Tikrit Journal of Pure Science Vol. 25 (6) 2020

TJPS

IASJ

المجلات الاكاديمية العراقية

Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php/j

i-Soft Separation Axioms in Soft Topological Spaces

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https://doi.org/10.25130/tjps.v25i6.322

ARTICLE INFO.

Article history:

-Received: 10 / 6 / 2020

-Accepted: 10 / 8 / 2020

-Available online: / / 2020

Keywords: Soft i-open sets, i-Soft Separation Axioms, Soft i-Continuous Mappings.

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ABSTRACT

In the current study the researchers have been introduced a modern kind of soft separation axioms which is named i-soft separation axioms by using the concept of soft i-open sets [17] in soft topological spaces, the relations among i-soft separation axioms and many examples about it are investigated. Further, they found that soft separation axioms imply i-soft separation axioms, but, the converses may not be true. Also, many theorems have been proved which are clarified the properties of i-soft separation axioms.

1. Introduction and Preliminaries

In 1964(see [1], [2]) (T_n spaces, n=0,1,2,3,4,5) for open sets by using (Klomogorov (respect. Frechet, Hausdorff, Vietors, Urysohn and Titus axioms)) have been studied. In 1963, 1965 (see [3], [4]), the concepts of semi-open sets, $\alpha -$ open sets have been introduced. The concept of soft sets and its properties has been introduced by Molodtsov and many other researchers in 1999, 2003, 2007, 2009, 2011, 2012 and 2015(see [5-12]. Chen, B. in 2013(see [13]) and Kannan, K. in 2012 (see [14]) introduced the concept of semi-open sets and soft $\alpha -$ open sets individually in soft topological spaces. Askandar, S. W. In 2012 (see [15]) and in 2016 (see [16]) have introduced the concept of i-open sets in ordinary topological spaces and i-separation axioms depends on i-open sets as (T_{ni} spaces, n=0, 1, 2, 3, 4, 5, 6). The purpose of this work is to introduce i-soft separation axioms by using soft i-open sets (see [17]). Throughout this work (X,τ,E),$(X,IOS(X_E))$and (Y,ρ,H)always are soft topological spaces *STS* (where *IOS* (X_E) is a family of all soft i-open sets in *X*) and we denotes by *SSs* to the soft sets, $int(K,E)$ and $Cl(K,E)$denotes soft interior and soft closure of the $SS(K,E)$Individually. The members of τ are called soft open sets *SOS* (X_E) and its complements are called soft closed sets *SCS* (X_E). $\emptyset_E, X_E$ Denote soft null and soft absolute sets.

This paper comprises of four segments. In the second one soft i-open set and its properties in *STSs* have been introduced. In the third segment the definitions of i-soft separation axioms spaces and the relations among them have been studied. Finally, in the fourth one, some important theorems have been proved to discuss the properties of this new kind of soft separation axioms spaces (see "Theorems 4.1, 4.2, 4.3, 4.4 and Theorem 4.5").

Definition 1.1: [11]. If (K,E) is a soft set over X and $x \in X$. It can be said that $x \tilde{\in} (K,E)$ whether $x \in K(e), \forall e \in E$.

Definition 1.2: [11]. Consider $x \in X$, as a soft set (x,E) over X , wherein $x_E(e) = \{x\}$, $\forall e \in E$ is denoted by x_E and was addressed as the singleton consider on soft point.

Definition 1.3: [11]. A soft set $(K,E) \tilde{\in} SS(X_E)$ named as a soft point inX_E is indicated by $K(e) = \phi$ $\forall e^C \in E - \{e\}$, and e_K if $\exists x \in X$ and $e \in E, K(e) \neq \phi$.The soft point e_K belongs to the soft set (G,E), $e_K \tilde{\in} (G,E)$, whether regarding the factor $e \in E$, $e_K \subseteq G(e)$.The group of X whole soft points is indicated by $SP(X)$.

Definition 1.4: [12]. The two "soft sets" $(G,A)$and (H,A) in $SS(X_A)$ are said to be soft disjoint, written $(G,A)\tilde{\cap}(H,A) = \phi_A$, if $G,(e) \cap H(e) = \phi$ $\forall e \in A$.

Definition 1.5: [12]. Two soft points e_G, e_H in X_A are $distinct$, written $e_G \neq e_H$, if there corresponding *SSs* (G, A), (H, A) are soft disjoint.

Definition 1.6: [18]. Let (X, τ, E) be *STS* and $(F, E) \tilde{\in} SS(X_E)$. Define $\tau_{(F,E)} = \{(G, E) \tilde{\cap} (F, E) : (G, E) \tilde{\in} \tau$ which considered soft topology on (F, E). The soft topology is called soft relative topology of τ on (F, E) and $((F, E), \tau_{(F,E)})$ is named soft subspace of (X, τ, E).

2. Soft i-Open Sets in Soft Topological Spaces.

Definition 2.1: Consider (F,E) as a soft set in (X,τ,E), therefore, (F,E) is said to be,

1. [17]. Soft $\text{i}-\text{open set}$ (*SIOS*), whether there is a soft open set $(G,E) \neq \phi, X$ where $(F,E) \tilde{\subseteq} Cl((F,E) \tilde{\cap} (G,E))$.
2. [5]. $\text{Soft semi}-\text{open set}$ (*SSOS*) if:

a. $(F,E) \tilde{\subseteq} Cl(Int(F,E))$.

b. Whether soft open set exist $(G,E) \neq \phi, X$ where $(G, E) \tilde{\subseteq} (F, E) \tilde{\subseteq} Cl(G, E)$.

3. [10], [9]. Soft $\alpha-\text{open set}$ $(S\alpha OS)$ if $(F, E) \tilde{\subseteq} Int(Cl(Int(F, E)))$.

The complement of *SIOS*, (resp., *SSOS* and $S\alpha OS$) known as soft $\text{i}-\text{closed set}$ (*SICS*) [17] (resp., $\text{soft semi}-\text{closed set}$ (*SSCS*) and soft $\alpha-$ closed set ($S\alpha CS$)). The intersection of all soft $\text{i}-$ closed sets (*SICSs*) over X containing (F, E) is called soft $\text{i}-\text{closure}$ of (F, E) and denoted by $i - Cl(F, E)$[17]. The union of whole (*SIOSs*) over X contained in (F, E) known as soft $\text{i}-\text{interior}$ of (F, E) and indicated by $i - Int(F, E)$[17]. The group of whole *SOSs* (resp., *SIOSs*, *SSOSs* and $S\alpha OSs$),(*SCSs*, *SICSs*, *SSCSs* and $S\alpha CSs$) in (X,τ,E) are indicated by $SOS(X_E)$(resp. $SIOS(X_E)$, $SSOS(X_E)$, $S\alpha OS(X_E)$, $SCS(X_E)$, $SICS(X_E)$, $SSCS(X_E)$ and $S\alpha CS(X_E)$).

Example2.1: Consider $X = \{2,4,6\}$, $\tau = \{\phi_E, (F_1, E), (F_2, E), X_E\}$, $E = \{m, l\}$.

Where$(F_1, E) = \{(m, \{2\}), (l, \{2\})\}$, $(F_2, E) = \{(m, \{2,6\}), (l, \{2,6\})\}$.

Consider, "$(F, E) = \{(m, \{2,4\}), (l, \{2,4\})\}$".

$SOS(X_E) = \{\emptyset, (F_1, E), (F_2, E), X_E\}$,

$SCS(X_E) = \{X_E, (F_1, E)^c$
$= \{(m, \{4,6\}), (l, \{4,6\})\}, (F_2, E)^c$
$= \{(m, \{4\}), (l, \{4\})\}, \phi_E \}$

$SIOS(X_E) = \{(\phi_E, (F_1, E), (F_2, E),$ $\{(m, \{6\}), (l, \{6\})\}$, $\{(m, \{2,4\}), (l, \{2,4\})\}$, $\{(m, \{4,6\}), (l, \{4,6\})\}$, $X_E\}$. Apparently, (F, E) is *SIOS* due to the existence *SOS* $(G, E) = (F_1, E)$ where $(F, E) \tilde{\subseteq} Cl((F, E) \tilde{\cap} (G, E))$, yet $(F, E)$is not *SOS*.

Theorem2.1: [17] Each "soft open set"(*SOS*) is a "soft i-open"(*SIOS*).

Theorem2.2: [17] Each "soft semi-open set"(*SSOS*) is a "soft i-open"(*SIOS*).

Definition2.2: A $SIOS(G, E)$ in (X, τ, E) considers $\text{soft i}-\text{open neighborhood}$ of $x \in X$ if $x \in G(e)$ $\forall e \in E$.

Definition2.3: Let (W, Z) be a *SS* in (X, τ, Z). A point $x \in X$ considers $\text{i}-\text{limit point}$ of (W, Z) if for each $\text{soft i}-\text{open neighborhood}$ (N, Z) of x, $(W, Z) \tilde{\cap} (N, Z)\tilde{\setminus}\{x\} \neq \emptyset_E$. In other words, a point $x \in X$ is $\text{i}-\text{limit point}$ of (W, Z)if for each *SIOS* (N, Z) containing x, $(W, Z) \tilde{\cap} (N, Z)\tilde{\setminus}\{x\} \neq \emptyset_E$. The set of whole $\text{i}-\text{limit points}$ of (W, Z) is called $\text{i}-\text{derived set}$ of (W, Z) and designated by $iD(W, Z)$. Obviously, a point $x \in X$ is not consider as $\text{i}-\text{limit point}$ of (W, Z) if there is a *SIOS* (N, Z) containing x wherein $(W, Z) \tilde{\cap} (N, Z)\tilde{\setminus}\{x\} = \emptyset_E$. A *SS* (W, Z) has been considered as *SICS* if $iD(W, Z)$ $\tilde{\subseteq} (W, Z)$ Wherein $iD(W, Z)$ $\tilde{\subseteq} W(e)$, $\forall e \in Z$.

Theorem2.3: Consider $(X, SIOS(X_Z))$ as *STS*, for *SSs* $(K, Z), (L, Z)$ in X, so the next phrases hold:

<u>i.</u> $iD\ (K, Z) \subseteq D\ (K, Z)$. Where $D\ (K, Z)$ is derived set of (K, Z)

<u>ii.</u> If $(K, Z) \tilde{\subseteq} (L, Z)$, *then* $iD\ (K, Z) \subseteq iD(L, Z)$.

<u>*iii.*</u> $iD(\ (K, Z) \tilde{\cup} (L, Z)) = iD\ (K, Z) \cup iD(L, Z)$.

<u>*iv.*</u> $iD(\ (K, Z) \tilde{\cap} (L, Z)) \subseteq iD\ (K, Z) \cap iD(L, Z)$.

<u>*v.*</u> If $x \in iD\ (K, Z)$ then $x \in iD(\ (K, Z)\tilde{\setminus}\{x\})$.

Proof

<u>i.</u> By "Theorem 2.1", we have $iD(K, Z) \subseteq D(K, Z)$.

<u>ii.</u> Let $x \in iD(K, Z)$ then for each *SIOS* (M, Z) containing x we get

$((K, Z) \tilde{\cap} (M, Z))\tilde{\setminus}\{x\} \neq \phi$ …… (1)

Since $(K, Z) \tilde{\subseteq} (L, Z)$, $\Longrightarrow$ $(K, Z) \tilde{\cap} (M, Z) \subseteq$ $(L, Z), \tilde{\cap}(M, Z)$ $\Longrightarrow$ $\big((K, Z) \tilde{\cap} (M, Z)\big) \tilde{\setminus} \{x\} \subseteq$ $((L, Z), \tilde{\cap} (M, Z)) \tilde{\setminus} \{x\} \neq \phi$.

From (i) we obtain, $((L, Z), \tilde{\cap} (M, Z)) \tilde{\setminus} \{x\} \neq \phi \Longrightarrow$ $x \in iD(L, Z)$,. hence $iD(K, Z) \subseteq iD(L, Z)$.

<u>iii.</u> Since(K, Z) $\tilde{\subseteq} (K, Z) \tilde{\cup} (L, Z)$, $(L, Z) \tilde{\subseteq} (K, Z) \tilde{\cup} (L, Z)$, By *(ii)* we get $iD(K, Z)$ $\subseteq iD((K, Z) \tilde{\cup} (L, Z),)$,

$iD(L, Z), \subseteq iD((K, Z) \tilde{\cup} (L, Z),)$.

$\Longrightarrow iD(K, Z)\ \cup iD(L, Z), \subseteq$
$iD((K, Z) \tilde{\cup} (L, Z),)$………*(*)*.

Now consider $x \notin iD(K, Z)$, $x \notin iD(L, Z)$, . Then there exists two *SIOSs* $(M^K_x, Z), (M^L_x, Z)$ containing x wherein $(K,Z) \tilde{\cap} (M^K_x, Z)) \tilde{\setminus} \{x\} = \phi$, $(L, Z) \tilde{\cap} (M^L_x, Z))$ $\tilde{\setminus} \{x\} = \phi$.

Let $(M, Z) = (M^K_x, Z) \tilde{\cap} (M^L_x, Z)$. Where $(M, Z)$is a *SIOS*, $(X, IOS(X_Z))$ is a *STS*).

$(((K, Z) \tilde{\cup} (L, Z),) \tilde{\cap} (M, Z)) \tilde{\setminus} \{x\} =$
$((K, Z) \tilde{\cap} (M, Z)) \tilde{\cup} ((L, Z),) \tilde{\cap} (M, Z)) \tilde{\setminus} \{x\}$
$=(((K, Z) \tilde{\cap} (M, Z)) \tilde{\setminus} \{x\}) \tilde{\cup} (((L, Z),) \tilde{\cap} (M, Z)) \tilde{\setminus} \{x\}) =$
$\emptyset \cup \emptyset = \emptyset$. Hence $x \notin iD\big((K, Z) \tilde{\cup} (L, Z),\big) \Longrightarrow$
$iD\big((K, Z) \tilde{\cup} (L, Z)\big) \subseteq iD(K, Z) \tilde{\cup} iD(L, Z)$…............(**)

From (*) and (**) we get, $iD\Big(\big((K, Z)\big) \tilde{\cup} (L, Z)\Big) =$
$iD\big((K, Z)\big) \tilde{\cup} iD(L, Z)$.

iv. Since $(K,Z)\,\tilde{\cap}\,(L,Z) \,\tilde{\subseteq}\, (F_1, E)$.
$(K,Z)\,\tilde{\cap}\,(L,Z) \,\tilde{\subseteq}\, (L,Z)$,
From (ii) we obtain that $iD((K,Z)\,\tilde{\cap}\,(L,Z)) \subseteq iD(K,Z)$, $iD((K,Z)\,\tilde{\cap}\,(L,Z)) \subseteq iD(L,Z)$,
Hence, $iD((K,Z)\,\tilde{\cap}\,(L,Z)) \subseteq iD(K,Z) \cap iD(L,Z)$.

v. Consider $x \in iD(K,Z) \Longrightarrow$ for each *SIOS* (M,Z) containing x.
We get, $((K,Z)\,\tilde{\cap}\,(M,Z))\tilde{\setminus}\{x\} \neq \phi \Longrightarrow ((K,Z)\,\tilde{\cap}\,(M,Z))\,\tilde{\cap}\,\{x\}^C$
$= \left((K,Z)\,\tilde{\cap}\,\{x\}^C\right)\tilde{\cap}\left((M,Z)\,\tilde{\cap}\,\{x\}^C\right) = \left((K,Z)\tilde{\setminus}\{x\}\right)\tilde{\cap}\left((M,Z)\right)\tilde{\setminus}\{x\} =$
$((K,Z)\tilde{\setminus}\{x\})\,\tilde{\cap}\,(M,Z))\tilde{\setminus}\{x\} \neq \phi \Longrightarrow$
$x \in iD((K,Z)\tilde{\setminus}\{x\})$.■

Theorem2.4: Consider $(X, SIOS(X_Z))$ as a "soft topological space"*(STS)*, for "soft sets"*(SSs)* $(P,Z),(Q,Z)$ in X, so the next phrases hold:

i. $iCl(X) = X$, $iCl(\phi) = \phi$.
ii. $iCl(P,Z)$ is a *SICS*.
iii. $(P,Z) \,\tilde{\subseteq}\, iCl(P,Z)$.
iv. $(P,Z) = iCl(P,Z)$ if and only if (P,Z) is a *SICS*.
v. $iCl(P,Z)$ is the smallest *SICS* containing (P,Z).
vi. $iCl(P,Z) = iCl(iCl(P,Z))$.
vii. $iCl\big((P,Z)\,\tilde{\cup}\,(Q,Z)\big) = iCl(P,Z)\,\tilde{\cup}\,iCl(Q,Z)$.
viii. $iCl(P,Z) = (P,Z)\,\tilde{\cup}\,iD(P,Z)$.

Proof: viii. By (iii) we obtain $(P,Z) \,\tilde{\subseteq}\, iCl(P,Z)$ ……… (1)
And by "theory (2.3)(ii)" $\Longrightarrow iD(P,Z) \subseteq iD(iCl(P,Z))$ ……… *(2)*
Since $iCl(P,Z)$ is a *SICS* $\Longrightarrow iD(iCl(P,Z)) \,\tilde{\subseteq}\, iCl(P,Z)$ …. *(3)*
From (2) and (3) we get $iD(P,Z) \,\tilde{\subseteq}\, iCl(P,Z)$ ………. *(4)*
From (1) and (4) we have $(P,Z)\,\tilde{\cup}\,iD(P,Z) \,\tilde{\subseteq}\, iCl(P,Z)$.
Now, let $x \,\tilde{\in}\, iCl(P,Z)$. If $x \,\tilde{\in}\, (P,Z)$, then the proof is obtained, If $x \,\tilde{\notin}\, (P,Z)$, each *SIOS*$(M,Z)$ containing x intersects (P,Z) at distinct point from x, so $x \in iD(P,Z)$, thus $iCl(P,Z) \,\tilde{\subseteq}\, (P,Z)\,\tilde{\cup}\,iD(P,Z)$. Which completes the proof. ■

Definition2.4: Consider two *STSs* (X,τ,E) and (Y,ρ,H) with the mappings, $u: X \longrightarrow Y$, $p: E \longrightarrow H$ and $f_{pu}: SS(X_E \longrightarrow SS(Y_H)$. Then:

1. [11]. If $f_{pu}(F,E) \,\tilde{\in}\, SOS(Y_H)$, $\forall (F,E) \,\tilde{\in}\, SOS(X_E)$, f_{pu} is named soft open mapping *SOM.*
2. [11]. If $f_{pu}(F,E) \,\tilde{\in}\, SCS(Y_H)$, $\forall (F,E) \,\tilde{\in}\, SCS(X_E)$, f_{pu} is named soft closed mapping *SCM.*
3. If $f_{pu}(F,E) \,\tilde{\in}\, SIOS(Y_H)$, $\forall (F,E) \,\tilde{\in}\, SOS(X_E)$, f_{pu} is named soft i − open mapping *SI-OM.*
4. If $f_{pu}(F,E) \,\tilde{\in}\, SICS(Y_H)$, $\forall (F,E) \,\tilde{\in}\, SCS(X_E)$, f_{pu} is named soft i − closed mapping *SI-CM.*
5. [11]. If $f_{pu}^{-1}(G,H) \,\tilde{\in}\, SOS(X_E)$, $\forall (G,H) \,\tilde{\in}\, SOS(Y_H)$, f_{pu} is named soft continuous mapping *SContM.*
6. If $f_{pu}^{-1}(G,H) \,\tilde{\in}\, SIOS(X_E)$, $\forall (G,H) \,\tilde{\in}\, SOS(Y_H)$, f_{pu} is named soft i-continuous mapping *SI-ContM.*
7. If $f_{pu}^{-1}(G,H) \,\tilde{\in}\, SICS(X_E)$ $\forall (G,H) \,\tilde{\in}\, SICS(Y_H)$, f_{pu} is named soft i − irresolute mapping *SI-IreM.*

Theorem2.5: Each *SContM* is *SI-ContM.*

Proof: Consider $f_{pu}: SS(X_E \longrightarrow SS(Y_H)$ as *SContM.* If (G,H) is a *SOS* in (Y,ρ,H) we have $f_{pu}^{-1}(G,H)$ is *SOS* in (X,τ,E)(by suppose). By "Theory (2.1)", we obtain $f_{pu}^{-1}(G,H)$ is a *SIOS* in (X,τ,E). Hence, f_{pu} is a *SI-ContM.*■

3. i-Soft Separation Axioms

Definition 3.1: [19]. Consider e_P, e_Q as any two distinct soft points in (X,τ,Z), then (X,τ,Z) is considered:

1. Soft semi − T_0 space, if there exist *SSOSs*(P,Z) *or* (Q,Z) wherein, $e_P \,\tilde{\in}\, (P,Z)$, $e_Q \notin (P,Z)$, $e_Q \,\tilde{\in}\, (Q,Z)$, $e_P \notin (Q,Z)$, for each e_P, e_Q *in* X.
2. Soft semi − T_1 space, if there exist two *SSOSs* (P,Z) *and* (Q,Z) Wherein, $e_P \,\tilde{\in}\, (P,Z)$, $e_Q \,\tilde{\notin}\, (P,Z)$, $e_Q \,\tilde{\in}\, (Q,Z)$, $e_P \,\tilde{\notin}\, (Q,Z)$, for each e_P, e_Q *in* X.
3. Soft semi − T_2 space, if there exist two disjoint *SSOSs*(P,Z) *and* (Q,Z) wherein, $e_P \,\tilde{\in}\, (P,Z)$, $e_Q \,\tilde{\in}\, (Q,Z)$, for each e_P, e_Q *in* X.

Definition 3.2: [20]. Consider x, y as any two distinct points in (X,τ,L), then (X,τ,L) is considered:

1. Soft T_0 space, if there exists a *SOS* (O,L) wherein either $x \,\tilde{\in}\, (O,L)$, $y \,\tilde{\notin}\, (O,L)$ or $y \,\tilde{\in}\, (O,L)$, $x \,\tilde{\notin}\, (O,L)$, for each x, y *in* X.
2. Soft T_1 space, if there exist two *SOSs* (O,L), (J,L) wherein, $x \,\tilde{\in}\, (O,L)$, $y \,\tilde{\notin}\, (O,L)$ and $y \,\tilde{\in}\, (J,L)$, $x \,\tilde{\notin}\, (J,L)$, for each x, y *in* X.
3. Soft T_2 space, if there exist two disjoint *SOSs*(O,L), (J,L) wherein, $x \,\tilde{\in}\, (O,L)$ and $y \,\tilde{\in}\, (J,L)$, for each x, y *in* X.

Definition 3.3: Consider x, y as any two distinct points in (X,τ,L) then (X,τ,L) is considered:

1. Soft i − T_0 space, if there exists a *SIOS* (O,L) wherein either, $x \,\tilde{\in}\, (O,L)$, $y \,\tilde{\notin}\, (O,L)$ or $y \,\tilde{\in}\, (O,L)$, $x \,\tilde{\notin}\, (O,L)$, for each x, y *in* X.
2. Soft i − T_1 (Individually, Soft semi − T_1 and soft Soft α − T_1 space), if there exist two *SIOSs* (Individually, *SSOSs* and *SαOSs*) (O,L), (J,L) wherein, $x \,\tilde{\in}\, (O,L)$, $y \,\tilde{\notin}\, (O,L)$ and $y \,\tilde{\in}\, (J,L)$, $x \,\tilde{\notin}\, (J,L)$ for each x, y *in* X.
3. Soft i − T_2 space, if there exist two disjoint *SIOSs* (O,L), (J,L) wherein $x \,\tilde{\in}\, (O,L)$ and $y \,\tilde{\in}\, (J,L)$, for each x, y *in* X.

Example3.1. Let $X = \{3,5\}$, $\tau = \{\phi_E, (F_1,E), X_E\}$, $E = \{s,r\}$"
Where, $(F_1,E) = \{(s,\{3\}),(r,\{3\})\}$, $SIOS(X_E) = \{\phi_E, (F_1,E), X_E\}$.
$3,5 \in X$ $(3 \neq 5)$ $\exists (F_1,E) \in SIOS(X_E)$ Wherein $3 \,\tilde{\in}\, (F_1,E)$, $5 \,\tilde{\notin}\, (F_1,E)$. Therefore; (X,τ,E) is Soft i − T_0 space.

Example3.2:
Let $X=\{7,8,9\}$
$,\tau=\{\phi_E,(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),X_E\}$
$E=\{w,z\}$"Where$(F_1,E)=\{(w,\{7\}),(z,\{7\})\}$, $(F_2,E)=\{(w,\{8\}),(z,\{8\})\}$, $(F_3,E)=\{(w,\{9\}),(z,\{9\})\}$."
$(F_4,E)=\{(w,\{7,8\}),(z,\{7,8\})\}$, $(F_5,E)=\{(w,\{7,9\}),(z,\{7,9\})\}$,
$(F_6,E)=\{(w,\{8,9\}),(z,\{8,9\})\}$. $SOS(X_E)=SIOS(X_E)=SSOS(X_E)=S\alpha OS(X_E)=\tau$.
$7,8\in X$ $(7\neq 8)$ $\exists(F_1,E),(F_2,E)\in SOS(X_E)$, $SIOS(X_E), SSOS(X_E), S\alpha OS(X_E)$.
Wherein
$7\tilde{\in}(F_1,E), 8\tilde{\notin}(F_1,E),\ 8\tilde{\in}(F_2,E), 7\tilde{\notin}(F_2,E)$.
$7,9\in X$ $(7\neq 9)\ \exists(F_1,E),(F_3,E)\in SOS(X_E)$, $SIOS(X_E), SSOS(X_E), S\alpha OS(X_E)$.
Wherein
$7\tilde{\in}(F_1,E), 9\tilde{\notin}(F_1,E)$,
$9\tilde{\in}(F_3,E), 7\tilde{\notin}(F_3,E)$.
$8,9\in X$ $(8\neq 9)\ \exists(F_2,E),(F_3,E)\in SOS(X_E)$, $SIOS(X_E), SSOS(X_E), S\alpha OS(X_E)$.
Wherein,
$8\tilde{\in}(F_2,E), 9\tilde{\notin}(F_2,E), 9\tilde{\in}(F_3,E), 8\tilde{\notin}(F_3,E)$.
Therefore; $(X,\tau,E)$is Soft T_1, Soft $\alpha-T_1$, Soft semi $-T_1$ and Soft $i-T_1$ space.

Definition3.4: (X,τ,E) is said to be Soft $i-$regular space (*SI-RS*) if it satisfies the next condition: If (F,E) is a *SICS* in $X$and $x\in X, x\tilde{\notin}(F,E)$
$\exists(G_1,E),(G_2,E)\in OS(X_E),(G_1,E)\tilde{\cap}(G_2,E)=\phi_E$
wherein, $(F,E)\tilde{\subseteq}(G_1,E), x\tilde{\in}(G_2,E)$.

Definition3.5: A Soft $i-T_1$ space is named Soft $i-T_3$ if it is *SIRS*.

Definition3.6: (X,τ,E) considers Soft $i-$normal space (*SI-NS*) if the next condition satisfied: if $(F_1,E),(F_2,E)$are two disjoint *SICSs* in X $\exists(G_1,E),(G_2,E)\in OS(X_E),(G_1,E)\tilde{\cap}(G_2,E)=\phi_E$.
Wherein $(F_1,E)\tilde{\subseteq}(G_1,E)$, $(F_2,E)\tilde{\subseteq}(G_2,E)$.

Definition3.7: A Soft $i-T_1$ space is named Soft $i-T_4$ if it is *SINS*.

Definition3.8: (X,τ,E) considers Soft $i-$completely regular space (*SI-CRS*) if the next condition satisfied: If (F_1,E) is a *SICS* in $X$and $x\in X, x\tilde{\notin}(F_1,E)$, there exists *SI-ContM* $f_{pu}:SS(X_E\longrightarrow SS(Y_H)$, $u:X\longrightarrow Y$, $p:E\longrightarrow H$, (X,τ,E) and (Y,ρ,H) are *STSs*, $Y=[0,1]$, $\rho=\{\emptyset_H,Y\}$Wherein $f_{pu}(F_1,E)=1_H$, $(u(x)=1\ \forall x\tilde{\in}(F_1,E))$, $f_{pu}(x_E)=0_H, (u(x)=0, x\tilde{\notin}(F_1,E))$.

Definition3.9: A Soft $i-T_1$ space is named soft $i-T_{(3\frac{1}{2})}$ if it is *SI-CRS*.

Definition3.10:
(X,τ,E) Considers Soft $i-$completely normal space (*SI-CNS*) if the next condition satisfied: If $(F_1,E),(F_2,E)\tilde{\in}SS(X_E),(F_1,E)\tilde{\cap}(F_2,E)=\phi_E$, $\exists(I_1,E),(I_2,E)\tilde{\in}IOS(X_E)$ s.t $(F_1,E)\tilde{\subseteq}(I_1,E)$, $(F_2,E)\tilde{\subseteq}(I_2,E)$
Wherein $(I_1,E)\tilde{\cap}(I_2,E)=\phi_E$.

Definition3.11: A Soft $i-T_1$ space is named Soft $i-T_5$ if it is *SI-CNS*.

Definition3.12:
(X,τ,E) considers Soft $i-$perefectly norma lspace (*SI-PNS*)if the next condition satisfied: If $(F_1,E),(F_2,E)$ are disjoint *SICSs* in X, there exists *SI-ContM* $f_{pu}:SS(X_E\longrightarrow SS(Y_H)$, $u:X\longrightarrow Y$, $p:E\longrightarrow H$, (X,τ,E) and (Y,ρ,H) are *STSs*, $Y=[0,1]$, $\rho=\{\emptyset_H,Y\}$. Wherein $f_{pu}^{-1}\{0_H\}=(F_1,E)$, $(u^{-1}(0)=x,\forall x\tilde{\in}(F_1,E))$, $f_{pu}^{-1}\{1_H\}=(F_2,E)$, $(u^{-1}(1)=x,\forall x\tilde{\in}(F_2,E))$.

Definition3.13:
A Soft $i-T_1$ space considers Soft $i-T_6$ if it is *SI-PNS*.

Example3.3:
Let $X=\{6,9\}$,$\tau=\{\phi_E,(F_1,E),(F_2,E),X_E\}$,
$E=\{q,r\}$
."Where,$(F_1,E)=\{(q,\{6\}),(r,\{6\})\}$, $(F_2,E)=\{(q,\{9\}),(r,\{9\})\}$, $SOS(X_E)=SIOS(X_E)=\tau$.
$SICS(X_E)$ $\{X_E,(F_1,E)^c=(F_2,E),(F_2,E)^c=(F_1,E),\phi_E\}$

1. $6,9\in X$ $(6\neq 9)$ $\exists(F_1,E),(F_2,E)\in SIOS(X_E)$.
Wherein $6\tilde{\in}(F_1,E), 9\tilde{\notin}(F_1,E)$,
$9\tilde{\in}(F_2,E), 6\tilde{\notin}(F_2,E)$. Therefore; (X,τ,E) is Soft $i-T_1$ space.
2. $6,9\in X$ $(6\neq 9)$ $\exists(F_1,E),(F_2,E)\in SIOS(X_E),(F_1,E)\tilde{\cap}(F_2,E)=\phi_E$.
Wherein $6\tilde{\in}(F_1,E)$, $9\tilde{\in}(F_2,E)$.
therefore; (X,τ,E) is Soft $i-T_2$ space.
3. (F_2,E) Is a *SICS* in X and $6\in X, 6\tilde{\notin}(F_2,E)$
$\exists(F_1,E),(F_2,E)\in SIOS(X_E),(F_1,E)\tilde{\cap}(F_2,E)=\phi_E$.
Wherein $(F_2,E)\tilde{\subseteq}(F_2,E), 6\tilde{\in}(F_1,E)$.therefore, (X,τ,E) is *SI-RS*.
4. (F_1,E)And(F_2,E) are *SICSs* in X, $\exists(F_1,E),(F_2,E)\in SIOS(X_E),(F_1,E)\tilde{\cap}(F_2,E)=\phi_E$.
Wherein $(F_2,E)\tilde{\subseteq}(F_2,E),(F_1,E)\tilde{\subseteq}(F_1,E)$.
therefore, $(X,\tau,E)$is *SI-NS*.
5. From (1) and (3) we obtain (X,τ,E) is Soft $i-T_3$ space.
6. From (1) and (4) we obtain (X,τ,E) is Soft $i-T_4$ space.
7. Let $f_{pu}:SS(X_E\longrightarrow SS(Y_H)$,be *SI-ContM*, $u:X\longrightarrow Y$, $p:E\longrightarrow H$, $Y=[0,1]$, $\rho=\{\emptyset_H,Y\}$, $(F_2,E)$is a *SICS* in X and $6\tilde{\notin}(F_2,E)$
Wherein $f_{pu}(F_2,E)=1_H$, $(u(x)=1\ \forall x\tilde{\in}(F_2,E))$, $f_{pu}(6_E)=0_H$, $(u(6)=0, 6\tilde{\notin}(F_2,E))$.
Therefore, (X,τ,E) is *SI-CRS*.

8. (X,τ,E) is soft $\mathrm{i}-\mathrm{T}_{(3\frac{1}{2})}$ which is obtained from (1) and (7).

9. Since $(F_1,E),(F_2,E)\tilde{\in} SS(X_E),(F_1,E)\tilde{\cap}(F_2,E)=\phi_E$ $\exists (F_1,E),(F_2,E)\tilde{\in} SIOS(X_E)$ s.t $(F_1,E)\tilde{\subseteq}(F_1,E),(F_2,E)\tilde{\subseteq}(F_2,E)$. Therefore, (X,τ,E) is *SI-CNS.*

10. (X,τ,E) is soft $\mathrm{i}-\mathrm{T}_5$ which is obtained from (1) and (9).

11. Consider $f_{pu}: SS(X_E \longrightarrow SS(Y_H)$ as, *SI-ContM* $u: X \longrightarrow Y$, $p: E \longrightarrow H$, $Y=[0,1]$, $\rho = \{\emptyset_H, Y\}$, since $(F_1,E),(F_2,E)$ are two *SICSs* and since $f_{pu}^{-1}\{0_H\} = (F_1,E)$, $(u^{-1}(0) = x, \forall x \tilde{\in} (F_1,E))$, $f_{pu}^{-1}\{1_H\} = (F_2,E)$, $(u^{-1}(1) = x, \forall x \tilde{\in} (F_2,E))$.Therefore, (X,τ,E) is *SI-PNS.*

12. (X,τ,E) is soft $\mathrm{i}-\mathrm{T}_6$ which is obtained by (1) and (9).

Theorem3.1:
Each soft $\mathrm{i}-\mathrm{T}_1$ space considers soft $\mathrm{i}-\mathrm{T}_0$.

Proof: By using "(Definition 3.3(1and2))" we get the required proof.

Theorem3.2: Each soft semi $-\mathrm{T}_1$ space considers soft $\mathrm{i}-\mathrm{T}_1$.

Proof: By using "(Definitions (3.1(2), 3.3(2))" and by "(Theorem 2.2) ", we get the required proof. The converse is not true, Indeed:

Example3.4:
Let $X=\{2,4,6,8\}, \tau = \{\phi_E, (F_1,E), (F_2,E), (F_3,E), X_E\}$, $E=\{k,w\}$.Where$(F_1,E) = \{(k,\{2\}),(w,\{2\})\}$, $(F_2,E) = \{(k,\{2,4\}),(w,\{2,4\})\}$, $(F_3,E) = \{(k,\{2,4,6\}),(w,\{2,4,6\})\}$." "$(F_4,E) = \{(k,\{2,6\}),(w,\{2,6\})\}$, $(F_5,E) = \{(k,\{2,8\}),(w,\{2,8\})\}$, $(F_6,E) = \{(k,\{2,6,8\}),(w,\{2,6,8\})\}$, $(F_7,E) = \{(k,\{2,4,8\}),(w,\{2,4,8\})\}$.
$SOS(X_E) = \tau$.
$SSOS(X_E) = S\alpha OS(X_E) =$
$\{\phi_E, (F_1,E), (F_2,E), (F_3,E),$"$\{(k,\{2,6\}),(w,\{2,6\})\}$, $\{(k,\{2,8\}),(w,\{2,8\})\}$,$\{(k,\{2,6,8\}),(w,\{2,6,8\})\}$, $\{(k,\{2,4,8\}),(w,\{2,4,8\})\} X_E\}$.
$SIOS(X_E) =$
$\{\phi_E, (F_1,E), (F_2,E), (F_3,E),$"$\{(k,\{4\}),(w,\{4\})\}$, $\{(k,\{6\}),(w,\{6\})\}$,$\{(k,\{2,6\}),(w,\{2,6\})\}$, $\{(k,\{4,6\}),(w,\{4,6\})\}, X_E\}$.

1. (X,τ,E) is not soft T_1 , because it is impossible to find two *SOSs* $(F_1,E),(F_2,E)$ Wherein $x_1 \tilde{\in} (F_1,E), x_2 \tilde{\notin} (F_1,E)$, $x_2 \tilde{\in} (F_2,E), x_1 \tilde{\notin} (F_2,E)$.
2. Similarly (X,τ,E) is not soft $\alpha-\mathrm{T}_1$.
3. (X,τ,E) Is not soft semi $-\mathrm{T}_1$
4. (X,τ,E) Is soft $\mathrm{i}-\mathrm{T}_1$. Since $\forall x_1,x_2 \in X\ (x_1 \neq x_2)$ $\exists (F_1,E),(F_2,E) \in SIOS(X_E)$ Wherein $x_1 \tilde{\in} (F_1,E), x_2 \tilde{\notin} (F_1,E)$, $x_2 \tilde{\in} (F_2,E), x_1 \tilde{\notin} (F_2,E)$.

Theorem3.3:
Each soft $\mathrm{i}-\mathrm{T}_2$ space considers soft $\mathrm{i}-\mathrm{T}_1$ and soft $\mathrm{i}-\mathrm{T}_0$

Soft i-T_2 ⟶ Soft i-T_1 ⟶ Soft i-T_0

Fig. 1: The Relations among soft $\mathrm{i}-\mathrm{T}_2$ space , soft $\mathrm{i}-\mathrm{T}_1$ **and** soft $\mathrm{i}-\mathrm{T}_0$.

Theorem3.4: Each *SI-CRS is a SI-R.S.*

Proof: By using "(Definitions (3.4 and 3.8)", we get the required proof.

Theorem3.5: Each soft $\mathrm{i}-\mathrm{T}_{3\frac{1}{2}}$ space considers soft $\mathrm{i}-\mathrm{T}_3$.

Proof: By using "(Definitions (3.5 and 3.9)", we get the required proof.

Theorem3.6: (X,τ,E) is named *SI-NS* if Whether it fulfills the next state: For each two separated *SICSs* $(F_1,E),(F_2,E)$ in X, and for each real numbers closed interval $[a,b]$ there exists *SI-ContM* $f_{pu}: SS(X_E \longrightarrow SS(Y_H)$, $u: X \longrightarrow Y$, $p: E \longrightarrow H$, $Y=[a,b]$, $\rho = \{\emptyset_H, Y\}$, $f_{pu}(F_1,E) = a_H$, $(u(x) = a\ \forall x \tilde{\in} (F_1,E))$, $f_{pu}(F_2,E) = b_H$, $(u(x) = b\ , \forall x \in (F_2,E)$.

Theorem3.7: Each soft $\mathrm{i}-\mathrm{T}_4$ space considers soft $\mathrm{i}-\mathrm{T}_{3\frac{1}{2}}$.

Proof: Consider (X,τ,E) satisfies soft $\mathrm{i}-\mathrm{T}_4$ space definition, which leads to soft $\mathrm{i}-\mathrm{T}_{3\frac{1}{2}}$ definition, the proof is complete "Theorem3.6". ■

Theorem3.8: Each soft $\mathrm{i}-\mathrm{T}_5$ space is soft $\mathrm{i}-\mathrm{T}_4$

Proof: Consider (X,τ,E) satisfies soft $\mathrm{i}-\mathrm{T}_5$ space definition, which leads to soft $\mathrm{i}-\mathrm{T}_4$ definition, hence the proof is complete (since each two discrete *SICSs* are separated).■

Theorem 3.9: Each soft subspace of soft $\mathrm{i}-\mathrm{T}_2$ space is a soft $\mathrm{i}-\mathrm{T}_2$.

Proof: Consider (X,τ,Z) is soft $\mathrm{i}-\mathrm{T}_2$ and (W,δ,Z) as soft subspace of X , x *and* y are two distinct points in W , we shall prove that x *and* y contained in disjoint *SIOSs* in soft subspace topology for W . Since x *and* y are distinct points of X , there exists two disjoint *SIOSs* of X , as, $(K_1,Z),(K_2,Z)$ Wherein $x \tilde{\in} (K_1,Z)$, $y \tilde{\in} (K_2,Z)$. Consider$(K_1,Z) \tilde{\cap} W$ and$(K_2,Z) \tilde{\cap} W$ are soft subsets of W .
Clearly:
1. $x \in W$ and $x \tilde{\in} (K_1,Z)$, so $x \tilde{\in}\ (K_1,Z) \tilde{\cap} W$. Similarly, $y \tilde{\in} (K_2,Z) \tilde{\cap} W$.

2. $(K_1,Z)\,\tilde{\cap}\,W$ And $(K_2,Z)\,\tilde{\cap}\,W$ are disjoint since $(K_1,Z),(K_2,Z)$ are disjoint.

3. $(K_1,Z)\,\tilde{\cap}\,W$ is *SIOS* relative over W, because it is the intersection with W of *SIOS* in X. Similarly, $(K_2,Z)\,\tilde{\cap}\,W$ is also *SIOS* over W. Hence, we get two disjoint *SIOSs* containing x and y, over the subspace topology of W. Therefore W is soft $\mathrm{i-T_2}$ space.■

Theorem3.10: Each soft subspace of *SI-RS* is *SI-RS*.

Proof: Consider (X,τ,L) is *SI-RS* and R as soft subspace, $x\in R$ and (M_1,L) soft i – closed subset in R. Right away x may be a side of the point over X, (M_1,L) is soft i – closed subset of X, wherein $(M_1,L)\tilde{\cap}R=(M_1,L)$. Such (M_1,L) exists by the way that soft subspace topology is defined. Obviously, whatever (M_1,L) is Picked dependent upon for those motivation, x impossible softly belongs to (M_1,L), because the only points in $(M_1,L)\tilde{\cap}R$ are in a SS not containing x. Since X is *SI-RS*, we can find *SIOSs* (J_1,L) and (J_2,L) in X wherein, $x\tilde{\in}(J_1,L)$, $(M_1,L)\tilde{\subseteq}(J_2,L)$ and $(J_1,L),(J_2,L)$ are soft disjoint. Now, $(J_1,L)\tilde{\cap}R$ and $(J_2,L)\tilde{\cap}R$ are disjoint soft i – open subsets of R, with $x\in(J_1,L)\tilde{\cap}R$ and $(M_1,L)\tilde{\subseteq}(J_2,L)\tilde{\cap}R$.■

Theorem3.11: Each soft i – closed subspace of *SI-NS* is *SI-NS*.

Proof: Since the subspace is already soft i – closed, soft i – closed subsets of it are already *SICSs* in the whole space. So we do not have to expand the soft i – closed subsets. Now we separate soft i – closed subsets in the whole space. We have disjointed *SIOSs* of the whole space. Now, simply intersect these *SIOSs* with the soft subspace, to get disjoint *SIOS* of the subspace separating the two disjoint *SICSs*.■

Corollaries3.1:

1. Each soft $\mathrm{i-T_3}$ (respect., soft $\mathrm{i-T_{3\frac{1}{2}}}$, soft $\mathrm{i-T_4}$, soft $\mathrm{i-T_5}$ and soft $\mathrm{i-T_6}$ space) is soft $\mathrm{i-T_1}$ but the converse is not necessary to be true because soft $\mathrm{i-T_1}$ space is not necessary to be *SI-RS* (respect.*SI-CRS*, *SI-NS*, *SI-CNS* and *SI-PNS*).

Proof: By using "(Definitions (3.5, 3.7, 3.9, 3.11, 3.13 and 3.3(2))"we get the required proof.

2. Each soft $\mathrm{T_0}$ space (respect., soft $\mathrm{T_1}$, soft $\mathrm{T_2}$) is soft $\mathrm{i-T_0}$ (respect. soft $\mathrm{i-T_1}$, soft $\mathrm{i-T_2}$ space) but the converse is not necessary to be true.

Proof: The proof is obtained from "Theory 2.1".

From above we have the next diagram as appear in the Figure 2:

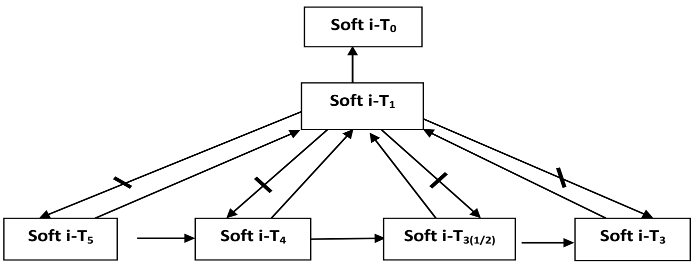

Fig. 2: The Relations among i-Soft Separation Axioms

Example3.5: Let $X=\{6,7\}$, $\tau=\{\phi_E,(F_1,E),X_E\}$, $E=\{q,r\}$."Where, $(F_1,E)=\{(q,\{6\}),(r,\{6\})\}$, $SOS(X_E)=SIOS(X_E)=\tau$.

$SICS(X_E)=\{X_E,(F_1,E)^c=\{(q,\{7\}),(r,\{7\})\},\phi_E\}$, $6,7\in X$ $(6\neq 7)$ $\exists(F_1,E)\tilde{\in}SIOS(X_E)$. Wherein $6\tilde{\in}(F_1,E),7\tilde{\notin}(F_1,E)$, Therefore; (X,τ,E) is Soft $\mathrm{i-T_0}$ space. But it is not Soft $\mathrm{i-T_1}$ space. Thus, (X,τ,E) is not Soft $\mathrm{i-T_2}$ space, also it is not *Soft* $\mathrm{i-T_3}$ *space*, it is not Soft $\mathrm{i-T_4}$ space, etc.

4. New Results

Theorem 4.1: (X,τ,E) is soft $\mathrm{i-T_0}$ if and only if every two different points of X have a different soft i – closure:

$\forall x,y\in X\ (x\neq y),\ iCl(x,E)\neq iCl(y,E)$.

Proof: 1.Let $x\neq y$ to need $iCl(x,E)\neq iCl(y,E)$. For each two different points x *and* y in X. since the two SSs $iCl(x,E),iCl(y,E)$ are different, there exist a point z in X belongs only to one of these two SSs and let $z\tilde{\in}iCl(x,E),z\tilde{\notin}iCl(y,E)$. If $x\tilde{\in}iCl(y,E)$ then $iCl(x,E)\tilde{\subseteq}iCl(iCl(y,E))=iCl(y,E)$. We have, $z\tilde{\in}iCl(y,E)\tilde{\subseteq}iCl(y,E)$, contradiction. Then $x\tilde{\notin}iCl(y,E)$, therefore; $iCl(y,E)^C$ is *SIOS* "Theorem 2.4" containing x not y.

1. Then again let X be soft $\mathrm{i-T_0}$ space and let x,y be two different points in X. By soft $\mathrm{i-T_0}$ space definition there exists *SIOS* (G,E) containing one of these two points not the other. Let $x\tilde{\in}(G,E),y\notin(G,E)$ then $(G,E)^C$ is *SICS* "Theorem 2.4"containing x not y. By $iCl(y,E))$ definition, we have $y\tilde{\in}iCl(y,E))$ but $x\tilde{\notin}iCl(y,E))$ because $x\tilde{\notin}(G,E)^C$. Therefore; $iCl(x,E))\neq iCl(y,E))$.■

Theorem 4.2: (X,τ,E) is soft $i-T_1$ space on the off chance that and just if each singleton *SS* belongs to it is *SICS*.

Proof: 1. Consider for each singleton *SS* belongs to (X,τ,E) is *SICS* and x,y be two different points in X. Then $(x,E)^C$ is *SIOS* containing y *not* x, $(y,E)^C$ is *SIOS* containing x *not* y. Therefore; (X,τ,E) is a soft $i-T_1$.

2. Consider (X,τ,E) is a soft $i-T_1$, $x\in X$. From soft $i-T_1$ definition we obtain, for each two different points in X $(x,y\in X,\ x\neq y)$ there exists *SIOS* (G_y,E) containing y not x wherein $y\,\tilde{\in}\,(G_y,E)\subseteq(x,E)^C$. Then, $(x,E)^C=\tilde{\bigcup}\{(y,E):(y,E)\tilde{\neq}(x,E)\}\tilde{\subseteq}(x,E)^C$. Therefore; $(x,E)^C$ is the union of *SIOSs*, then it is *SIOS*. Then (x,E) is *SICS* $\forall x$ in X.■

Theorem 4.3: (X,τ,E) is *SI-RS* if and only if $\forall x\in X$ and for each *SIOS* (G,E) containing x there exists a *SIOS* (G^*,E) wherein $x\,\tilde{\in}\,(G^*,E)$ and $iCl(G^*,E)\tilde{\subseteq}(G,E)$.

Proof: 1. Consider (X,τ,E) as *SI-RS* and let $x\,\tilde{\in}\,(G,E)$ where (G,E) is *SIOS* in X. Then $(F,E)=X_E\,\tilde{\setminus}\,(G,E)$ is *SICS* not contains x. By *SI-RS* definition, there exist two discrete *SIOSs* (G_x,E) and (G_F,E) wherein $x\,\tilde{\in}\,(G_x,E)$ and $(F,E)\tilde{\subseteq}(G_F,E)$. Since $(G_x,E)\tilde{\subseteq}(G_F,E)^C$, then $iCl(G_x,E)\tilde{\subseteq}iCl(G_F,E)^C=(G_F,E)^C\tilde{\subseteq}(F,E)^C=(G,E)$. Therefore; $x\,\tilde{\in}\,(G_x,E)$ and $iCl(G_x,E)\tilde{\subseteq}(G,E)$. Then (G,E) is *SIOS* which we need.

2. Consider the condition above is true and we will show (X,τ,E) is *SI-RS*. Let $x\,\tilde{\notin}\,(F,E)$ where (F,E), is *SICS*. Then $x\,\tilde{\in}\,(F,E)^C$ where $(F,E)^C$ is *SIOS* in X. Then there exists SIOS (G^*,E) wherein $x\,\tilde{\in}\,(G^*,E)$ and $iCl(G^*,E)\tilde{\subseteq}(F,E)^C$. Obviously, (G^*,E) and $(iCl(G^*,E))^C$ are discrete *SIOSs* wherein $x\,\tilde{\in}\,(G^*,E),(F,E)\tilde{\subseteq}(iCl(G^*,E))^C$. Therefore; (X,τ,E) is *SI-RS*.■

Theorem 4.4: (X,τ,E) is *SI-NS* if and only if for each *SICS* (F,E) and for each *SIOS* (G,E) containing (F,E) there exists *SIOS* (G^*,E) wherein $(F,E)\tilde{\subseteq}(G^*,E)$ and $iCl(G^*,E)\tilde{\subseteq}(G,E)$.

Proof: 1. Consider (X,τ,E) be *SI-NS* be and let (F,E) be *SICS* contained in *SIOS* (G,E) then $(K,E)=X_E\,\tilde{\setminus}\,(G,E)$ is *SICS*, where (K,E) *and* (F,E) are discrete *SSs*. By *SI-NS* definition there exist two *SIOSs* (G_K,E) *and* (G_F,E) wherein $(K,E)\tilde{\subseteq}(G_K,E)$ *and* $(F,E)\tilde{\subseteq}(G_F,E)$. Since $(G_F,E)\tilde{\subseteq}X_E\,\tilde{\setminus}\,(G_K,E)$ then $iCl(G_F,E)\tilde{\subseteq}iCl(X_E\,\tilde{\setminus}\,(G_K,E))=X_E\,\tilde{\setminus}\,(G_K,E)\ =\tilde{\subseteq}\ X_E\,\tilde{\setminus}\,(K,E)=(G,E)$. Therefore, $x\,\tilde{\in}\,(G_x,E)$ and $iCl(G_x,E)\tilde{\subseteq}(G,E)$. Then (G_F,E) is the wanted *SIOS*.

2. Think about the condition above is valid; we will demonstrate that (X,τ,E) is *SI-NS*. Consider (F_1,E) and (F_2,E) as two discrete *SICSs* in X, then $(F_1,E)\tilde{\subseteq}(X_E\,\tilde{\setminus}\,(F_2,E))$ where $X_E\,\tilde{\setminus}\,(F_2,E)$, is *SIOS* in X. Then there exists *SIOS* (G^*,E) wherein $(F_1,E)\tilde{\subseteq}(G^*,E)$ and $iCl(G^*,E)\tilde{\subseteq}(X_E\,\tilde{\setminus}\,(F_2,E))$. obviously (G^*,E) and $X_E\,\tilde{\setminus}\,iCl(G^*,E)$ are discrete *SIOSs* wherein $(F_1,E)\tilde{\subseteq}(G^*,E)$, $(F_2,E)\tilde{\subseteq}(X_E\,\tilde{\setminus}\,iCl(G^*,E)$. Hence, (X,τ,E) is *SI-NS*.■

Theorem 4.5: (X,τ,E) is *SI-NS* if and only if for any disjoint *SICSs* $(F_1,E),(F_2,E)$, there exists *SI-ContM*, $f_{pu}:SS(X_E)\longrightarrow SS(Y_H)$(where $u:X\longrightarrow Y$, $p:E\longrightarrow H$, $Y=[0,1]$, $\rho=\{\emptyset_E,Y\}$), wherein $f_{pu}(F_1,E)=\{0_H\}$, ,$(u(x)=0,\forall x\,\tilde{\in}\,(F_1,E))$, $f_{pu}(F_2,E)=\{1_H\}$, $(u(x)=1,\forall x\,\tilde{\in}\,(F_2,E))$.

Proof: Let (X,τ,E) be *SI-NS* and let $(F_1,E),(F_2,E)$ be two *SICSs* in X. Set (F_{10},E) to be (F_1,E), and set (F_{11},E) to be X. Let $(F_{1/1/2},E)$ be a set containing (F_{10},E) whose soft $i-$closure is contained in (F_{11},E). When all is said in done, inductively characterize for every normal number n and for every single regular number $a\prec 2^{n-1}$, $(F_{1(\frac{2a+1}{2^n})},E)$ to be a soft set containing $(F_{1(\frac{a}{2^{n-1}})},E)$ whose soft $i-$closure is contained within the complement of $(F_{1(\frac{a+1}{2^{n-1}})},E)$. This defines (F_{1k},E) where k is a rational number in the interval $[0,1]$ expressible in the form $\dfrac{a}{2^n}$ where a *and* n are entire numbers.

Now define the mapping $f_{pu}:SS(X_E)\longrightarrow SS(Y_H)$ to be $f_{pu}(k_E)=\inf\{x:k\,\tilde{\in}\,(F_{1x},E)\}$, $(u(k)=inf\{x:k\in(F_{1x}(e),\forall e\in E)\}$. Consider any element x within *SNS* X, and consider any open interval (a,b) around $f_{pu}(x)$. There exists rational numbers c and d in that expressible open interval in the form $\dfrac{k}{2^n}$ where k and n are whole numbers, wherein $c<f_{pu}(x)<d$. *If* $c<0$, then replace it with 0, and if $d>1$, then replace it with 1. Then the intersection of the complement of the set (F_{1c},E) and the set (F_{1d},E) is soft open neighborhood of $f(x)$ with image with (a,b) (a, b), obtaining the map is SContM. Since each SContM is SI-ContM "Theorem 2.5" we obtain, f_{pu} is *SI-ContM*.

Conversely, considers for any two disjoint *SICSs*, there is *SI-ContM*

$f_{pu}: SS(X_E) \longrightarrow SS(Y_H)$(where $u: X \longrightarrow Y$, $p: E \longrightarrow H$, $Y=[0,1]$, $\rho = \{\emptyset_E, Y\}$), wherein $f_{pu}(x)=\{0_H\}$, $(u(x) = 0, \forall x \tilde{\in} (F_1, E))$, $f_{pu}(x)=\{1_H\}$, $(u(x) = 1, \forall x \tilde{\in} (F_2, E))$. Since the disjoint *SS* $([0,0.5), E)$ and $((0.5,1], E)$ are *SIOSs* and under those soft topology subspace, the inverses $f_{pu}^{-1}([0,0.5))$, which contains X, and $f_{pu}^{-1}([0.5,1])$, which contains Y, are also *SIOSs* and disjoint. ■

Remark4.1: Consider (X^*, τ^*, E) as a partial *STS* of (X, τ, E) and (F, E) be *SS* in X^* then $\tau^* \subseteq \tau \subseteq IOS(X_E)$ if and only if $X^* \in \tau$.

Theorem4.6: (X, τ, L) is *SI-CNS* in the event that and just if partial *STS* of it is *SI-NS*.

Proof:

1. Consider (X, τ, L) as *SI-CNS* and let (X^*, τ^*, E) be a partial *STS* of X. Let (W, L), (N, L) be two discrete *SICSs* in X, then:

$(W,L)\tilde{\cap} iCl(N,L) = iCl^*(W,L)\tilde{\cap} iCl(N,L) = X^* \tilde{\cap} iCl(W,L) \tilde{\cap} iCl(N,L)$
$= iCl^*(W,L)\tilde{\cap} iCl^*(N,L) = (W,L)\tilde{\cap}(N,L) = \phi$. Then (W, L), (N, L) are separated *SSs* in X. By *SI-CNS* definition there exists two *SIOSs* (J_1, L), (j_2, L) wherein $(W,L)\tilde{\subseteq}(J_1,L)$, $(N,L)\tilde{\subseteq}(J_2,L)$ then $X^* \tilde{\cap}(J_1,L), X^* \tilde{\cap}(J_2,L)$ are discrete *SIOSs* in X^*. Where $(W,L)\tilde{\subseteq} X^* \tilde{\cap}(J_1,L), (N,L) \tilde{\subseteq} X^* \cap (J_2,L)$. Therefore; (X^*, τ^*, E^*) is *SI-NS*.

4. Then again, consider each partial *STS* of (X, τ, L) as *SI-NS* and prove that X is *SI-CNS*. Let $(B_1, L), (B_2, L)$ be separated sets in X and let *SIOS* $[iCl(B_1, L)\tilde{\cap} iCl(B_2, L)]^C = X^*$ be a partial *STS* of X, this space is *SI-NS* (by suppose) and $X^* \tilde{\cap} iCl(B_1,L), X^* \tilde{\cap} iCl(B_2,L)$ are two discrete *SICSs* in X^*. At that point there exist two discrete *SIOSs* $(J_{B1}, L), (J_{B2}, L)$ in X^* wherein $X^* \tilde{\cap} iCl(B_1,L) \tilde{\subseteq} (J_{B1},L), X^* \tilde{\cap} iCl(B_2,L) \tilde{\subseteq} (J_{B2},L)$.

Since, X^* is *SIOS* in X, then $(J_{B1}, L), (J_{B2}, L)$ are *SIOSs* in X too "Remark 4.1".Then $(B_1,L) \tilde{\subseteq} X^* \tilde{\cap} iCl(B_1,L) \tilde{\subseteq} (J_{B1},L), (B_2,L) \tilde{\subseteq} X^* \tilde{\cap} iCl(B_2,L) \tilde{\subseteq} (J_{B2},L)$.

Therefore; (X, τ, E) is *SI-CNS*. ■

Theorem 4.7: If (X, τ, E) is *SI-NS*, and then it considers as *SI-CRS* in also just if it is *SI-RS*.

Proof: It is enough to prove each *SI-NS* and *SI-RS* space is *SI-CRS* "Theory 3.4". Let $x \tilde{\notin} (F, E)$ where, (F, E) is *SICS* in X, then $x \tilde{\in} (F, E)^C$ where $(F, E)^C$ is *SIOS*. Then there exists *SIOS*(G^*, E) wherein $x \tilde{\in} (G^*, E)$ and $iCl(G^*, E) \tilde{\subseteq} (F, E)^C$ "Theory 4.3". Since (F, E) and $iCl(G^*, E)$ are discrete *SICSs* in *SI-NS* (X, τ, E) and by "Theorem3.6", there exists *SI-ContM* $f_{pu}: SS(X_E) \longrightarrow SS(Y_H)$wherein $f_{pu}(F, E) = \{1_H\}$, $(u(x) = 1, \forall x \tilde{\in} (F, E))$, $f_{pu}(iCl(G^*, E)) = \{0_H\}$, $(u(x) = 0, \forall x \tilde{\in} iCl(G^*, E))$and since $x \tilde{\in} (G^*, E)$ then $f_{pu}(x, E) = \{0_H\}$. Therefore; (X, τ, E) is *SI-CRS*.

Remark4.2: The "Definition 2.4(7)" is also true for *SIOSs* by taking the soft complements of it.

Theorem4.8: Let (X, τ, E) be *STS* and (Y, δ, H) is soft $i-T_2$ space. If $f_{pu}: SS(X_E) \longrightarrow SS(Y_H)$ is injective 1-1 and *SI-IreM*, then X is soft $i-T_2$ space.

Proof: Consider $x, y \in X$ such that $x \neq y$. Since f_{pu} is $1-1$, then $f_{pu}(x_E) \neq f_{pu}(y_E)$. Since (Y, δ, H) is soft $i-T_2$, at that point there exist two *SIOSs* $(G_1, H), (G_2, H)$ in Y wherein $f_{pu}(x) \tilde{\in} (G_1, H), f_{pu}(y) \tilde{\in} (G_2, H)$ and $(G_1, H)\tilde{\cap}(G_2, H) = \phi$. Since f_{pu} is *SI-IreM* then $f^{-1}(G_1, H), f^{-1}(G_2, H)$ are two *SIOSs* in X. $x \tilde{\in} f^{-1}(G_1, H), y \tilde{\in} f^{-1}(G_2, H)$. $f^{-1}(G_1, H) \tilde{\cap} f^{-1}(G_2, H) = \phi$. Hence X is soft $i-T_2$ space. ■

Theorem4.9: Let (X, τ, E) be *STS* and (Y, δ, E) is soft $i-T_2$ space. If $f_{pu}: SS(X_E) \longrightarrow SS(Y_H)$ is injective (one-one) and *SI-ContM*, then X is soft $i-T_2$ space.

Proof: Similarly as in"Theory4.8", and using f_{pu} as *SI-ContM* instead of *SI-IreM*. ■

Theorem4.10: Let (X, τ, E) and (Y, δ, H) be *STSs* and Y is *SI-RS*. If $f_{pu}: SS(X_E) \longrightarrow SS(Y_H)$ is *SICM*, *SI-IreM* and 1-1, then X is *SI-RS*.

Proof: Let (F, E)be *SCS* in X, $x \tilde{\notin} (F, E)$.Since f_{pu} is *SICM*, then $f_{pu}(F, E)$ is *SICS* in Y. $f_{pu}(x_E) = y_E \tilde{\notin} f_{pu}(F, E)$. But Y is *SI-RS*, then there are two *SIOSs*$(G_1, H), (G_2, H)$ in Y wherein $f_{pu}(F, E) \tilde{\subseteq} (G_1, H)$ $y \tilde{\in} (G_1, H)$ and $(G_1, H)\tilde{\cap}(G_2, H) = \phi$. Since f_{pu} is *SI-IreM* and 1-1, so $f_{pu}^{-1}(G_1, H), f_{pu}^{-1}(G_2, H)$ are *SIOSs* in X and $x \tilde{\in} f_{pu}^{-1}(G_1, H), (F, E) \tilde{\subseteq} f_{pu}^{-1}(G_2, H)$, $f_{pu}^{-1}(G_1, H) \tilde{\cap} f_{pu}^{-1}(G_2, H) = \phi$. Hence X is *SI-RS*. ■

Theorem4.11: Consider $f_{pu}: SS(X_E) \longrightarrow SS(Y_H)$ as *SICM* and *SI-IreM* from (X, τ, E) into(Y, δ, H). If Y is *SI-NS*, so is X.

Proof: Let $(F_1, E), (F_2, E)$ be *SICSs* in X wherein, $(F_1, E)\tilde{\cap}(F_2, E) = \phi$. Since $f_{pu}$is *SICM*, then $f_{pu}(F_1, E), f_{pu}(F_2, E)$ are two *SICSs* in Y and $f_{pu}(F_1, E) \tilde{\cap} f_{pu}(F_2, E) = \phi$. Since Y is *SI-NS* and f_{pu} is *SI-IreM*, then there are two *SIOSs* $(G_1, H), (G_2, H)$ in Y wherein

$f_{pu}(F_1,E)\tilde{\subseteq}(G_1,H), f_{pu}(F_2,E)\tilde{\subseteq}(G_2,H)$ and $(G_1,H)\tilde{\cap}(G_2,H)=\phi$, also $f_{pu}^{-1}(G_1,H), f_{pu}^{-1}(G_2,H)$ are two *SIOSs* in X and $(F_1,E)\tilde{\subseteq} f_{pu}^{-1}(G_1,H),(F_2,E)\tilde{\subseteq} f_{pu}^{-1}(G_2,H)$, $f_{pu}^{-1}(G_1,H)\tilde{\cap} f_{pu}^{-1}(G_2,H)=\phi$. Hence X is *SI-NS*.■

Acknowledgment: The Authors are very grateful to the University of Mosul/ College of Education for Pure Sciences for their provided facilities, which helped to improve the quality of this work.

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بديهيات الفصل الناعمة من النمط –i في فضاءات تبولوجية ناعمة

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الملخص

في الدراسة الحالية ادخل الباحثان نوعا حديثا من بديهيات الفصل الناعمة اسمياه بديهيات الفصل الناعمة من النمط–i باستخدام تعريف المجموعات المفتوحة الناعمة من النمط–i في فضاءات تبولوجية ناعمة (انظر[17])، العلاقات بين بديهيات الفصل الناعمة من النمط–i والعديد من الامثلة تم اعطائها، علاوة على ذلك، الباحثان وجدا بان بديهيات الفصل الناعمة تؤدي الى بديهيات الفصل الناعمة من النمط–i، ولكن العكس ليس من الضروري ان يكون صحيحا. ايضا، تم برهان العديد من المبرهنات حول بديهيات الفصل الناعمة من النمط–i.