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Dividing Graceful Labeling of Certain Tree Graphs

Abdullah Zahraa O. , Arif Nabeel E. , F. A. Fawzi

Department of Mathematic College of Computer Science and Mathematics , Tikrit University , Tikrit , Iraq

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Corresponding Author:

Name: Abdullah Zahraa O.

E-mail: Zahraaoth90@gmail.com

Tel:

ABSTRACT

A tree is a connected acyclic graph on n vertices and m edges. graceful labeling of a tree defined as a simple undirected graph $G(V,E)$ with order n and size m, if there exist an injective mapping $f: V(G) \to \{0,1,2,3,\dots,m\}$ that induces a bijective mapping $f^*: E(G) \to \{1,2,3,\dots\dots,m\}$ defined by $f^*(u,v) = |f(u) - f(v)|$ for each $(u,v) \in E(G)$ and $u, v \in V(G)$. In this paper we introduce a new type of graceful labeling denoted dividing graceful then discuss this type of certain tree graphs .

1- Introduction

A graph $G = (V,E)$ consists of two finite sets: $V(G)$, the vertex set of the graph G, which is nonempty set of elements called vertices, the number of these vertices called order, and $E(G)$ the edge set of graph (may be empty set) , which is set of elements called edge, the number of these edges called size. A graph then can be thought of as drawing or diagram consisting of collection of vertices together with edges joining certain pairs of these vertices [1]. Let $G(V,E)$ be a simple undirected graph with order n and size m, if there exist an injective mapping $f: V(G) \to \{0,1,2,3,\dots,m\}$ that induces a bijective mapping $f^*: E(G) \to \{1,2,3,\dots,m\}$ defined by $f^*(u,v) = |f(u) - f(v)|$ for each $[u,v] \in E(G)$ and $u, v \in V(G)$, then f is called graceful labeling of graph G [2]. In 2002, Kourosh, E. Introduced a graceful with some operation graphs [1]. In 2012 Uma, R. and Murugesan, N. discussed graceful labeling of some simple graphs and their properties [3]. In 2014 Pradham, P. and Kumar, K. discussed the graphs which are obtained by adding the pendant edge to the vertices of K_n or P_2 or both in $P_2 + K_n$ are graceful [2]. In 2014 , Munia, A. and et. al.. discussed a new class of graceful tree [4]. In 2014, Vaithilingam, K. discussed difference labeling of some graph families [5]. In 2018, Selvarajan, T. M. and Subramoniam, R. discussed prime graceful labeling [6]. In this paper discuss a new type of graceful denoted by dividing graceful then study it for certain tree graphs.

2- The concepts

Definition 2.1 [2]: Let G be a simple undirected graph with order n and size m , if there exist an injective mapping $f: V(G) \to \{0,1,2,3,\dots,m\}$ that induces a bijective mapping $f^*: E(G) \to \{1,2,3,\dots,m\}$ defined by $f^*(u,v) = |f(u) - f(v)|$ for each $(u,v) \in E(G)$ and $u, v \in V(G)$, then f is called graceful labeling of graph G .

Definition 2.2 [10]**:** A closed path is called a cycle . thus, the degree of each vertex of a cycle graph is two and denoted by C_n .

Definition 2.3 [9]: A tree is a connected graph without any cycles .

Example: Graceful labeling of tree (Fig. 2.1) .

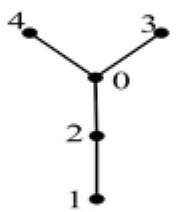


Fig. 2.1 Tree .

Definition 2.4 [9]: A binary tree is defined as a tree in which there is exactly one vertex of degree two and each of the remaining vertices is of degree one or three .

Example: Graceful labeling of binary tree (Fig. 2.2) .

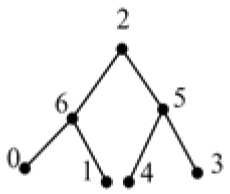


Fig. 2.2 Binary tree

Definition 2.5 [7] : A graph is called a path if the degree $d(v)$ of every vertex , v is ≤ 2 and there are no more than two end vertices.

Example: Graceful labeling of path (Fig. 2.3) .

Fig. 2.3 path.

Definition 2.6 [8] : A caterpillar is a tree such that if one removes all of its leaves, the remaining graph is a path this path can be termed as back bone of the caterpillar.

Example: Graceful labeling of the caterpillar (Fig. 2.4) .

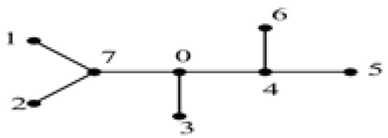


Fig. 2.4 The caterpillar

Definition 2.7 [4] : In graph theory , a tree with one internal vertex and k leaves is said to be star $S_{1,k}$ that happen to be complete bipartite graph $K_{1,k}$.

Example: Graceful labeling of star (Fig. 2.5)

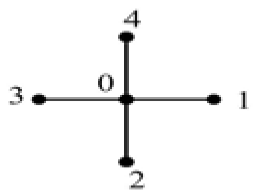


Fig. 2.5 Star.

Definition 2.8 [7]: A spider is a tree having a unique node center with degree greater than 2 and all other have degrees less than or equal to 2 .

Example: Graceful labeling of spider (Fig. 2.6) .

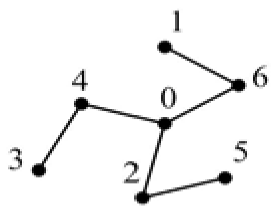


Fig. 2.6 Spider .

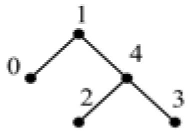
Definition 2.9: A graceful labeling of graph $G\,(V, E)$ with order n and size m is a one-to-one mapping $\varphi: \mathrm{V(G)} \rightarrow \{0,1,2,3, \dots, m\}$ with the following property .If we define for any edge $e = [u, v] \in \mathrm{E(G)}$ with condition $\varphi^{*}(u, v) = \left\lceil \frac{f(u)f(v)+m}{m} \right\rceil$, $then\ \varphi^{*}: \mathrm{E(G)} \rightarrow \{1,2,3, \dots, m\}$, this graceful is called dividing graceful .

3- Main Results

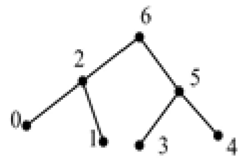
3.1 Dividing graceful labeling of a binary tree and path.

Examples 3.1.1: Dividing graceful labeling of a binary tree of diameter (3,4,5,6).

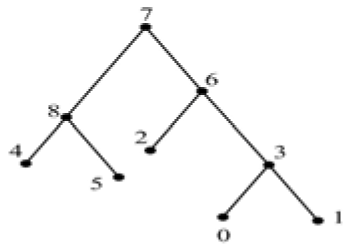
$a: diam = 3$



$b: diam = 4$



$c: diam = 5$



$d: diam\ = 6$

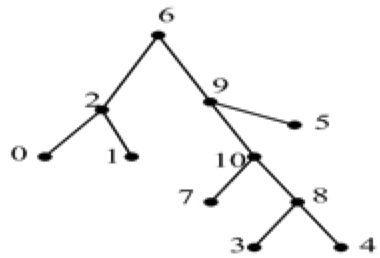
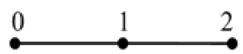


Fig. 3.1 Binary trees.

Examples 3.1.2 : Dividing graceful labeling of path diameter (2,3,4,5,6).

$a: diam = 2$



$b: diam = 3$

$c: diam = 4$

$d: diam = 5$

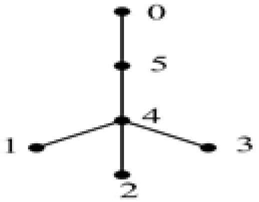
$e: diam = 6$

Fig. 3.2 Path.

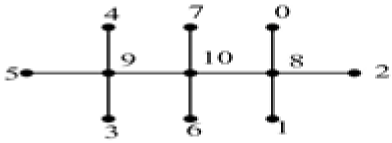
3.2 Dividing graceful labeling of Caterpillar and $DS_{1,k}$.

Examples 3.2.1: Dividing graceful labeling of Caterpillar of diameter (3,4,5,6,7).

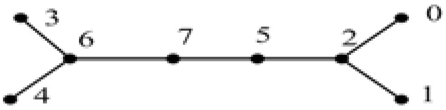
a: $diam = 3$



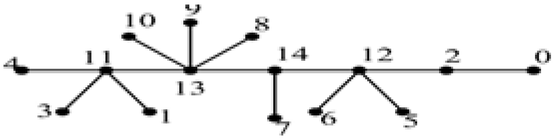
b: $diam = 4$



c: $diam = 5$



d: diam =6



e: $diam = 7$

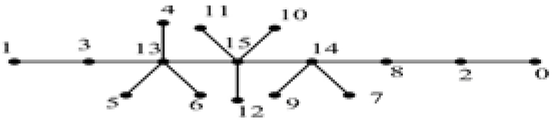
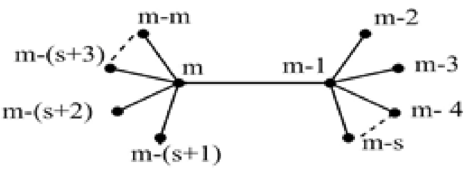


Fig. 3.3 Caterpillar trees.

Observation 3.2.2: Two stars (with same order n and size m) joining by one edge (u , v) such that identify u and v with maximum degrees of their stars denoted a mate of $DS_{1,k}$, and $DS_{1,k}$ is a special case of caterpillar .

Theorem 3.2.3: There is dividing graceful labeling of $DS_{1,k}$.

Proof: By using definition **2.9** , for some arbitrary labeling of vertices of $DS_{1,k}$ shown as follows :



Example 3.2.4: Dividing graceful labeling of $DS_{1,k}$.

Fig. 3.4 DS_6

3.3 Dividing graceful labeling of Star.

Theorem 3.3.1 : Dividing graceful labeling of any Star (S_n) is

$$f(v) = \begin{cases} m-1 & if\ \deg(u) = m \\ other\ wise & if\ \deg(u) = 1 \end{cases}$$

proof: By using definition **2.9** , it is clear that when we label the center vertex of any star by $m-1$ we can label other vertices by any one from set of $\{0,1,2,\dots,m\}$.

Example 3.3.2: Dividing graceful labeling of S_5 and S_6 .

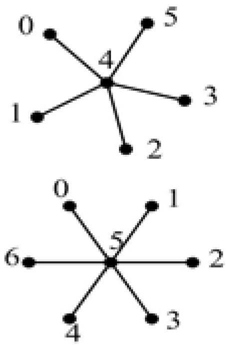
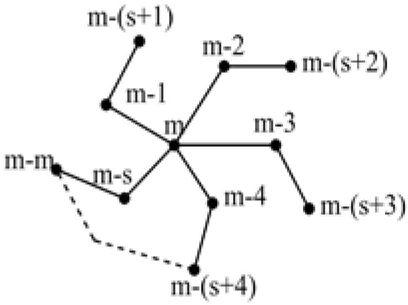


Fig. 3.5 S_5 *and* S_6

3.4 Dividing graceful labeling for Spider.

Theorem 3.4.1: There is dividing graceful labeling of a spider (SP_n) with diameter equal 4 .

Proof : By using definition **2.9** , for some arbitrary labeling of vertices of (SP_n) is as follows :



Example 3.4.2: The spider (SP_9) *is* dividing graceful labeling.

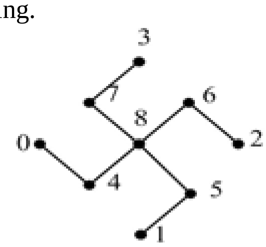


Fig. 3.6 SP_9

4- Conclusion

Our work we introduce a new type of graceful labeling denoted dividing graceful labeling, then infer the graph $G(V, E)$ with order n and size m is a one-to-one mapping $\varphi: V(G) \to \{0,1,2,3,\dots,m\}$ with the following property .If we define for any edge $e = [u,v] \in E(G)$ with condition $\varphi^*(u,v) = \left\lceil \frac{f(u)f(v)+m}{m} \right\rceil$, *then* $\varphi^*: \mathrm{E(G)} \to \{1,2,3,\dots,m\}$, it with some graphs such as binary tree with diameter 3,4,5 and 6 , path , caterpillar , star , $DS_{1,k}$ and spider with diameter equal 4 .

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الترقيم المقسم لبيانات أشجار معينة

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الملخص

ان عملية الترقيم معرفة للبيان البسيط ذو الرتبة n والحجم m على انه تطبيق متباين $f: V(G) \rightarrow \{0,1,2,3,\dots,m\}$ وكذلك تطبيق شامل $f^*: E(G) \rightarrow \{1,2,3,\dots,m\}$ معرفة بالشكل $f^*(u,v) = |f(u) - f(v)|$ لكل $(u,v) \in E(G)$ و $u, v \in V(G)$. في بحثنا هذا قدمنا نوع جديد من الترقيم اسميناه الترقيم المقسم وناقشناه لمجموعة معينة من بيانات الاشجار .