond x=0$ imply $x=y$ for all $x, y \in X$.
2.5 Definition [2]

Let $(X ; *, \diamond)$ be a pseudo $B H$-algebra and let $\emptyset \neq I \subseteq X . I$ is called a pseudo subalgebra of $X$ if
$x * y, x \diamond y \in I$ whenever $x, y \in I$. I is called a pseudo ideal of $X$ if it satisfies:
I. $\quad 0 \in I$,
II. $\quad x * y, x \triangleright y \in I$ and $y \in I$ imply $x \in I, \forall x, y \in$ $X$.
3 Pseudo - BG algebra
3.1 Definition

A pseudo- $\boldsymbol{B} \boldsymbol{G}$ algebra is a structure ( $\boldsymbol{X}, *, \varnothing, \mathbf{0}$ ), where * and $\diamond$ are two binary operation on a nonempty set $\boldsymbol{X}$ and satisfying the following axioms: for all $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$,
P. $1 x * 0=x \diamond 0=x$
P. $2 x * x=x \diamond x=0$
P. $3(x * y) \diamond(0 * y)=(x \diamond y) *(0 \diamond y)=x$

### 3.2 Properties

Let ( $\mathrm{X} ; *, \diamond, 0$ ) be a pseudo $-B G$ algebra then the following holds:
I. If $x * y=x \diamond y=0$ then $x=y$ for any $x, y \in X$
II. If $(y * y) \diamond(0 * y)=(y \diamond y) *(0 \diamond y)$ then $(0 \diamond y)=(0 * y)$

## Proof: I

If $x * y=0$ and $x \diamond y=0$ then $(x * y) \diamond(0 *$ $y)=(x \diamond y) *(0 \diamond y)$ Ву P3
We have that $0 \diamond(0 * y)=0 *(0 \diamond y)$ then $(y * y) \diamond(0 * y)=(y \diamond y) *(0 \diamond y)$ we
obtion $x=y$

## Proof: II

If $(y * y) \diamond(0 * y)=(y \diamond y) *(0 \diamond y)$. Hence by (P.2) $0 \diamond(0 * y)=0 *(0 \diamond y)$ since $y * y=y \diamond$ $y=0$ then $0 \diamond((y * y) * y)=0 *((y \diamond y) \diamond y)$, which implies that $0 \diamond y=0 * y$

### 3.3 Example

Let $\mathrm{X}=\{0,1\}$ we define the $(X ; *, 0,0)$ as follows:
$x * y=x+y-2 x . y$
And $x \diamond y=|x-y|$
For all $a, b \in X$ satisfy $P 1, P 2$ and $P 3$
Hence $(X ; *, 0,0)$ is pseudo - $B G$ - Algebra

### 3.4 Example

Let $\mathrm{X}=\{0,1,2\}$ we define the $(X ; *, 0,0)$ as follows:
Let $a * b=|a-b|(\sqrt{2})^{a b|a-b||b-2|} \quad$ and $\quad a \vee b=$ $|\mathrm{a}-\mathrm{b}|(3-\mathrm{b})^{\frac{(\mathrm{ab}|\mathrm{a}-\mathrm{b}|)}{(3-\mathrm{b})}}$
For all $a, b \in X$ satisfy $P 1, P 2$ and $P 3$
Hence $(X ; *, \downarrow, 0)$ Is pseudo - BG algebra

### 3.5 Properties

Let $(X ; *, \diamond 0)$ be a pseudo BG-algebra. Then
I. the right cancellation law holds in X , i.e., $x * y=$ $z \diamond y$ implies $x=z$,
II. $0 *(0 * x)=0 \diamond(0 \diamond x)=x$ for all $\mathrm{x} \in \mathrm{X}$,
III. If $0 * x=0 \diamond y$, then $x=y, \forall x, y \in X$,
IV. $(x *(0 * x)) \diamond x=x, \forall x, y \in X$.

## Proof:

I. Assume that $x * y=z \diamond y$. Then
$x=(x * y) \diamond(0 * y)=(z \diamond y) *(0 \diamond y)=$
$z$.
II. In axiom (P.3) for definition 3.1, replacing $y$ by $x$, we
have that $(x * x) \diamond(0 * x)=(x \diamond x) \star$ $(0 \diamond x)=x$ since by (P.2) $0 \diamond(0 * x)=0 *$ $(0 \diamond x)=x$.
III. If $0 * x=0 \diamond y$, then $x=(x * x) *(0 *$ $x)=(y * y) *(0 * y)=y$ by the axiom (P3) for pseudo BG-algebra.
IV. $(x *(0 * x)) \diamond x=(x *(0 * x)) *$
$(0 *(0 * x))=x$ by the axiom (P3) and
Proposition 3.5-(II).

### 3.6 Pseudo Sub-BG algebra

Let $X$ be pseudo BG- algebra then $I$ is called a pseudo sub $\quad B G$-algebra of $X$ if $I \subseteq X$ and $x *$ $y$ or $x \diamond y \in I$ when ever $x, y \in I$.

### 3.7 Ideal Pseudo -BG - algebras

In a pseudo - BG algebras, we have a set $I$ and $\emptyset \neq I \subseteq X$ then $I$ is pseudo ideal of $X$ if it satisfies,

1. $0 \in I$
2. $x * y, x \diamond y \in I$ and $y \in I$ imply $x \in I$ for all $x, y \in X$. Obviously $\{0\}$ and $X$ are pseudo ideal.

### 3.8 Definition

In pseudo $-\boldsymbol{B G}$-algebras Define the relation " $\leq$ " on X by ( $\mathrm{x} \leq \boldsymbol{y} \leftrightarrow \boldsymbol{x} * \boldsymbol{y}=\mathbf{0}$ ) or (equivalent $\boldsymbol{x} \oslash \boldsymbol{y}=\mathbf{0}$ ).

### 3.9 Proposition

Let $I$ be a pseudo ideal of a pseudo $-B G$ algebra $X$, if $x \in I$ and $y \leq x$, then $y \in I$.
Proof: Assume that $x \in I$ and $y \leq x$.Then $y * x=0$ and $y \diamond x=0$. By definition (2.4) $0 \in I ; x * y, x \diamond$ $y \in I$ and $y \in I$ imply $x \in I$ for all $x, y \in X$, we have $y \in I$.

### 3.10 Proposition

If $J$ is a pseudo ideal of a pseudo $-B G$ algebra $X$, then
i. $\forall x_{1}, x_{2}, x_{3} \in X, x_{1}, x_{2} \in J, x_{3} * x_{2} \leq x_{1} \rightarrow$ $x_{3} \in J$.
ii. $\quad \forall a, b c \in X, a, b, \in J, c \diamond b \leq a \rightarrow c \in J$.

Proof: If $J$ is a pseudo ideal and let $x_{1}, x_{2}, x_{3} \in X$.
Such that. $x_{1}, x_{2} \in J$ and $x_{3} * x_{2} \leq x_{1}$. Then $\left(x_{3} *\right.$ $\left.x_{2}\right) \diamond x_{1}=0 \in J$. Since $x_{1} \in J$ and we have $x_{3} * x_{2} \in$
$J$. Since $x_{2} \in J$ and $J$ is a pseudo ideal of $X$, then $x_{3} \in J$.

### 3.11 Proposition

Let A be pseudo ideal of a pseudo $-B G$ algebra $X$.
If $B$ is a pseudo ideal of $A$, then it is a pseudo ideal of $X$.

## Proof:

Since $B$ is a pseudo ideal of $A$, we have $0 \in B$. Let $y, x * y, x \diamond y \in B$ for some $x \in X$. If $\in A$, then $x \in B$, since $B$ is a pseudo ideal of $A$. If $x \in X-A$, then $y, x * y, x \diamond y \in B \subseteq A$ and so $x \in A$ because $A$ is a pseudo ideal of $X$. Thus $x \in B$ since $B$ is a pseudo ideal of $A$.This competes the proof.

### 3.12 Definition

An element w of a pseudo - BG-algebra $X$ is called a pseudo a tom if for every $x \in X, x \leq w$ implies $x=w$. Obviously, 0 is a pseudo atom of $X$.

### 3.13 Lemma

A non-zero element $a \in X$ is a pseudo atom of $X$ if $\{0, a\}$ is a pseudo ideal of $X$.

### 3.14 Definition

A non-empty subset $A$ of a pseudo $-B G-$ algebra $X$ is called a pseudo strong ideal of $X$ if it satisfies definition (3.6)
(PI3) $(x * y) \diamond z, y \in A$ imply $x * z \in A$;
(PI3') $(x \diamond y) * z, y \in A$ imply $x, y, z \in X$.
Note that if $X$ is a pseudo$B G$ - algebra satisfying $x * y=x \diamond y$ for all $x, y \in X$, then the notation of a pseudo strong ideal and a strong ideal consider .

### 3.15 Proposition

Every pseudo strong ideal is a pseudo ideal.
Proof: Putting $z=0$ in by definition (3.14), we have $x * y, x \diamond y, y \in A$ implied $x \in A$.

## 4 Sime Pseudo - $B G$ algebra

### 4.1 Definition

Let $X$ is a non-empty set, " $*^{\prime \prime}$ and " $\diamond$ " are two binary operation satisfying the following axioms:
P. $1 \quad x * x=x \diamond x=0$
P. $2 x * 0=x \diamond 0=x$
P. $3(x * y) \diamond(0 * y)=(x \diamond y) *(0 \diamond y)$

Then $(X ; *, 0,0)$ is semi pseudo $B G$ - algebra

### 4.2 Example

Let $\mathrm{X}=\{0,1,2\}$ we define the $(X ; *, 0,0)$ as follows

| $*$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{2}$ | $\mathbf{2}$ | $\mathbf{2}$ | $\mathbf{0}$ |


| $\diamond$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ |
| :--- | :--- | :--- | :--- |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{2}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{2}$ | $\mathbf{2}$ | $\mathbf{0}$ | $\mathbf{0}$ |

Then it is easy to show that $(X ; *, 0)$ and $(X ; 0,0)$ are not BG-algebras and ( $X ; *, 0,0$ ) is not a pseudo BGalgebra because $(2 * 1) \diamond(0 * 1)=(2 \diamond 1) *$ $(0 \diamond 1) \neq 2$, but $(X ; *, 0,0)$ is a semi pseudo BGalgebra.

### 4.3 Lemma

Let ( $\mathrm{X} ; *, 0,0$ ) be a semi pseudo $-B G$ - algebra if $(y * y) \diamond(0 * y)=(y \diamond y) *(0 \diamond y)$ then $(0 \diamond y)=$ ( $0 * y$ )
Proof: since $(y * y) \diamond(0 * y)=(y \diamond y) *(0 \diamond y)$ by P. 1 we get $0 \diamond(0 * y)=0 *(0 \diamond y)$.

### 4.4 Lemma

Let ( $\mathrm{X} ; *, \diamond, 0$ ) be a semi pseudo $-B G$ - algebra then

1. $(x \diamond 0) \diamond 0=x$ and $(x * 0) * 0=x \forall x \in X$.
2. If $(0 * x)=(0 * y)$ and $(0 \diamond x)=(0 \diamond y)$, then $x=y \forall x, y \in X$.
3. $(x *(0 * x) * x=x$ and $(x \diamond(0 \diamond x) \diamond x=x$

## Proof: it is clearer

### 4.5 Definition

$I$ is called a semi pseudo sub $B G$-algebra of $X$ if $I \subseteq$ $X$ and $x * y$ or $x \diamond y \in I$ when ever $x, y \in I$.

### 4.6 Definition

In a semi pseudo - $B G$ algebras, let $\emptyset \neq I \subseteq X$ then $I$ is pseudo ideal of $X$ if it satisfies,
3. $0 \in I$
4. $x * y, x \diamond y \in I$ and $y \in I$ implies $x \in I$ for all $x, y \in X$. Obviously $\{0\}$ and $X$ are semi pseudo ideal.

### 4.7 Theorem

The intersection two semi pseudo - BG - subalgebra is also semi pseudo -BG- subalgebra.
Proof: let $I \subset X$ and $J \subset X$ are semi Pseudo - BG Algebra
Since $0 \in I \& 0 \in J$ then $0 \in I \cap J$ and $I \& J \subset X$ then $\emptyset \neq I \cap J \in X$.
Let $x * y, x \diamond y \in I \cap J$ then $x * y, x \diamond y \in I$, and $x * y, x \diamond y \in J$
Since $I$ and $J$ are ideal semi Pseudo - BG - Algebra then $y \in I$ and $y \in J$ implies $x \in I$ and $\in J$, then $y \in I \cap J$ and implies $x \in I \cap J$.

### 4.8 Definition

In semi pseudo - $\boldsymbol{B} \boldsymbol{G}$-algebras Define the relation " $\leq$ " on X by $(\mathrm{x} \leq \boldsymbol{y} \leftrightarrow \boldsymbol{x} * \boldsymbol{y}=\mathbf{0})$ or (equivalent $x \diamond y=0$ ).

### 4.9 Theorem

Let ( $X, *, \Delta, 0$ ) be a semi pseudo $\boldsymbol{- B G}$-algebras. If $x *(y * z)=x \diamond(y \diamond z), \forall x, y, z \in X$ then $0 * x=$ $x=0 \diamond x, \forall x \in X$.
Proof: Let $x \in X$, where $x=x * 0=x *(x * x)=$ $(x * x) * x=0 * x$
and where $0 \diamond x=x$.

### 4.10 Theorem

Every semi pseudo - $\boldsymbol{B} \boldsymbol{G}$-algebras ( $X, *, \infty, 0$ ) satisfy the associative law is a group under each operation " * " and " $\circ$ ".
Proof: Putting $x=y=z$ in the associative law $(x * y) * z=x \diamond(y \diamond z)$ and using $0 * x=x * x=$ $x$. This means $0 \in X, \forall x \in X$ has a sets inverse the element of $X$ itself by definition 3.1 P2. There for $(X, *)$ and $(X, \diamond)$ are a group.

### 4.11 Definition

An element $a$ of a semi pseudo $\boldsymbol{- B G}$-algebras. $x$ is said to be minimal if $\forall x \in X$ the following implication $x \leq a \rightarrow x=a$

### 4.12 Property

Let $x$ be a semi pseudo $-\boldsymbol{B} \boldsymbol{G}$-algebras and let $a \in X$. If $a$ is minimal then
$x \diamond(x * a)=a \quad$ and $\quad x *(x \diamond a)=a$
Proof: let $a$ is minimal [by $x \leq 0 \rightarrow x=0$ ]
$x \diamond(x * a) \leq a \forall x \in X$. Since $a$ is minimal then $x \diamond(x * a)=a$

## 5 Homomorphism

### 5.1 Definition

Let $X$ and $y$ be a semi pseudo BG-Algebra. A mapping $f: X \rightarrow Y$ is called a homomorphism of semi pseudo BG-Algebra if
$f(x * y)=f(x) * f(x)$ and $f(x \diamond y)=f(x) \diamond$ $f(y), \forall x, y \in X$
Note that: if $f: X \rightarrow Y$ is homomorphism of semi pseudo, then $f(0 x)=0 y$, where $0 x$ and $0 y$ are zero elements of $x$ and $y$ respectively

### 5.2 Example

Let ( $X, *, \diamond, 0$ ) be a semi pseudo BG-Algebra then the function $f: X \rightarrow Y$ such that $f(x)=x * 0$, for any $x \in X$ is a homomorphism of semi pseudo BGAlgebra
$f(x) * f(y)=(x * 0) *(y * 0)=(x * y) * 0=$
$f(x * y)$. And $f(x) \diamond f(y)=(x * 0) \diamond(y * 0)=$ $(x \diamond y)=(x \diamond y) * 0=f(x \diamond y)$.

### 5.3 Theorem

Let $f: X \rightarrow Y$ be a homomorphism of semi pseudo BG-Algebra
I. If $B$ is a semi pseudo ideal of $y \rightarrow f^{-1}(B)$ is a pseudo ideal of $x$
II. If $f$ is surjective and $I$ is a semi pseudo ideal of $x$, then $f(I)$ is a semi pseudo ideal of $y$.
Proof: I. $0_{y} \in f^{-1}(B)$, let $y \in f^{-1}(B)$ and let $x_{1}, x_{2} \in X$ be such that
$x_{1} * y \in f^{-1}(B)$ and $x_{2} \triangleright y \in f^{-1}(B)$. To show $x_{1}, x_{2} \in f^{-1}(B)$ ?
$x_{1} * y \in f^{-1}(B) \quad \rightarrow \exists b_{1} \in B$ and $\quad b_{1} \in B \quad$ such that
$f\left(x_{1} * y\right)=b_{1}$ and $f\left(x_{2} \diamond y\right)=b_{2}$ also $y \in$
$f^{-1}(B), \exists b \in B$
Such that
$y=f^{-1}(b) \Rightarrow f(y)=b$
$f\left(x_{1}\right) * f(y)=f\left(x_{1} * y\right)=b_{1}$
$b_{1} \in B \Rightarrow f\left(x_{1}\right) \in B$
$f\left(x_{2}\right) \diamond f(y)=f\left(x_{2} \diamond y\right)=b_{2}$
$b_{2} \in B \Rightarrow f\left(x_{2}\right) \in B$
Since $B$ is semi pseudo ideal
$\therefore f\left(x_{1}\right) \in B \quad \& f\left(x_{2}\right) \in B$ in $Y \quad \Rightarrow \quad x_{1} \in$ $f^{-1}(B) \& x_{2} \in f^{-1}(B)$ in $X$
$f^{-1}(B)$ is a semi pseudo ideal in $X$.
II. Assume that f is surjective and let I be a semi pseudo ideal of $x$ obriousely, $0_{y} \in f(I)$. For every $y \in f(I)$. Let $a, b \in Y$ be such that $a * y \in f(I)$ and $b \diamond y \in f(I)$, then there exist $x_{*}, x_{\circ} \in I$ such that $f\left(x_{*}\right)=a * y$ and $f\left(x_{\circ}\right)=b \diamond y$. Since $y \in f(I)$ there exist $x y \in I, y=f\left(x_{y}\right)$. Also $f$ is surjective. $\exists x_{a}, x_{b} \in X$ such that $f\left(x_{a}\right)=a$ and $f\left(x_{b}\right)=$

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$b, f\left(x_{a} * x_{y}\right)=f\left(x_{a}\right) * f\left(x_{y}\right)=a * y \in f(I)$ and
$f\left(x_{b} * x_{y}\right)=f\left(x_{b}\right) * f\left(x_{y}\right)=b * y \in f(I)$
Which implies that $x_{a} * x_{y} \in I$ and $x_{b} \diamond x_{y} \in I$, since $I$ is semi pseudo ideal of $X$ we get $x_{a}, x_{b} \in I$ and $a=f\left(x_{a}\right), b=f\left(x_{b}\right) \in f(I) f(I)$ is semi pseudo ideal of $X$.

### 5.4 Corollary

Let $f: X \rightarrow Y$ be a homomorphism of semi pseudo ideal. Then $\operatorname{ker}(f)=\{x \in X ; f(x)=0\}$ is a semi pseudo ideal of $X$.

### 5.5 Property

Let $\quad f:\left(X, *_{1},{ }^{\circ} 1,0\right) \longrightarrow\left(Y, *_{2},{ }_{2}, 0\right)$ be a homomorphism of semi pseudo BG-Algebra. Then $x *_{1} y, x \diamond_{1} y \in \operatorname{ker}(f)$ if $f(x)=f(y), \forall x, y \in X$
Proof: Assume that $f(x)=f(y)$. Then $f(x) *_{2} f(y)=f\left(x *_{1} y\right)=0$ and $f(x) \otimes_{2} f(y)=$ $f\left(x \otimes_{1} y\right)=0$ Hence $x *_{1} y, x \otimes_{1} y \in \operatorname{ker}(f)$.

### 5.6 Theorem

Let $f: X \rightarrow Y$ is homomorphism of Semi pseudo ideal. Then $f$ is monomorphism if $f \operatorname{ker}(f)=\{0\}$.

### 5.7 Theorem

Let $X, Y, Z$ be a Semi pseudo ideal and $h: X \rightarrow Y$ be an onto homomorphism of Semi ideal and $g: Y \rightarrow Z$ be a homomorphism of semi pseudo BG-algebra. If $\operatorname{ker}(h) \subseteq \operatorname{ker}(g), \exists$ a unique homomorphism of Semi pseudo ideal $f: X \rightarrow Z$ satisfy $f \circ h=g$.

## Conclusion

we introduced the notion new types of algebras pseudo BG- algebra, pseudo sub BG -algebra, Pseudo Ideal and pseudo strong Ideal of Pseudo-BGAlgebras. We state some Proposition and examples which determine the relationships between these notions and some types of ideal and we introduced the notion semi pseudo BG- algebra, pseudo sub BG -algebra, Pseudo Ideal and pseudo strong Ideal of semi pseudo-BG-Algebras.

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\begin{aligned}
& \text { BG - تركيب الجبر الزائف - BG الجبر شببة الزائف } \\
& \text { ارام خليل ابراهيم باجلان باجلان , راستي رحيم محمدامين, شليان ، التربية ، جامعة كرمان ، عدنان علي }
\end{aligned}
$$

الملخص
في هذا البحث ، قدمنا مفهوم أنواع جديدة من الجبر الزائف -BG الجبر، الجبر شبه الزائف BG، الجبر المثالي الزائف والمثال القوي الزائف. نذكر بعض المقترحات والأمثلة التي تحدد العلاقات بين هذه المفاهيم وبعض أنواع المثالية وقدمنا فكرة شبه زائفة -BG الجبر، شبه زائف - BG الجبر . مثالي شبه زائف -BG - الجبر . لقد بحثا في مفهوم جديد للجبر يسمى شبه الجبر الزائف -BG. نذكر بعض المقترحات والأمثلة التي تحدد العلاقات بين هذه المفاهيم وبعض أنواع المثل العليا المحددة الحد الأدنى وتماثل الثكل والنواة.

