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ON VIASMAN-GRAY MANIFOLD WITH GENERALIZED CONHARMONIC CURVATURE TENSOR

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ABSTRACT

The current study deals with the generalized conhormonic curvature tensor of Vaisman - Gray manifold. The aim of this paper to calculate the components of generalized Ricci tensor and generalized Riemannian tensor of VG-manifold in the adjoint G-structure space to find Generalized conharmonic Curvature tensor of VG-manifold, one of the almost hermitian manifold structures is donated by $W_1 \oplus W_4$, where W_1 and W_4 respectively denoted to the nearly kahler manifold and locally conformal kahler manifold have been studied .

1. Introduction

In (1957) Y. Ishi [6] studies conharmonic transformation which is a conformal transformation.
One of the representative work of differential geometry is an almost Hermitian structure. In particular, the problem of classification of this structurer. Gray and Hervella [5] foundd that the action of the unitary group $U(n)$ on the space of all tensorrs of type (3,0)decomposed this space into sixteen classes. In 1993, Banaru [3] succeeded in re-classifying the sixteen classes of almost Herm manifold by using the structure and virtual tensors, which were named Kirichnko's tensors [2]. Among the sixteen classes of almost hermitian manifold, there are eight which are invariants under the conformal transformation metric.
This research tackles the almost kahler and nearly kahler manifold so it found their important geometrical properties. In 2006 Krolikowski proved that there is an 4-dimensional almost kahler manifold of locally conformally flat with a metric of a special form [8]. In 2010, Lafta studied conhormonic curvature tensor of classes almost kahler and nearly kahler manifolds [9]. In 2016 [13] Habeeb M. Abood and Yasir A. Abdulameer where studied the flatnesss of conharmonic curvaturee tensor of VG-manifold in using the method of an adjoint G-structure space and I have proved that the compounds of conharmonic curvature tonsor of VG-manifold and Riemannian curvature tensor and Ricci tensor of VG-manifold in the adjoint G-structure space. In 2018 [1] Ali A. Shihab and Dhabia`a M. Ali where studied generalized conharmonic curvature tensor of nearly Kahler manifold. In the study also concentrates generalized conharmonic curvature tensor of Vaisman - Gray manifold.

2. Preliminaries

Let M be a smooth manifold of dimension $2n$, $C^\infty(M)$ is algebra of smooth function on M; X(M) is the module of smooth vector fields on manifold of M ; $g = <.,>$ is Riemannian metrics, ∇ is Riemannian connection of the metrics g on M; d is the operator of exterior differentiation. In the further all manifold tensor field, etc. objects are assumed smooth a classes $C^\infty(M)$.
we concentrate our attention on generalized conharmonic tensor of Vaisman-Gray manifold, where Vaisman Gray manifold is considered as one of the most important classes of almost hermitian manifold which is denoted by $W_1 \oplus W_4$ and represents a generalization of the W_1 and W_4 classes.The space W_1 is called nearly Kähler manifold

(NK -manifold) and $W_4$is called locally conformal Kahler manifold (LCK-manifold).

Definition 1 [5]

A Vaisman-Gray structure is an G-structure{ J , g = < . , . > } such that:

$\nabla_X(F) \times (X,Y) = \frac{-1}{2(n-1)}\{< X,X > \delta F(Y) - < X,Y > \delta F(X) - < JX,Y > \delta F(JX)\}$ where ∇ is the Riemannian connection of g, $F(X,Y) = < JX,Y >$ is the Kähler form, δ is a coderivative and $X,Y,Z \in X(M)$. An AH-structure (J , g = < . , . >) is called a structure of class W_1 or nearly Kahler (NK - structure)

if its Kähler form is a killing form, or equivalently, $\nabla_X(J) = 0$; $X \in X(M)$. An AH – structure (J , g = < . , . >) is called a structure of class W_4or locally conformal Kahler structure (LCK -structure) if. $\nabla_X(F)(Y,Z) = \frac{-1}{2(n-1)}\{< X,Y > \delta F(Z) - < X,Z > \delta F(Y) - < X,JY > \delta F(JZ) + X,JZ > \delta F(JY)\}$.

A manifold M with Vaisman-Gray structure is called a Vaisman-Gray manifold (VG -manifold).

Theorem 2 [7]

The collection of the structure equations of VG - manifold in the adjoint G-structure space has the following forms:

i) $d\omega^a = \omega^a_b \wedge \omega^b + B^{ab}_c \omega^c \wedge \omega_b + B^{abc} \omega_b \wedge \omega_c$;

ii) $d\omega_a = -\omega^b_a \wedge \omega_b + B^c_{ab} \omega_c \wedge \omega^b + \omega_{abc} \omega^b \wedge \omega_c$;

iii) $d\omega^a_b \omega^a_c \wedge \omega^c_b + \left(2B^{adh}B_{hbc} + A^{ad}_{bc}\right)\omega^c \wedge \omega_d + \left(B^{ah}{}_{[c}B_{d]bh} + A^a_{bcd}\right)\omega^c \wedge \omega^d + (B_{bh}{}^{[c}B^{d]ah} + A^{acd}_b)\ \omega_c \wedge \omega_d$,

where ω^i is the components of mixture form, ω^i_j is the components of Riemannian relation of metric g,$\{A^a_{bcd}$,$A^{acd}_b\}$ is some functions on adjoint G-structure space and A^{ad}_{bc} is system functions in the adjoint G-structure space which are symmetric by the lower and upper indices and are called components of holomorphic sectional curvature tensor. the next theorem gives the components of Riemannian curvature tensor of VG-manifold.

Theorem 3 [10]

In the adjoint G-structure space, the components of Riemannian curvature tensor of VG-manifold are given by the following forms:

i) $R_{abcd} = 2(B_{ab[cd]} + \alpha_{[a}B_{b]cd})$;

ii)$R_{\hat{a}bcd} = 2A^a_{bcd}$;

iii)$R_{\hat{a}\hat{b}cd} = 2(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]})$;

iv)$R_{\hat{a}bc\hat{d}} = A^{ad}_{bc} + B^{adh}B_{hbc} - B^{ah}{}_cB_{hb}{}^d$,

where$\{\alpha^a{}_b, \alpha_a{}^b, \alpha_{ab}, \alpha^{ab}\}$ are some functions on adjoint G-structure space such that:

$d\alpha_a + \alpha_b\omega^b_a = \alpha^b_a\omega_b + \alpha_{ab}\omega^b$ and $d\alpha^a - \alpha^b\omega^a_b = \alpha^a_b\omega^b + \alpha^{ab}\omega_b$.

Definition 4 [12]

A tensor of type (2,0) which is defined as is $r_{ij} = R^k_{ijk} = g^{kl}R_{kijl}$ is called a Ricci tensor.

Definition 5 [10]

In the adjoint G-structure space, the components of Ricci tensor of Vaisman- Gray manefold are given as the folowing forms:

1) $r_{ab} = \frac{1-n}{2}(\alpha_{ab} + \alpha_{ba} + \alpha_a + \alpha_b)$

2) $r_{\hat{a}b} = r^a_b = 3B^{cah}B_{cbh} + A^{ca}_{cb} + \frac{n-1}{2}(\alpha^a\alpha_b - \alpha^h\alpha_h) - \frac{1}{2}\alpha^h{}_h\delta^a_b + (n-2)\alpha^a{}_b$

Definition 6 [6]

Let (M,J, g) be a Vaisman-Gray manifold, the Conharmonic curvature tensor of AH- manifold M of type [4, 0] which is dafined as the following form:

$T_{ijkl} = R_{ijkl} - \frac{1}{2(n-1)}[r_{il}g_{jk} - r_{jl}g_{ik} + r_{jk}g_{il} - r_{ik}g_{jl}]$ ……………..…..**(1)**

Where r, R and g are respectively Ricci tonsor, Riemanian curveture tensor and Riemanian metric. and satisfies all the properties of algebraic curvature tensor:

$$\left.\begin{array}{c} 1)\ T[X,Y,Z,W] = -T[Y,\ X,Z,W];\\ 2)\ T[X,Y,Z,W] = -T(X,Y,W,Z);\\ 3)\ T[X,Y,Z,W] + T[Y,\ Z,X,\ W] + T[Z,\ X,\ Y,\ W] = 0;\\ 4)\ T\ [X,Y,Z,W] = T[Z,W,X,Y]; \end{array}\right\}$$

……..(2)

$\forall X,Y,Z,W \in X(M)$

Remark 7 [2]

From the Banarues classification of AH-menifold, the class VG-manifold satisfies the following conditions:

$B^{abc} = -B^{bac}$, $B^{ab}_c = \alpha^{[a}\delta^{b]}_c$.

Definition 8 [4]

A generalized Riemannian curveture tensor on AH-manfold M is called a tensor of kind (4, 0) whose is defined as the following format:

$(HR)(X,Y,Z,W) = \frac{1}{16}\{3[\mathrm{R}(\mathrm{X},\mathrm{Y},\mathrm{Z},\mathrm{W}) + \mathrm{R}.(\mathrm{JX},\mathrm{JY},\mathrm{Z},\mathrm{W}) + \mathrm{R}.(\mathrm{X},\mathrm{Y},\mathrm{JZ},\mathrm{JW}) + \mathrm{R}(\mathrm{JX},\mathrm{JY},\mathrm{JZ},\mathrm{JW})] - \mathrm{R}.(\mathrm{X},\mathrm{Z},\mathrm{JW},\mathrm{JY}) - \mathrm{R}(\mathrm{JX},\mathrm{JZ},\mathrm{W},\mathrm{Y}) - \mathrm{R}.(\mathrm{X},\mathrm{W},\mathrm{JY},\mathrm{JZ}) - \mathrm{R}(\mathrm{JX},\mathrm{JW},\mathrm{Y},\mathrm{Z}) + \mathrm{R}.(\mathrm{JX},\mathrm{Z},\mathrm{JW},\mathrm{Y}) + \mathrm{R}(\mathrm{X},\mathrm{JZ},\mathrm{W},\mathrm{JY}) + \mathrm{R}.(\mathrm{JX},\mathrm{W},\mathrm{Y},\mathrm{JZ}) + \mathrm{R}(\mathrm{X},\mathrm{JW},\mathrm{JY},\mathrm{Z})\}$,

where R(X , Y , Z , W) is the Riemannian curvature tensor$\mathrm{R}(\mathrm{X},\mathrm{Y},\mathrm{Z},\mathrm{W}) \in \mathrm{T_p}(\mathrm{M})$ and satisfies the following properties :

1) $(\mathrm{HR})(\mathrm{X},\mathrm{Y},\mathrm{Z},\mathrm{W}) = -(\mathrm{HR})(\mathrm{Y},\mathrm{X},\mathrm{Z},\mathrm{W}) = -(\mathrm{HR})(\mathrm{X},\mathrm{Y},\mathrm{W},\mathrm{Z})$;

2) $(\mathrm{HR})(\mathrm{X},\mathrm{Y},\mathrm{Z},\mathrm{W})) = (\mathrm{HR})(\mathrm{Z},\mathrm{W},\mathrm{X},\mathrm{Y})$;

3) $(\mathrm{HR})(\mathrm{X},\mathrm{Y},\mathrm{Z},\mathrm{W}) + (\mathrm{HR})(\mathrm{X},\mathrm{Z},\mathrm{W},\mathrm{Y}) + (\mathrm{HR})(\mathrm{X},\mathrm{W},\mathrm{Y},\mathrm{Z}) = 0$;

Definition 9

A generalized Conharmonic curvature tensor (GT-curvature) tensor of Vaisman-Grey manifold (VG- manifold) M of type (4, 0) which is defined as the following form:

$(HR)(X,Y,Z,W) = \frac{1}{16}\{3[\mathrm{R}.(\mathrm{X},\mathrm{Y},\mathrm{Z},\mathrm{W}) + \mathrm{R}.(\mathrm{JX},\mathrm{JY},\mathrm{Z},\mathrm{W}) + \mathrm{R}.(\mathrm{X},\mathrm{Y},\mathrm{JZ},\mathrm{JW}) + \mathrm{R}.(\mathrm{JX},\mathrm{JY},\mathrm{JZ},\mathrm{JW})] - \mathrm{R}.(\mathrm{X},\mathrm{Z},\mathrm{JW},\mathrm{JY}) - \mathrm{R}.(\mathrm{JX},\mathrm{JZ},\mathrm{W},\mathrm{Y}) - \mathrm{R}.(\mathrm{X},\mathrm{W},\mathrm{JY},\mathrm{JZ}) - \mathrm{R}.(\mathrm{JX},\mathrm{JW},\mathrm{Y},\mathrm{Z}) + \mathrm{R}.(\mathrm{JX},\mathrm{Z},\mathrm{JW},\mathrm{Y}) + \mathrm{R}.(\mathrm{X},\mathrm{JZ},\mathrm{W},\mathrm{JY}) + \mathrm{R}.(\mathrm{JX},\mathrm{W},\mathrm{Y},\mathrm{JZ}) + \mathrm{R}.(\mathrm{X},\mathrm{JW},\mathrm{JY},\mathrm{Z})\}$

Consider this equation in the adjoint G-structre space we get:

$GT_{abcd}=\frac{1}{16}\{3[T_{abcd}+T_{\hat{a}\hat{b}cd}+T_{ab\hat{c}\hat{d}}+T_{\hat{a}\hat{b}\hat{c}\hat{d}}]-T_{ac\hat{d}\hat{b}}-T_{\hat{a}\hat{c}db}-T_{ad\hat{b}\hat{c}}-T_{\hat{a}\hat{d}bc}+T_{\hat{a}c\hat{d}b}+T_{a\hat{c}d\hat{b}}+T_{\hat{a}db\hat{c}}+T_{a\hat{d}\hat{b}c}\}$.

Theorem 10

The components of the generalized Riemannian curvature tensor of VG-manifold in the adjoint G-structure space are given as the following forms:

1) $GT_{\hat{a}b\hat{c}d}=\left\{-A^{ac}_{bd}+B^{ah}{}_{b}B_{hd}{}^{c}+\frac{1}{2}\alpha^{[a}_{[b}\delta^{c]}_{d]}\right\}$

2) $GT_{\hat{a}bc\hat{d}}=\left\{A^{ad}_{bc}-B^{ah}{}_{c}B_{hb}{}^{d}+\frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]}\right\}$

Proof

By using Theorem (3) and definition (8), we calculation the compounds of generalized Riemannian tensor as follows:

1) For $i=a$, $j=b$, $k=c$ and $l=d$,

$GT_{abcd}=\frac{1}{16}\{3[T_{abcd}-T_{abcd}-T_{abcd}+T_{abcd}]+T_{acdb}+T_{acdb}+T_{adbc}+T_{adbc}-T_{acdb}-T_{acdb}-T_{adbc}-T_{adbc}\}$

$GT_{abcd}=0$

2) For $i=\hat{a}$, $j=b$, $k=c$ and $l=d$

$GT_{\hat{a}bcd}\frac{1}{16}\{3[T_{\hat{a}bcd}+T_{\hat{a}bcd}-T_{\hat{a}bcd}-T_{\hat{a}bcd}]+T_{\hat{a}cdb}-T_{\hat{a}cdb}+T_{\hat{a}dbc}-T_{\hat{a}dbc}+T_{\hat{a}cdb}-T_{\hat{a}cdb}+T_{\hat{a}dbc}-T_{\hat{a}dbc}\}$

$GT_{\hat{a}bcd}=0$

3) For $i=a, j=\hat{b}, k=c$ and $l=d$,

$GT_{a\hat{b}cd}=\frac{1}{16}\{3[T_{a\hat{b}cd}+T_{a\hat{b}cd}-T_{a\hat{b}cd}-T_{a\hat{b}cd}]-T_{acd\hat{b}}+T_{acd\hat{b}}-T_{ad\hat{b}c}+T_{ad\hat{b}c}-T_{acd\hat{b}}+T_{acd\hat{b}}-T_{ad\hat{b}c}+T_{ad\hat{b}c}\}$

$GT_{a\hat{b}cd}=0$

4) For $i=a, j=b, k=\hat{c}$ and $l=d$,

$GT_{ab\hat{c}d}=\frac{1}{16}\{3[T_{ab\hat{c}d}-T_{ab\hat{c}d}+T_{ab\hat{c}d}-T_{ab\hat{c}d}]+T_{a\hat{c}db}-T_{a\hat{c}db}-T_{adb\hat{c}}+T_{adb\hat{c}}-T_{a\hat{c}db}+T_{a\hat{c}db}+T_{adb\hat{c}}-T_{adb\hat{c}}\}$

$GT_{ab\hat{c}d}=0$

5) For $i=a, j=b, k=c$ and $l=\hat{d}$

$GT_{abc\hat{d}}=\frac{1}{16}\{3[T_{abc\hat{d}}-T_{abc\hat{d}}+T_{abc\hat{d}}-T_{abc\hat{d}}]-T_{ac\hat{d}b}+T_{ac\hat{d}b}+T_{a\hat{d}bc}-T_{a\hat{d}bc}+T_{ac\hat{d}b}-T_{ac\hat{d}b}-T_{a\hat{d}bc}+T_{a\hat{d}bc}\}$.

$GT_{abc\hat{d}}=0$

6) *For* $i=\hat{a}, j=\hat{b}, k=c$ *and* $l=d$

$GT_{\hat{a}\hat{b}cd}=\frac{1}{16}\{3[T_{\hat{a}\hat{b}cd}-T_{\hat{a}\hat{b}cd}-T_{\hat{a}\hat{b}cd}+T_{\hat{a}\hat{b}cd}]-T_{\hat{a}cd\hat{b}}-T_{\hat{a}cd\hat{b}}-T_{\hat{a}d\hat{b}c}-T_{\hat{a}d\hat{b}c}+T_{\hat{a}cd\hat{b}}+T_{\hat{a}cd\hat{b}}+T_{\hat{a}d\hat{b}c}+T_{\hat{a}d\hat{b}c}\}$.

$GT_{\hat{a}\hat{b}cd}=0$

7) For $i=\hat{a}, j=b, k=\hat{c}$ and $l=d$

$GT_{\hat{a}b\hat{c}d}=\frac{1}{16}\{3[T_{\hat{a}b\hat{c}d}+T_{\hat{a}b\hat{c}d}+T_{\hat{a}b\hat{c}d}+T_{\hat{a}b\hat{c}d}]+T_{\hat{a}\hat{c}db}+T_{\hat{a}\hat{c}db}-T_{\hat{a}db\hat{c}}-T_{\hat{a}db\hat{c}}+T_{\hat{a}\hat{c}db}+T_{\hat{a}\hat{c}db}-T_{\hat{a}db\hat{c}}-T_{\hat{a}db\hat{c}}\}GT_{\hat{a}b\hat{c}d}=\frac{1}{16}\{12\,T_{\hat{a}b\hat{c}d}+4T_{\hat{a}\hat{c}db}-4T_{\hat{a}db\hat{c}}\}$

$GT_{\hat{a}b\hat{c}d}=\frac{1}{4}\{3\,T_{\hat{a}b\hat{c}d}+T_{\hat{a}\hat{c}db}-T_{\hat{a}db\hat{c}}\}$

$GT_{\hat{a}b\hat{c}d}=\frac{1}{4}\{-3\,T_{\hat{a}bd\hat{c}}+T_{\hat{a}\hat{c}db}-T_{\hat{a}db\hat{c}}\}$

$GT_{\hat{a}b\hat{c}d}=\frac{1}{4}\left\{-3(A^{ac}_{bd}+B^{ach}B_{hbd}-B^{ah}{}_{d}B_{hb}{}^{c}+2\left(-B^{ach}B_{hdb}+\alpha^{[a}_{[b}\delta^{c]}_{d]}\right)-A^{ac}_{db}-B^{ach}B_{hdb}+B^{ah}{}_{b}B_{hd}{}^{c}\right\}$

$GT_{\hat{a}b\hat{c}d}=\frac{1}{4}\left\{-3A^{ac}_{bd}-3B^{ach}B_{hbd}+3B^{ah}{}_{d}B_{hb}{}^{c}-2B^{ach}B_{hdb}+2\alpha^{[a}_{[b}\delta^{c]}_{d]}-A^{ac}_{db}-B^{ach}B_{hdb}+B^{ah}{}_{b}B_{hd}{}^{c}\right\}$

By using Remark (7) and according to the equality $B^{abc}=-B^{bac}$,

we get:

$GT_{\hat{a}b\hat{c}d}=\frac{1}{4}\left\{-3A^{ac}_{bd}-3B^{ach}B_{hbd}+3B^{ah}{}_{d}B_{hb}{}^{c}+2B^{ach}B_{hdb}+2\alpha^{[a}_{[b}\delta^{c]}_{d]}-A^{ac}_{bd}+B^{ach}B_{hbd}+B^{ah}{}_{b}B_{hd}{}^{c}\right\}$

$GT_{\hat{a}b\hat{c}d}=\frac{1}{4}\left\{-4A^{ac}_{bd}+4B^{ah}{}_{b}B_{hd}{}^{c}+2\alpha^{[a}_{[b}\delta^{c]}_{d]}\right\}$

$GT_{\hat{a}b\hat{c}d}=\left\{-A^{ac}_{bd}+B^{ah}{}_{b}B_{hd}{}^{c}+\frac{1}{2}\alpha^{[a}_{[b}\delta^{c]}_{d]}\right\}$

8) For $i=\hat{a}, j=b, k=c$ and $l=\hat{d}$

$GT_{\hat{a}bc\hat{d}}=\frac{1}{16}\{3*[T_{\hat{a}bc\hat{d}}+T_{\hat{a}bc\hat{d}}+T_{\hat{a}bc\hat{d}}+T_{\hat{a}bc\hat{d}}]-T_{\hat{a}c\hat{d}b}-T_{\hat{a}c\hat{d}b}+T_{\hat{a}\hat{d}bc}+T_{\hat{a}\hat{d}bc}-T_{\hat{a}c\hat{d}b}-T_{\hat{a}c\hat{d}b}+T_{\hat{a}\hat{d}bc}+T_{\hat{a}\hat{d}bc}"\}$. $GT_{\hat{a}bc\hat{d}}=\frac{1}{16}\{12T_{\hat{a}bc\hat{d}}+4T_{\hat{a}\hat{d}bc}-4T_{\hat{a}c\hat{d}b}\}$

$GT_{\hat{a}bc\hat{d}}=\frac{1}{4}\{3T_{\hat{a}bc\hat{d}}+T_{\hat{a}\hat{d}bc}-T_{\hat{a}c\hat{d}b}\}$

$GT_{\hat{a}bc\hat{d}}=\frac{1}{4}\{3T_{\hat{a}bc\hat{d}}+T_{\hat{a}\hat{d}bc}+T_{\hat{a}cb\hat{d}}\}GT_{\hat{a}bc\hat{d}}=\frac{1}{4}\left\{3(A^{ad}_{bc}+B^{adh}B_{hbc}-B^{ah}{}_{c}B_{hb}{}^{d}+2\left(-B^{adh}B_{hbc}+\alpha^{[a}_{[b}\delta^{d]}_{c]}\right)+A^{ad}_{cb}+B^{adh}B_{hcb}-B^{ah}{}_{b}B_{hc}{}^{d}\right\}$

$GT_{\hat{a}bc\hat{d}}=\frac{1}{4}\left\{3A^{ad}_{bc}+3B^{adh}B_{hbc}-3B^{ah}{}_{c}B_{hb}{}^{d}-2B^{adh}B_{hbc}+2\alpha^{[a}_{[b}\delta^{d]}_{c]}+A^{ad}_{cb}+B^{adh}B_{hcb}-B^{ah}{}_{b}B_{hc}{}^{d}\right\}$

By using Remark (7) and according to the equality $B^{abc}=-B^{bac}$,

we get:

$GT_{\hat{a}bc\hat{d}}=\frac{1}{4}\left\{3A^{ad}_{bc}+3B^{adh}B_{hbc}-3B^{ah}{}_{c}B_{hb}{}^{d}-2B^{adh}B_{hbc}+2\alpha^{[a}_{[b}\delta^{d]}_{c]}+A^{ad}_{bc}-B^{adh}B_{hbc}-B^{ah}{}_{b}B_{hc}{}^{d}\right\}$

$GT_{\hat{a}bc\hat{d}}=\frac{1}{4}\left\{4A^{ad}_{bc}-4B^{ah}{}_{c}B_{hb}{}^{d}+2\alpha^{[a}_{[b}\delta^{d]}_{c]}\right\}$

$GT_{\hat{a}bc\hat{d}}=\left\{A^{ad}_{bc}-B^{ah}{}_{c}B_{hb}{}^{d}+\frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]}\right\}$.

Definition 11 [11]

A tensor of type (2 , 0) which is defined as $r(GT)_{ij}=(GT)^{k}_{ijk}$ is called a generalized Ricci tensor.

Theorem 12

The components of generalized Ricci tensor of VG-manifold in the adjoint G-structure space are given as the following form:

$$r(GT)_{\hat{a}b}=(GT)_{c\hat{a}b\hat{c}}=-A^{ac}_{bc}+B^{ah}{}_{b}B_{hc}{}^{c}+\frac{1}{2}\alpha^{[a}_{[b}\delta^{c]}_{c]}$$

Proof

By using Theorem (10), we can get the components of generalized Ricci tensor as follows:

1) For $i = a$, $j = b r(GT)_{ab} = (GT)^{k}_{abk} = (GT)^{c}_{abc} + (GT)^{\hat{c}}_{ab\hat{c}} = (GT)_{\hat{c}abc} = (GT)_{cab\hat{c}} = 0$

2) For $i = \hat{a}$, $j = \hat{b}$

$r(GT)_{\hat{a}\hat{b}} = (GT)^{k}_{\hat{a}\hat{b}k} = (GT)^{c}_{\hat{a}\hat{b}c} + (GT)^{\hat{c}}_{\hat{a}\hat{b}\hat{c}} = (GT)_{\hat{c}\hat{a}\hat{b}c} = (GT)_{c\hat{a}\hat{b}\hat{c}} = 0$

3) For $i = \hat{a}$, $j = b$

$r(GT)_{\hat{a}b} = (GT)^{k}_{\hat{a}bk} = (GT)^{c}_{\hat{a}bc} + (GT)^{\hat{c}}_{\hat{a}b\hat{c}} = (GT)_{\hat{c}\hat{a}bc} + (GT)_{c\hat{a}b\hat{c}}$

$r(GT)_{\hat{a}b} = (GT)_{c\hat{a}b\hat{c}} = -A^{ac}_{bc} + B^{ah}{}_{b}B_{hc}{}^{c} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{c]}_{c]}$.

Theorem 13

The components from the generalized conharmonic curvature of VG-manifold in the adjoint G-structure are given as follows:

$$GT_{ijkl} = R_{ijkl} - \frac{1}{2(n-1)}[g_{ik}r(GT)_{jl} + g_{jl}r(GT)_{ik} - g_{il}r(GT)_{jk} - g_{jk}r(GT)_{il}]$$

Then

1) $GT_{\hat{a}\hat{b}cd} = 0 - \frac{1}{2(n-1)}\Big\{(\delta^{a}_{c})\Big(-A^{bk}_{dk} + B^{bh}{}_{d}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[b}_{[d}\delta^{k]}_{k]}\Big) + (\delta^{b}_{d})\Big(-A^{ak}_{ck} + B^{ah}{}_{c}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[a}_{[c}\delta^{k]}_{k]}\Big) - (\delta^{a}_{d})\Big(-A^{bk}_{ck} + B^{bh}{}_{c}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[b}_{[c}\delta^{k]}_{k]}\Big) - (\delta^{b}_{c})\Big(-A^{ak}_{dk} + B^{ah}{}_{d}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[a}_{[d}\delta^{k]}_{k]}\Big)\Big\}$

2) $GT_{\hat{a}b\hat{c}d} = -A^{ac}_{bd} + B^{ah}{}_{b}B_{hd}{}^{c} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{c]}_{d]} + \frac{1}{2(n-1)}\Big\{(\delta^{a}_{d})\Big(-A^{ck}_{bk} + B^{ch}{}_{b}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[c}_{[b}\delta^{k]}_{k]}\Big) + (\delta^{c}_{b})\Big(-A^{ak}_{dk} + B^{ah}{}_{d}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[a}_{[d}\delta^{k]}_{k]}\Big)\Big\}$

3) $GT_{\hat{a}bc\hat{d}} = -A^{ad}_{bc} + B^{ah}{}_{b}B_{hc}{}^{d} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]} - \frac{1}{2(n-1)}\Big\{(\delta^{a}_{c})\Big(-A^{dk}_{bk} + B^{dh}{}_{b}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[d}_{[b}\delta^{k]}_{k]}\Big) + (\delta^{d}_{b})\Big(-A^{ak}_{ck} + B^{ah}{}_{c}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[a}_{[c}\delta^{k]}_{k]}\Big)\Big\}$

Proof:

1) *For* $i = a$ ، $j = b$ ، $k = c$ *and* $l = d$

$GT_{abcd} = R_{abcd} - \frac{1}{2(n-1)}[g_{ac}r(GT)_{bd} + g_{bd}r(GT)_{ac} - g_{ad}r(GT)_{bc} - g_{bc}r(GT)_{ad}]GT_{abcd} = 0 - \frac{1}{2(n-1)}\{(0)(0) + (0)(0) - (0)(0) - (0)(0)\} = 0$

$GT_{abcd} = 0$

2) *For* $i = \hat{a}, j = b, k = c$ *and* $l = d$

$GT_{\hat{a}bcd} = R_{\hat{a}bcd} - \frac{1}{2(n-1)}[g_{\hat{a}c}r(GT)_{bd} + g_{bd}r(GT)_{\hat{a}c} - g_{\hat{a}d}r(GT)_{bc} - g_{bc}r(GT)_{\hat{a}d}]GT_{\hat{a}bcd} = 0 - \frac{1}{2(n-1)}\Big\{(\delta^{a}_{c})(0) + (0)\Big(-A^{ak}_{ck} + B^{ah}{}_{c}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[a}_{[c}\delta^{k]}_{k]}\Big) - (\delta^{a}_{d})(0) - (0)\Big(-A^{ak}_{dk} + B^{ah}{}_{d}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[a}_{[d}\delta^{k]}_{k]}\Big)\Big\} = 0$

$GT_{\hat{a}bcd} = 0$

3) *For* $i = a, j = \hat{b}, k = c$ *and* $l = d$

$GT_{a\hat{b}cd} = R_{a\hat{b}cd} - \frac{1}{2(n-1)}[g_{ac}r(GT)_{\hat{b}d} + g_{\hat{b}d}r(GT)_{ac} - g_{ad}r(GT)_{\hat{b}c} - g_{\hat{b}c}r(GT)_{ad}]$

$GT_{abcd} = 0 - \frac{1}{2(n-1)}\Big\{(0)\Big(-A^{bk}_{dk} + B^{bh}{}_{d}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[b}_{[d}\delta^{k]}_{k]}\Big) + (\delta^{b}_{d})(0) - (0)\Big(-A^{bk}_{ck} + B^{bh}{}_{c}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[b}_{[c}\delta^{k]}_{k]}\Big) - (\delta^{b}_{c})(0)\Big\} = 0$

$GT_{a\hat{b}cd} = 0$

4) *For* $i = a, j = b, k = \hat{c}$ *and* $l = d$

$GT_{ab\hat{c}d} = R_{ab\hat{c}d} - \frac{1}{2(n-1)}[g_{a\hat{c}}r(GT)_{bd} + g_{bd}r(GT)_{a\hat{c}} - g_{ad}r(GT)_{b\hat{c}} - g_{b\hat{c}}r(GT)_{ad}]GT_{ab\hat{c}d} = 0 - \frac{1}{2(n-1)}\Big\{(\delta^{c}_{a})(0) + (0)\Big(-A^{ck}_{ak} + B^{ch}{}_{a}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[c}_{[a}\delta^{k]}_{k]}\Big) - (0)\Big(-A^{ck}_{bk} + B^{ch}{}_{b}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[c}_{[b}\delta^{k]}_{k]}\Big) - (\delta^{c}_{b})(0)\Big\} = 0$

$GT_{ab\hat{c}d} = 0$

5) *For* $i = a, j = b, k = c$ *and* $l = \hat{d}$

$GT_{abc\hat{d}} = R_{abc\hat{d}} - \frac{1}{2(n-1)}[g_{ac}r(GT)_{b\hat{d}} + g_{b\hat{d}}r(GT)_{ac} - g_{a\hat{d}}r(GT)_{bc} - g_{bc}r(GT)_{a\hat{d}}]GT_{abc\hat{d}} = 0 - \frac{1}{2(n-1)}\Big\{(0)\Big(-A^{dk}_{bk} + B^{dh}{}_{b}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[d}_{[b}\delta^{k]}_{k]}\Big) + (\delta^{d}_{b})(0) - (\delta^{d}_{a})(0) - (0)\Big(-A^{dk}_{ak} + B^{dh}{}_{a}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[d}_{[a}\delta^{k]}_{k]}\Big)\Big\} = 0$

$GT_{abc\hat{d}} = 0$

6) *For* $i = \hat{a}, j = \hat{b}, k = c$ *and* $l = d$

$GT_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} - \frac{1}{2(n-1)}[g_{\hat{a}c}r(GT)_{\hat{b}d} + g_{\hat{b}d}r(GT)_{\hat{a}c} - g_{\hat{a}d}r(GT)_{\hat{b}c} - g_{\hat{b}c}r(GT)_{\hat{a}d}]$

$GT_{\hat{a}\hat{b}cd} = 0 - \frac{1}{2(n-1)}\Big\{(\delta^{a}_{c})\Big(-A^{bk}_{dk} + B^{bh}{}_{d}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[b}_{[d}\delta^{k]}_{k]}\Big) + (\delta^{b}_{d})\Big(-A^{ak}_{ck} + B^{ah}{}_{c}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[a}_{[c}\delta^{k]}_{k]}\Big) - (\delta^{a}_{d})\Big(-A^{bk}_{ck} + B^{bh}{}_{c}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[b}_{[c}\delta^{k]}_{k]}\Big) - (\delta^{b}_{c})\Big(-A^{ak}_{dk} + B^{ah}{}_{d}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[a}_{[d}\delta^{k]}_{k]}\Big)\Big\}$

7) *For* $i = \hat{a}, j = b, k = \hat{c}$ *and* $l = d$

$GT_{\hat{a}b\hat{c}d} = R_{\hat{a}b\hat{c}d} - \frac{1}{2(n-1)}[g_{\hat{a}\hat{c}}r(GT)_{bd} + g_{bd}r(GT)_{\hat{a}\hat{c}} - g_{\hat{a}d}r(GT)_{b\hat{c}} - g_{b\hat{c}}r(GT)_{\hat{a}d}]$

$GT_{\hat{a}b\hat{c}d} = -A^{ac}_{bd} + B^{ah}{}_{b}B_{hd}{}^{c} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{c]}_{d]} - \frac{1}{2(n-1)}\Big\{(0)(0) + (0)(0) - (\delta^{a}_{d})\Big(-A^{ck}_{bk} + B^{ch}{}_{b}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[c}_{[b}\delta^{k]}_{k]}\Big) - (\delta^{c}_{b})\Big(-A^{ak}_{dk} + B^{ah}{}_{d}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[a}_{[d}\delta^{k]}_{k]}\Big)\Big\}$

$GT_{\hat{a}b\hat{c}d} = -A^{ac}_{bd} + B^{ah}{}_{b}B_{hd}{}^{c} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{c]}_{d]} + \frac{1}{2(n-1)}\Big\{(\delta^{a}_{d})\Big(-A^{ck}_{bk} + B^{ch}{}_{b}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[c}_{[b}\delta^{k]}_{k]}\Big) + (\delta^{c}_{b})\Big(-A^{ak}_{dk} + B^{ah}{}_{d}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[a}_{[d}\delta^{k]}_{k]}\Big)\Big\}$

8) *For* $i = \hat{a}, j = b, k = c$ *and* $l = \hat{d}$,

$GT_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} - \frac{1}{2(n-1)}[g_{\hat{a}c}r(GT)_{b\hat{d}} + g_{b\hat{d}}r(GT)_{\hat{a}c} - g_{\hat{a}\hat{d}}r(GT)_{bc} - g_{bc}r(GT)_{\hat{a}\hat{d}}]$

$GT_{\hat{a}bc\hat{d}} = -A^{ad}_{bc} - B^{ah}{}_{cb}B_{hb}{}^{d} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]} - \frac{1}{2(n-1)}\Big\{(\delta^{a}_{c})\Big(-A^{dk}_{bk} + B^{dh}{}_{b}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[d}_{[b}\delta^{k]}_{k]}\Big) + (\delta^{d}_{b})\Big(-A^{ak}_{ck} + B^{ah}{}_{c}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[a}_{[c}\delta^{k]}_{k]}\Big) - (0)(0) - (0)(0)\Big\}$

$$GT_{\hat{a}bc\hat{d}} = -A^{ad}_{bc} - B^{ah}{}_{cb}B_{hb}{}^{d} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]} - \frac{1}{2(n-1)}\left\{(\delta^{a}_{c})\left(-A^{dk}_{bk} + B^{dh}{}_{b}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[d}_{[b}\delta^{k]}_{k]}\right) + (\delta^{d}_{b})\left(-A^{ak}_{ck} + B^{ah}{}_{c}B_{hk}{}^{k} + \frac{1}{2}\alpha^{[a}_{[c}\delta^{k]}_{k]}\right)\right\}.$$

Conclusion

The components of the generalized Riemannian curvature tensor of Viasman-Grey manifold**,** the components of generalized Ricci tensor of Viasman-Grey manifold, and the components of the generalized conharmonic curvature of Viasman-Grey manifold .

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تنسر الانحناء الكونهورمني المعمم لمنطوي فايسمان– كراي

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الملخص

الهدف من هذا البحث حساب مركبات تنسر ريجي المعمم وتنسر الانحناء الريماني المعمم لمنطوي فايسمان في فضاء $G-$ للوصول الى إيجاد تنسر الانحناء الكونهورمني المعمم لمنطوي فايسمان– كراي, واحدة من بنيات المنطوي الهرميتي التقريبي التي يرمز لها بالرمز W_1 , W_4 حيث $W_1 \oplus W_4$ على التوالي ترمز لمنطوي كوهلر التقريبي ومنطوي كوهلر المتطابق محليا التي تم دراستها.