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On Completely Normal Spaces

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ABSTRACT

The purpose of this paper is to introduce a new class of completely normal spaces namely($\alpha^{*\prime}$, **semi**$^{*\prime}$) completely normal space . The relationship between them was studied and we investigate some characterizations of them. At last we give more of examples to explain the subject and study the topological property and hereditary property of these types.

1-Introduction

In 1965, O. Njasted [1] defined a subset $A^{*\prime}$of a space $X^{*\prime}$ to be $\alpha^{*\prime}$- open if $A^{*\prime} \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A^{*\prime})))$, which plays an important role in the field of general topology. In 1963, N. Levine [2] defined semi$^{*\prime}$-open which is a subset $A^{*\prime}$of a space $X^{*\prime}$ satisfies $A^{*\prime} \subseteq \mathrm{cl}(\mathrm{int}(A^{*\prime}))$, or equivalently if there exists an open set O such that $O \subseteq A^{*\prime} \subseteq \mathrm{cl}O$. In 1955, Kelley [3] introduced the concept of normal space, completely normal space and study the relationship between them.

2-Prliminaris:

We recall and generalize more of definitions with examples to explain the subject.

Definition 2.1 [6]

Let $(X^{*\prime},\tau^{*\prime})$ be a topological space and $A^{*\prime} \subseteq X^{*\prime}$ then:

1) The closure of $A^{*\prime}$is denoted by $\mathrm{cl}(A^{*\prime})$ defined as $\mathrm{cl}(A^{*\prime}) = \cap\{B^{*\prime}: B^{*\prime}$ is a closed and $A^{*\prime} \subseteq B^{*\prime}\}$.

2) The interior of $A^{*\prime}$is denoted by $\mathrm{int}(A^{*\prime})$ defined as $\mathrm{int}(A^{*\prime}) = \cup\{B^{*\prime}: B^{*\prime}$ is an open and $A^{*\prime} \supseteq G^{*\prime}\}$.

Definition 2.2

A subset $A^{*\prime}$ of space $X^{*\prime}$ is said to be:

1) $\alpha^{*\prime}$ -open if $A^{*\prime} \subseteq \mathrm{int}\left(\mathrm{cl}\left(\mathrm{int}(A^{*\prime})\right)\right)$ [1].

2) semi$^{*\prime}$-open if $A^{*\prime} \subseteq \mathrm{cl}\left(\mathrm{int}(A^{*\prime})\right)$[2]..

The complements of the above mentioned open sets are called their respective closed sets.

Remark 2.3 [5] Every open set in $X^{*\prime}$ is $\alpha^{*\prime}$-open set in $X^{*\prime}$ but not conversely.

Remark 2.4 [2,4] Every $\alpha^{*\prime}$-open set in $X^{*\prime}$ is semi$^{*\prime}$-open set in $X^{*\prime}$ but not conversely.

Definition 2.5: The topological space $(X^{*\prime},\tau^{*\prime})$ is called $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ normal space if for any two disjoint closed sets $A^{*\prime},B^{*\prime} \subseteq X^{*\prime}$, there exist two disjoint $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ open sets $U^{*\prime}, V^{*\prime} \subseteq X^{*\prime}$with $A^{*\prime} \subseteq U^{*\prime}$and $B^{*\prime} \subseteq V^{*\prime}$.

Theorem 2.6: Every normal space is $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ normal space but not conversely.

Theorem 2.7: Every $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$normal space is a topological property.

Definition 2.8

A topological space $(X^{*\prime},\tau^{*\prime})$ is said to be:

1) $T_1^{*\prime}$- space if for every $a^{*\prime} \neq b^{*\prime}$in $X^{*\prime}$ there exist two open sets one containing $a^{*\prime}$ but not $b^{*\prime}$and other containing $b^{*\prime}$ but not $a^{*\prime}$ [3].

2) [8] $\alpha^{*\prime}T_1^{*\prime}$- space if for every $a^{*\prime} \neq b^{*\prime}$in $X^{*\prime}$ there exist two $\alpha^{*\prime}$- open sets one containing $a^{*\prime}$ but not $b^{*\prime}$and other containing $b^{*\prime}$ but not $a^{*\prime}$ [8].

3) $\mathrm{semi}^{*\prime}T_1^{*\prime}$- space if for every $a^{*\prime} \neq b^{*\prime}$in $X^{*\prime}$ there exist two semi$^{*\prime}$- open sets one containing $a^{*\prime}$ but not $b^{*\prime}$and other containing $b^{*\prime}$ but not $a^{*\prime}$.

Theorem 2.9 Every $(\alpha^{*\prime},\mathrm{semi}^{*\prime})T_1^{*\prime}$- space is a topological property.

Theorem 2.10 [7] A topological space $(X^{*\prime},\tau^{*\prime})$ is completely normal space if and only if every subspace of $X^{*\prime}$is normal space.

Theorem 2.11 [7] Every completely normal space is normal space.

Theorem 2.12 [3] A completely normal space is topological property.

Definition 2.13 [3] A topological space $(X^{*\prime},\tau^{*\prime})$ is called $T_5^{*\prime}$- space if it is both $T_1^{*\prime}$- space and completely normal space.

Remark 2.14 [3] Every $T_5^{*\prime}$- space is $T_4^{*\prime}$- space but not conversely.

Theorem 2.15 [3] A $T_5^{*\prime}$- space is topological property.

3-Some Types of Completely Normal Spaces:

In this section we introduce the class of completely normal space with their properties**.**

Definition 3.1 Let $(X^{*\prime},\tau^{*\prime})$ be a topological space is called $(\alpha^{*\prime}, semi^{*\prime})$ completely normal space if and only if given a pair of separated sets $A^{*\prime},B^{*\prime} \subseteq X^{*\prime}$, there exist two disjoint $(\alpha^{*\prime}, semi^{*\prime})$ open sets $U^{*\prime}, V^{*\prime} \subseteq X^{*\prime}$ such that $A^{*\prime} \subseteq U^{*\prime}, B^{*\prime} \subseteq V^{*\prime}$.

In the following examples we can show three types of completely normal space :

Example 3.2:

Let $(X^{*\prime},\tau^{*\prime})$ be a topological space where $X^{*\prime}=\{1,2,3,4,5\}$and $\tau^{*\prime}=\{\emptyset, X^{*\prime},\{1\},\{1,5\},\{2,4\},\{1,2,4\}, \{1,3,5\}, \{1,2,4,5\}\}$:

(Case one) {2,4},{3}are two separated closed sets by two disjoint $\alpha^{*\prime}$-open sets {1,3},{2,4}in $X^{*\prime}$. Hence $(X^{*\prime},\tau^{*\prime})$ is $\alpha^{*\prime}$ -completely normal space.

(Case two) {1},{2,4}are two separated open sets by two disjoint $\alpha^{*\prime}$-open sets {1,3},{2,4}in $X^{*\prime}$. Hence $(X^{*\prime},\tau^{*\prime})$ is $\alpha^{*\prime}$-completely

(Case three) {3}is closed set and{2,4}is open set are separated by two disjoint $\alpha^{*\prime}$-open sets {1,3},{2,4}in $X^{*\prime}$. Hence $(X^{*\prime},\tau^{*\prime})$ is $\alpha^{*\prime}$-completely

Example 3.3

Let $(X^{*\prime},\tau^{*\prime})$ be a topological space where $X^{*\prime}=\{1,2,3,4\}$and $\tau^{*\prime}=\{\emptyset, X^{*\prime},\{1\},\{4\},\{2\},\{1,4\}, \{2,4\}, \{1,2\}, \{2,3,4\}, \{1,2,4\}\}$:

(Case one) {1},{3}are two separated closed sets by two disjoint $semi^{*\prime}$-open sets {1,2},{3,4}in $X^{*\prime}$. Hence $(X^{*\prime},\tau^{*\prime})$ is $semi^{*\prime}$-completely normal space.

(Case two) {2},{4}are two separated open sets by two disjoint $semi^{*\prime}$-open sets {1,2},{3,4}in $X^{*\prime}$. Hence $(X^{*\prime},\tau^{*\prime})$ is $semi^{*\prime}$-completely

(Case three) {1}is closed set and{2}is open set are separated by two disjoint $semi^{*\prime}$-open sets {1,2},{3,4}in $X^{*\prime}$. Hence $(X^{*\prime},\tau^{*\prime})$ is $semi^{*\prime}$ -completely

Theorem 3.4

Every $(\alpha^{*\prime}, semi^{*\prime})$ completely normal space is $(\alpha^{*\prime}, semi^{*\prime})$ normal space.

Proof: Let $A^{*\prime}$, $B^{*\prime}$ be two disjoint closed sets in$(\alpha^{*\prime}, semi^{*\prime})$ completely normal space $(X^{*\prime},\tau^{*\prime})$. As $cl(A^{*\prime})= A^{*\prime}$and $cl(B^{*\prime})= B^{*\prime}$we get $A^{*\prime} \cap B^{*\prime} =\emptyset$**.**

Thus $A^{*\prime}$and $B^{*\prime}$ are separated sets in $X^{*\prime}$.

Hence by definition, there exist two disjoint $(\alpha^{*\prime}, semi^{*\prime})$ open sets $U^{*\prime}$and $V^{*\prime}$such that $A^{*\prime} \subseteq U^{*\prime}, B^{*\prime} \subseteq V^{*\prime}$.

This shows that $(X^{*\prime},\tau^{*\prime})$ is $(\alpha^{*\prime}, semi^{*\prime})$ normal space.

Remark 3.5

Converse of the theorem (3.4) need not be true by later example after theorem (3.10) which is needed in this example.

Theorem 3.6

1) Every completely normal space is $\alpha^{*\prime}$- completely normal space.

2) Every $\alpha^{*\prime}$- completely normal space is $semi^{*\prime}$-completely normal space.

Proof:

1) Let $(X^{*\prime},\tau^{*\prime})$ be a completely normal space and let $A^{*\prime},B^{*\prime}$be two separated subsets of $X^{*\prime}$.

So there exist two disjoint open sets $U^{*\prime}$, $V^{*\prime} \subseteq X^{*\prime}$such that $A^{*\prime} \subseteq U^{*\prime}, B^{*\prime} \subseteq V^{*\prime}$.

Since every open set is $\alpha^{*\prime}$- open set . Then $U^{*\prime}$, $V^{*\prime}$be two disjoint $\alpha^{*\prime}$- open sets subsets of $X^{*\prime}$such that $A^{*\prime} \subseteq U^{*\prime}, B^{*\prime} \subseteq V^{*\prime}$.

Hence $(X^{*\prime},\tau^{*\prime})$ is $\alpha^{*\prime}$- completely normal space.

2) Obviously.

Remark 3.7

Converse of the theorem (3.6) need not be true by the following examples:

Example 3.8

1) In example (3.2) show that $\alpha^{*\prime}$-completely normal space need not be completely normal space.

2) In example (3.3) show that $semi^{*\prime}$-completely normal space need not be $\alpha^{*\prime}$-completely normal space.

Theorem 3.9

An $(\alpha^{*\prime}, semi^{*\prime})$ completely normal space is hereditary property.

Proof:

Let $(X^{*\prime},\tau^{*\prime})$ be an $(\alpha^{*\prime}, semi^{*\prime})$ completely normal space.

And let $(X^{*\prime\prime},\tau^{*\prime\prime})$ be a subspace of $(X^{*\prime},\tau^{*\prime})$ to prove that $(X^{*\prime\prime},\tau^{*\prime\prime})$ is $(\alpha^{*\prime}, semi^{*\prime})$completely normal space. Let $A^{*\prime}$and $B^{*\prime}$ be any two separated sets in $(X^{*\prime\prime},\tau^{*\prime\prime})$.

Claim that $A^{*\prime}$and $B^{*\prime}$ are separated subsets of $(X^{*\prime\prime},\tau^{*\prime\prime})$. Since $X^{*\prime\prime} \subseteq X^{*\prime}$then $A^{*\prime}$and $B^{*\prime}$are separated sets in$(X^{*\prime},\tau^{*\prime})$ **.**

$A^{*\prime} \cap cl(B^{*\prime})=\emptyset$ and $B^{*\prime} \cap cl(A^{*\prime})=\emptyset$.

$A^{*\prime} \cap [cl(B^{*\prime}) \cap X^{*\prime\prime}]=\emptyset$ and $B^{*\prime} \cap [cl(A^{*\prime}) \cap X^{*\prime\prime}]=\emptyset$.

$[A^{*\prime} \cap X^{*\prime\prime}] \cap cl(B^{*\prime})=\emptyset$ and$[B^{*\prime} \cap X^{*\prime\prime}] \cap cl(A^{*\prime})=\emptyset$.

Since $(X^{*\prime},\tau^{*\prime})$ is an $(\alpha^{*\prime}, semi^{*\prime})$completely normal space.

There exist $U^{*\prime}$and $V^{*\prime}$be two disjoint $(\alpha^{*\prime}, semi^{*\prime})$ open sets subsets of $(X^{*\prime},\tau^{*\prime})$ such that $A^{*\prime} \subseteq U^{*\prime}, B^{*\prime} \subseteq V^{*\prime}$ and $U^{*\prime} \cap V^{*\prime}=\emptyset$.

Define $U^{*\prime\prime}=U^{*\prime} \cap X^{*\prime\prime}$and $V^{*\prime\prime}=V^{*\prime} \cap X^{*\prime\prime}$.

Then $U^{*\prime\prime},V^{*\prime\prime}$be two disjoint $(\alpha^{*\prime}, semi^{*\prime})$ open sets subsets of $(X^{*\prime\prime},\tau^{*\prime\prime})$.

$U^{*\prime\prime} \cap V^{*\prime\prime}=(U^{*\prime} \cap V^{*\prime}) \cap X^{*\prime\prime}=\emptyset$, $A^{*\prime} \subseteq U^{*\prime\prime}, B^{*\prime} \subseteq V^{*\prime\prime}$.

This shows that $(X^{*\prime\prime},\tau^{*\prime\prime})$ is $(\alpha^{*\prime}, semi^{*\prime})$ completely normal space.

Theorem 3.10

A topological space $(X^{*\prime},\tau^{*\prime})$ is $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ completely normal space if and only if every subspace of $X^{*\prime}$is$(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ normal space.

Proof:

Suppose $(X^{*\prime},\tau^{*\prime})$ is $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ completely normal space and let $(X^{*\prime\prime},\tau^{*\prime\prime})$ be a subspace of $(X^{*\prime},\tau^{*\prime})$. By theorem (3.9) and theorem (3.4). We get $(X^{*\prime\prime},\tau^{*\prime\prime})$ is $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$normal space.

Conversely:

Assume that $(X^{*\prime},\tau^{*\prime})$ be a topological space such that each subspace of $(X^{*\prime},\tau^{*\prime})$ is $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ normal space.

To prove $(X^{*\prime},\tau^{*\prime})$ is $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ completely normal space.

Let $A^{*\prime}$and $B^{*\prime}$ be separated subsets of $(X^{*\prime},\tau^{*\prime})$.

Then $A^{*\prime}\cap cl(B^{*\prime})=\emptyset$ and $B^{*\prime}\cap cl(A^{*\prime})=\emptyset$.

Define $X^{*\prime\prime}=X^{*\prime}-[\,cl(A^{*\prime})\cap cl(B^{*\prime})]=[\,X^{*\prime}-cl(A^{*\prime})]\cap[X^{*\prime}-cl(B^{*\prime})]$.

Let $\tau^{*\prime\prime}=\{U^{*\prime}\cap X^{*\prime\prime}: U^{*\prime}\in\tau^{*\prime}\}$be the relative topology on $X^{*\prime\prime}$.

As $cl(A^{*\prime})$ is closed in $(X^{*\prime},\tau^{*\prime})$, $cl(A^{*\prime})\cap X^{*\prime\prime}$is closed in $X^{*\prime\prime}$,

similarly $cl(B^{*\prime})\cap X^{*\prime\prime}$ is closed in $X^{*\prime\prime}$.

Further

$[cl(A^{*\prime})\cap X^{*\prime\prime}]\cap[cl(B^{*\prime})\cap X^{*\prime\prime}]$

$=[\,cl(A^{*\prime})\cap cl(B^{*\prime})]\cap X^{*\prime\prime}$

$=[\,cl(A^{*\prime})\cap cl(B^{*\prime})]\cap[X^{*\prime}-(\,cl(A^{*\prime})\cap cl(B^{*\prime}))]=\emptyset$.

Thus $cl(A^{*\prime})\cap X^{*\prime\prime}$and $cl(B^{*\prime})\cap X^{*\prime\prime}$are disjoint closed subsets of $X^{*\prime\prime}$.

As $X^{*\prime\prime}$is $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ normal space, there exist $U^{*\prime\prime},V^{*\prime\prime}$be two disjoint $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ open subsets of $(X^{*\prime\prime},\tau^{*\prime\prime})$.

Such that $[cl(A^{*\prime})\cap X^{*\prime\prime}]\subseteq U^{*\prime\prime}$and $[cl(B^{*\prime})\cap X^{*\prime\prime}]\subseteq V^{*\prime\prime}$and $U^{*\prime\prime}\cap V^{*\prime\prime}=\emptyset$.

$U^{*\prime\prime}=U^{*\prime}\cap X^{*\prime\prime}$and $V^{*\prime\prime}=V^{*\prime}\cap X^{*\prime\prime}$for some $U^{*\prime},V^{*\prime}$be two disjoint $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ open subsets of $(X^{*\prime},\tau^{*\prime})$.

Claim that $A^{*\prime}\subseteq U^{*\prime}, B^{*\prime}\subseteq V^{*\prime}$.

1) $A^{*\prime}\cap cl(B^{*\prime})=\emptyset$

$\longrightarrow A^{*\prime}\subseteq[X^{*\prime}-cl(B^{*\prime})],\ cl(A^{*\prime})\cap cl(B^{*\prime})\subseteq cl(B^{*\prime})$.

$\longrightarrow X^{*\prime}-[\,cl(A^{*\prime})\cap cl(B^{*\prime})]\supseteq[X^{*\prime}-cl(B^{*\prime})]$

$\longrightarrow\ X^{*\prime\prime}\supseteq[X^{*\prime}-cl(B^{*\prime})]\supseteq A^{*\prime}$.

Thus $A^{*\prime}\subseteq X^{*\prime\prime}$. Similarly we get $B^{*\prime}\subseteq X^{*\prime\prime}$.

2) $A^{*\prime}\cap X^{*\prime\prime}=A^{*\prime}\cap[X^{*\prime}-cl(A^{*\prime})\cap cl(B^{*\prime})]$

$=A^{*\prime}\cap[X^{*\prime}-cl(A^{*\prime})]\cup[X^{*\prime}-cl(B^{*\prime})]$

$=[\,A^{*\prime}\cap[X^{*\prime}-cl(A^{*\prime})]]\cup[\,A^{*\prime}\cap[X^{*\prime}-cl(B^{*\prime})]]$

$=\emptyset\cup A^{*\prime}=A^{*\prime}$.

$[A^{*\prime}\subseteq cl(A^{*\prime})\longrightarrow A^{*\prime}\cap[X^{*\prime}-cl(A^{*\prime})]=\emptyset \quad\longrightarrow A^{*\prime}\cap cl(B^{*\prime})=\emptyset$

$\longrightarrow A^{*\prime}\subseteq[X^{*\prime}-cl(B^{*\prime})\longrightarrow A^{*\prime}(X^{*\prime}-cl(B^{*\prime}))=A^{*\prime}]$.

Thus $A^{*\prime}\cap X^{*\prime\prime}=A^{*\prime}$, similarly we get $B^{*\prime}\cap X^{*\prime\prime}=B^{**\prime}$.

3) $A^{*\prime}\subseteq X^{*\prime\prime}\longrightarrow A^{*\prime}=A^{*\prime}\cap X^{*\prime\prime}\subseteq cl(A^{*\prime})\cap X^{*\prime\prime}\subseteq U^{*\prime\prime}\subseteq U^{*\prime}$.

Thus $A^{*\prime}\subseteq U^{*\prime}$,similarly we get $B^{*\prime}\subseteq V^{*\prime}$.

Thus for given any pair of separated sets in $A^{*\prime}$and $B^{*\prime}$in $(X^{*\prime},\tau^{*\prime})$.

There exist two $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ open subsets $U^{*\prime}$, $V^{*\prime}$ of $(X^{*\prime},\tau^{*\prime})$.

Such that $A^{*\prime}\subseteq U^{*\prime}$, $B^{*\prime}\subseteq V^{*\prime}$and $U^{*\prime}\cap V^{*\prime}=\emptyset$.

Hence $(X^{*\prime},\tau^{*\prime})$ is $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ completely normal space.

Example 3.11

Let $X^{*\prime}=\{1,2,3,4\}$and

$\tau^{*\prime}=\{\emptyset,X^{*\prime},\{1\},\{1,5\},\{2,4\},\{1,2,4\},\{1,3,5\},\{1,2,4,5\}\}$.

Let $Y^{*\prime}=\{1,2,3\}$ and $\tau^{*\prime}_{Y^{*\prime}}=\{\emptyset,Y^{*\prime},\{1,2\},\{1,3\},\{1\}\}$, then $(Y^{*\prime},\tau^{*\prime}_{Y^{*\prime}})$ be a subspace of $(X^{*\prime},\tau^{*\prime})$. Since $\{2\},\{3\}$ are two disjoint closed sets in $Y^{*\prime}$which not separated by two disjoint $\alpha^{*\prime}$-open sets in $Y^{*\prime}$, hence $(X^{*\prime},\tau^{*\prime})$ is $\alpha^{*\prime}$- normal space but $(Y^{*\prime},\tau^{*\prime}_{Y^{*\prime}})$ is not $\alpha^{*\prime}$-normal space. Then $(X^{*\prime},\tau^{*\prime})$ is not $\alpha^{*\prime}$-completely normal space by theorem (3.4).

Example 3.12

Let$X^{*\prime}=\{1,2,3,4\}$ and $\tau^{*\prime}=\{\emptyset,X^{*\prime},\{1\},\{2\},\{4\},\{1,2\},\{2,4\},\{1,4\},\{1,2,4\},\{2,3,4\}\}$.

Let $Y^{*\prime}=\{1,2,3\}$ and $\tau^{*\prime}_{Y^{*\prime}}=\{\emptyset,Y^{*\prime},\{2,3\},\{1,3\},\{3\}\}$, then $(Y^{*\prime},\tau^{*\prime}_{Y^{*\prime}})$ be a subspace of $(X^{*\prime},\tau^{*\prime})$. Since $\{1\},\{2\}$ are two disjoint closed sets in $Y^{*\prime}$which not separated by two disjoint $\mathrm{semi}^{*\prime}$-open sets in $Y^{*\prime}$, hence $(X^{*\prime},\tau^{*\prime})$ is $\mathrm{semi}^{*\prime}$-normal space but $(Y^{*\prime},\tau^{*\prime}_{Y^{*\prime}})$ is not $\mathrm{semi}^{*\prime}$- normal space. Then $(X^{*\prime},\tau^{*\prime})$ is not $\mathrm{semi}^{*\prime}$-completely normal space by theorem (3.4).

Theorem 3.13

An $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ completely normal space is topological property.

Proof:

Let $(X^{*\prime},\tau^{*\prime})$ be an $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ completely normal space.

Let $(X^{*\prime\prime},\tau^{*\prime\prime})$ be any topological space and let $f^{*\prime}$: $(X^{*\prime},\tau^{*\prime})\longrightarrow(X^{*\prime\prime},\tau^{*\prime\prime})$ be a homeomorphism.

To prove that $X^{*\prime\prime}$is an $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$ completely normal space.

Let $A^{*\prime}$and $B^{*\prime}$be two separated sets in $X^{*\prime\prime}$.Hence $A^{*\prime}\cap cl(B^{*\prime})=\emptyset$ and $B^{*\prime}\cap cl(A^{*\prime})=\emptyset$.

Since $f^{*\prime}$ is continuous* function $cl(f^{*\prime^{-1}}(A^{*\prime}))\subseteq f^{*\prime^{-1}}(cl(A^{*\prime}))$

and $cl(f^{*\prime^{-1}}(B^{*\prime}))\subseteq f^{*\prime^{-1}}(cl(B^{*\prime}))$

Hence $f^{*\prime^{-1}}(A^{*\prime}))\cap cl(f^{*\prime^{-1}}(B^{*\prime}))\subseteq$

$f^{*\prime^{-1}}(cl(A^{*\prime}))f^{*\prime^{-1}}(cl(B^{*\prime}))$

$=f^{*\prime^{-1}}(A^{*\prime}\cap cl(B^{*\prime}))=f^{*\prime^{-1}}(\emptyset)=\emptyset$.

Therefore $f^{*\prime^{-1}}(A^{*\prime})\cap cl(f^{*\prime^{-1}}(B^{*\prime}))=\emptyset$……….(1)

Similarly, we can show that $cl(f^{*\prime^{-1}}(A^{*\prime}))\cap f^{*\prime^{-1}}(B^{*\prime})=\emptyset$………..(2)

from (1) and (2) we get $f^{*\prime^{-1}}(A^{*\prime})$ and $f^{*\prime^{-1}}(B^{*\prime})$ are separated sets in $(X^{*\prime},\tau^{*\prime})$.

As $X^{*\prime}$is an $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$completely normal space there exist disjoint $(\alpha^{*\prime},\mathrm{semi}^{*\prime})$open sets say $U^{*\prime}$, $V^{*\prime}$ in $(X^{*\prime},\tau^{*\prime})$ such that $f^{*\prime^{-1}}(A^{*\prime})\subseteq U^{*\prime}$and $f^{*\prime^{-1}}(B^{*\prime})\subseteq V^{*\prime}$.

As $f^{*\prime}$ is onto, $A^{*\prime}=f^{*\prime}(f^{*\prime^{-1}}(A^{*\prime}))$ and $B^{*\prime}=f^{*\prime}(f^{*\prime^{-1}}(B^{*\prime}))$.

Hence $A^{*\prime} = f^{*\prime}(f^{*\prime^{-1}}(A^{*\prime})) \subseteq f^{*\prime}(U^{*\prime})$ and $B^{*\prime} = f^{*\prime}(f^{*\prime^{-1}}(B^{*\prime})) \subseteq f^{*\prime}(V^{*\prime})$.
Further $f^{*\prime}(U^{*\prime}) \cap f^{*\prime}(V^{*\prime}) = f^{*\prime}(U^{*\prime} \cap V^{*\prime}) = \emptyset$ (since $f^{*\prime}$ is one to one).
Further $f^{*\prime}$ is an $(\alpha^{*\prime}, semi^{*\prime})$ open function $\longrightarrow f^{*\prime}(U^{*\prime}), f^{*\prime}(V^{*\prime}) \in X^{*\prime\prime}$.
Thus for two disjoint separated sets $A^{*\prime}$ and $B^{*\prime}$ in $X^{*\prime\prime}$. There exist $f^{*\prime}(U^{*\prime}), f^{*\prime}(V^{*\prime}) \in X^{*\prime\prime}$ one containing $A^{*\prime}$ and the other containing $B^{*\prime}$.
Hence $(X^{*\prime\prime}, \tau^{*\prime\prime})$ is an $(\alpha^{*\prime}, semi^{*\prime})$ completely normal space.

Now, we introduce some separation axioms:

Definition 3.14
Let $(X^{*\prime}, \tau^{*\prime})$ be a topological space is said to be:
1) $\alpha^{*\prime}T_5^{*\prime}$-space if it is both $\alpha^{*\prime}T_1^{*\prime}$- space and $\alpha^{*\prime}$-completely normal space.
2) $semi^{*\prime}T_5^{*\prime}$-space if it is both $semi^{*\prime}T_1^{*\prime}$-space and $semi^{*\prime}$-completely normal space.

Proposition 3.15
1) Every $T_5^{*\prime}$- space is $\alpha^{*\prime}T_5^{*\prime}$- space.
2) Every $\alpha^{*\prime}T_5^{*\prime}$- space is $semi^{*\prime}T_5^{*\prime}$- space.

Theorem 3.16
Every $(\alpha^{*\prime}, semi^{*\prime})\ T_5^{*\prime}$- space is $(\alpha^{*\prime}, semi^{*\prime})T_4^{*\prime}$- space.

Proof:
Let $(X^{*\prime}, \tau^{*\prime})$ be an $(\alpha^{*\prime}, semi^{*\prime})\ T_5^{*\prime}$-space.
Since every $(\alpha^{*\prime}, semi^{*\prime})$ completely normal space is $(\alpha^{*\prime}, semi^{*\prime})$ normal space.
By theorem(3.4) we get $(X^{*\prime}, \tau^{*\prime})$ is an $(\alpha^{*\prime}, semi^{*\prime})T_4^{*\prime}$- space.

Remark 3.17
Converse of theorem (3.16) need not true, see examples (3.11)(3.12)

Theorem 3.18
1) An $(\alpha^{*\prime}, semi^{*\prime})T_5^{*\prime}$- space is topological property.
2) An $(\alpha^{*\prime}, semi^{*\prime})T_5^{*\prime}$- space is hereditary property.

Proof: Obviously

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الفضاءات السوية الكاملة

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قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

الملخص

الغرض من هذا البحث هو تقديم صنف جديد من الفضاءات السوية الكاملة أسميناه $(\alpha^{*\prime}, semi^{*\prime})$ الفضاء السوي الكامل من النمط $(\alpha^{*\prime}, semi^{*\prime})$. درسنا العلاقات بينهما وتحرينا بعض خصائصهما. وأخيرا أعطينا كثير من الأمثلة لأجل ايضاح الموضوع ودرسنا الصفات التبولوجيه والوراثية لهذه الأنواع.