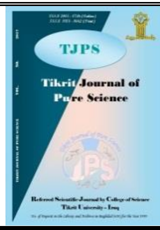
IASJ

المجلات الاكاديمية العراقية

Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php/j



Chromatic Number and some Properties of Pseudo-Von Neumann Regular graph of Cartesian Product of Rings

Nermen J. Khalel , Nabeel E. Arif

Department of Mathematic, College of Computer Science and Mathematics, Tikrit, Tikrit University, Iraq

https://doi.org/10.25130/tjps.v25i3.262

ARTICLE INFO.

Article history:

-Received: 6 / 8 / 2019

-Accepted: 24 / 9 / 2019

-Available online: / / 2020

Keywords: Graph, Chromatic Number, Commutative Ring.

Corresponding Author:

Name: Nermen J. Khalel

E-mail: nrnjamal88@gmail.com

Tel:

ABSTRACT

Let R be a commutative ring, the Pseudo – Von Neumann regular graph of the ring R is define as a graph whose vertex set consists of all elements of R and any two distinct vertices a and b are adjacent if and only if $a = a^2b$ or $b = b^2a$, this graph is denoted by *P-VG(R)*, in this work we got some new results about chromatic number of Pseudo-Von Neumann regular graph of cartesian product of rings.

1- Introduction

Beck [1] studied coloring of commutative rings and studied chromatic number of it is graph such that two different elements x and y are adjacent iff $xy = 0$, Bhavanari et.al. studied prime graph of a ring with some properties of its graph [2], and he studied cartesian product of prime graph with Srinivasulu [3] Kalita [4] computed chromatic number of prime graph of some finite ring, Patra k. et.al [5]. found chromatic number of prime graph of some rings of Z_n , where n=$\prod_{i=1}^{r} p_i^{\alpha_i}$, Elizabeth [6] studied colorings of zero divisor graphs of commutative rings . The study obtains the chromatic number of Pseudo-Von Nemann regular graph of cartesian product of rings.

2- Primer lay

Definition 2.1: let R be a ring and $a \in R$, a is called regular element if there exist $b \in R$ such that $a=aba$, if any element in R is regular then R is regular ring , if R is commutative then $a = a^2b$ and we say that R is Von Neumann regular ring.

Definition 2.2: A graph G is defined by an ordered pair ($V(G)$, $E(G)$), when $V(G)$ is a non empty set whose elements are called vertices and $E(G)$ is a set (may be empty) of unordered pairs of distinct vertices of $V(G)$. the element of $E(G)$ are called edges of the graph G . we denote by $\overline{uv}$, an edge between two end vertices u and v .

Definition 2.3: A simple graph that has no loops or multiple edges .

Definition 2.4: A graph H is said to be a subgraph of a graph G if all the edges and all the vertices of H are in G , and it is denoted by $H \subset G$.

Definition 2.5: A path is a graph G that contains a list $v_1, v_2, \dots, v_n$ of vertices of G s.t. for $1 \le i \le p - 1$, there is an edge $\overline{v_i v_{i+1}}$ in G and these are the only edges in G .

Definition 2.6: let v_1 and v_2 be two vertices , $d(v_1, v_2)$ is called the distance from v_1 to v_2 if it is the shortest path from v_1 to v_2.

Definition 2.7: A close path is called cylce , the degree of each vertex of a cycle graph is two , a cycle with n vertices denoted by C_n .

Definition 2.8: Let $G(V, E)$ be a graph and $C \subset G$, is called clique if the induced subgraph of G induced by C is a complete graph .

The clique is called maximal if there is no clique with more vertices .

Theorem 2.9 :

For circular graph C_n one has

$$\mathcal{X}(C_n)=\begin{cases} 2 & when\ n\ is\ even \\ 3 & when\ n\ is\ odd \end{cases}$$

Definition 2.10: A h-coloring of the vertex set of a graph G is a function γ: $V(G) \to \{1,2,\dots,h\}$ such that $\gamma(v_1) \ne \gamma(v_2)$ whenever v_1 is adjacent to v_2 , if a h-coloring of G exists , then G is called h- colorable.

Definition 2.11: The chromatic number of G is defined as $\mathcal{X}(G)$ = min { h : G is h- colorable }

where $\mathcal{X}(G)= h$, G is called h- chromatic.

Definition 2.12: The Cartesian product $G \times K$ of graphs G and K is a graph such that:

- The vertex set of $G \times K$ is the Cartesian product $V(G) \times V(K)$ and

- The two vertices (u, v) and (s, t) are adjacent in $G \times K$ if and only if either $u = s$ and v is adjacent to t in K or $v = t$ and u is adjacent to s in G.

3- Main Results

Definition 3.1[14]: Let R be a commutative ring. A graph G (V , E) is said to be (Pseudo -Von Neumann regular graph) of R if $V(G) = R$ and $E(G) = \{ \overline{ab} / a = a^2b$ or $b = b^2a$ and $a \neq b \}$ denoted by *P-VG(R)* , shortly *P*-Von Neumann regular graph .

Example 3.2:

$Z_2 = \{0,1\}$

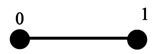


Fig. 1: ***P-VG***(Z_2)

$Z_3 = \{0,1,2\}$

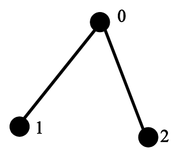


Fig. 2: ***P-VG***(Z_3)

$Z_4 = \{0,1,2,3\}$

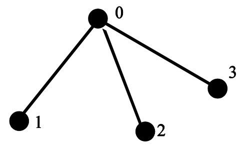


Fig. 3: ***P-VG***(Z_4)

$Z_5 = \{0,1,2,3,4\}$

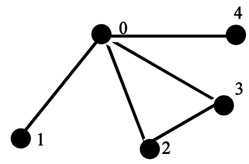


Fig. 4: ***P-VG***(Z_5)

Definition 3.2 :

Let $R = R_1 \times R_2$, then the *P-VG*(R) is define as the vertices set = { (a,b) :$a \in R_1$ and $b \in R_2$ } then (a,b) and (u,v) are adjacent in *P-VG*(R) if and only if a adjacent to u in *P-VG*(R_1) and b adjacent to v in *P-VG*(R_2) , and (0 ,0) adjacent to all vertices .

Example 3.3:

1- Let $R = Z_2 \times Z_2$

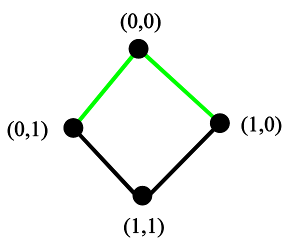


Fig. 5-i: ***P-VG***(Z_2) × ***P-VG***(Z_2)

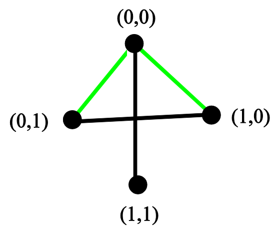


Fig. 5-ii: ***P-VG***$(Z_2 \times Z_2)$

P-VG(Z_2) × *P-VG*(Z_2) ∩ *P-VG*$(Z_2 \times Z_2)$= 3- star graph.

2- Let $R = Z_3 \times Z_3$

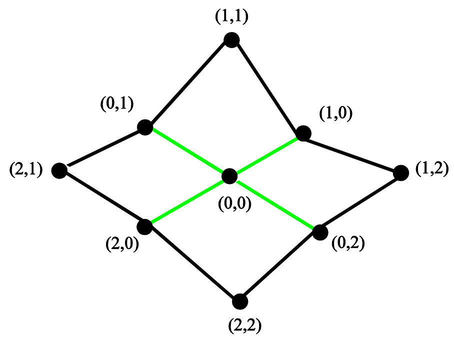


Fig. 6-i: P-VG(Z_3) ×P-VG(Z_3)

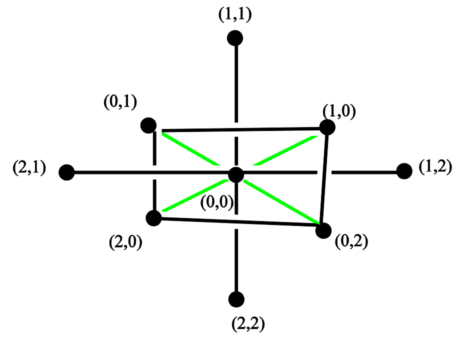


Fig. 6-ii: P-VG$(Z_3 \times Z_3)$

P-VG(Z_3) ×*P-VG*(Z_3) ∩ *P-VG*$(Z_3 \times Z_3)$ = 5- star graph.

In general *P-VG*(Z_n) ×*P-VG*(Z_n) ∩*P-VG*$(Z_n \times Z_n)$ = (2n-1) - star graph and the below theorem show that .

Theorem 3.4:

let R =Z_n , then the intersection of cartesian product of P-VG (Z_n) and *P-VG* $(Z_n \times Z_n)$ is equal to (2n-1)-star graph i.e.

P-VG(Z_n) ×*P-VG*(Z_n) ∩*P-VG*$(Z_n \times Z_n)$ = (2n-1) - star graph.

Proof :

Since$\overline{(0,0)(0,a)} \in$ E (*P-VG*(Z_n) ×*P-VG*(Z_n) ∩*P-VG*$(Z_n \times Z_n)$).

Then *P-VG*(Z_n) ×*P-VG*(Z_n) ∩*P-VG*$(Z_n \times Z_n) \neq \emptyset$.

Now, let H= { (a,b) , a = 0 or b = 0 } and $E(H)$ = $\{\overline{(0,0)(a,b)}, (a,b) \neq (0,0)$ }:

H is a subgraph and in the same time H is a star graph by the set $E(H)$.

Now, we need to prove this star has 2n-1 of vertices.

The study focuses on a number have of vertices are equal to 2(*n*-1) because the set of vertices of *K* graph { (0,1), … ,(0,*n*-1) , (1,0), … ,(*n*-1,0)} and (0,0) is a center of star graph then *H* has 2(*n*-1) +1 = 2*n*-1 vertices, i.e. *H* has order 2*n*-1

H = (2*n*-1) – star graph ⊂ *P-VG*(Z_n) ×*P-VG*(Z_n) and it is in the same time is a sub graph from *P-VG*$(Z_n \times Z_n)$.

Hence *H* ⊂ *P-VG*(Z_n) ×*P-VG*(Z_n) ∩*P-VG*$(Z_n \times Z_n)$.

Let $\overline{(a,b)(u,v)}$ ∈ *P-VG*(Z_n) ×*P-VG*(Z_n) ∩*P-VG*$(Z_n \times Z_n)$, and *a*,*b*,*u*,*v* ≠0.

Implies that $\overline{(a,b)(u,v)}$ ∈ *P-VG*(Z_n) ×*P-VG*(Z_n) and $\overline{(a,b)(u,v)}$ ∈ *P-VG*$(Z_n \times Z_n)$.

Then (either *a*=*u* and $\overline{bv}$ ∈ *E*(*P-VG*(Z_n) or *b*=*v* and $\overline{au}$ ∈ *E*(*P-VG*(Z_n) and ($\overline{au}$, $\overline{bv}$ ∈ *E*(*P-VG*(Z_n))

Hence there are two cases:

Case i: if (*a*=*u* and $\overline{bv}$ ∈ *E*(*P-VG*(Z_n) and $\overline{au}$ ∈ *E*(*P-VG*(Z_n)) , this is a contradiction by the definition of *P-VN-* Regular graph).

Case ii : if *b*=*v* and $\overline{au}$ ∈ *E*(*P-VG*(Z_n) and $\overline{bv}$ ∈ *E*(*P-VG*(Z_n))

,also this is a contradiction by the definition of *P-VN-* Regular graph).

Then *P-VG*(Z_n) ×*P-VG*(Z_n) ∩*P-VG*$(Z_n \times Z_n)$= *K*= (2*n*-1) - star graph.

lemma 3.5:

Let *R*= Z_p ,$p > 3$ be a prime number then $\mathcal{X}$ (*P-VG*(*R*))=3.

Proof : since *P-VG*$(Z_p$) has only cycle C_3 then $\mathcal{X}$ (*P-VG*(*R*))=3.

Theorem 3.6:

Let *R*= Z_{p^k} ,$p > 3$ be a prime number, *k* be a positive integer then $\mathcal{X}$ (*P-VG*(*R*))=3.

Proof : since *P-VG*$(Z_{p^k}$) has only cycle C_3 then $\mathcal{X}$ (*P-VG*(*R*))=3.

Corollary 3.7 :

Let *R*=Z_n, then $\mathcal{X}$ (*P-VG*(*R*))=

$$\begin{cases} 2 & n = 2,3,4,8 \\ 4 & if\ any\ vertex\ has\ invrse. \\ 3 & other\ wise \end{cases}$$

proof :

if $R = Z_n$, n = 2,3,4,8 then *P-VG*(*R*) is a star graph and $\mathcal{X}$ (*P-VG*(*R*)) =2.

if any vertex $a \in R$ has inverse then $(\overline{pa}, \overline{pa^{-1}}, \overline{aa^{-1}})$ is a cycle C_3 in *P-VG*(*R*), and since all vertices in *P-VG*(*R*) are adjacent to vertex 0 then $\mathcal{X}$ (*P-VG*(*R*)) =4 now in other wise

Case 1 : if *n* is prime number then $\mathcal{X}$ (*P-VG*(*R*)) =3 (by lemma 3.5)

Case 2 : if *n*=p^k where *p* is prime number and *k* a positive integer then $\mathcal{X}$ (*P-VG*(*R*)) =3 since *P-VG*(*R*) has only$\frac{(p-1)n}{2p} - 1$ of cycle C_3.

Case 2 : if *n*=*pk* where *p* is prime number and *k* a positive integer, if any vertex in *R* has no inverse then $\mathcal{X}$ (*P-VG*(*R*)) =3.

Theorem 3.8:

Let $R = Z_p \times Z_p \times ... \times Z_p$, (*n* times of Z_p) $p > 3$ is prime number then $\mathcal{X}$ (*P-VG(R)*)= $\mathcal{X}$ (*P-VG(* Z_p)) +1.

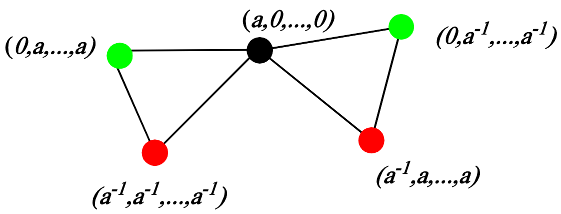
Proof :

Let (*a*,0,…,0) ∈ *R* and $a \neq 0$, then (*a*,0,0,…,0) is adjacent each to vertices

$(a^{-1}, a^{-1}, ..., a^{-1}), (0, a^{-1}, a^{-1}, ..., a^{-1}), (0, a, a, ..., a)$ and $(a^{-1}, a, a, ..., a)$ only. since $a \in Z_p$ and Z_p is a field.

But $(a^{-1}, a, a, ..., a)$ and $(0, a^{-1}, a^{-1}, ..., a^{-1})$ are adjacent and also

$(0, a, a, ..., a)$ and $(a^{-1}, a^{-1}, ..., a^{-1})$ are adjacent



Then, the graph has cycle of length 3 , therefore , we colored it by 3 colors .

But by definition of *P-VG-r*egular graph of cartesian product of a rings all vertices are adjacent to (0 ,0,…,0) then $\mathcal{X}$ (*P-VG*(*R*))=4

$\mathcal{X}$ (*P-VG(* Z_p)) +1 = 3+1 = 4 = $\mathcal{X}$ (*P-VG(R)*).

Theorem 3.9:

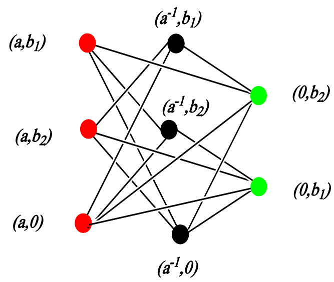
Let $R = Z_m \times Z_n$ and $n, m \geq 3$, then

1- $\mathcal{X}$ (*P-VG(R)*) = $\mathcal{X}$ (*P-VG(*Z_m*)*) + $\mathcal{X}$ (*P-VG(*Z_n) − 2 if

i- *m* and *n* are a prime.

ii- *m* is prime and *n* not prime and $\mathcal{X}$ (*P-VG(*Z_n*)*) =3

iii- *m* and *n* are not a prime and $\mathcal{X}$ (*P-VG(*Z_m*)*) = $\mathcal{X}$ (*P-VG(*Z_n*)*)=3



2- $\mathcal{X}$ (*P-VG(R)*) = $\mathcal{X}$ (*P-VG(*Z_m)) + $\mathcal{X}$ (*P-VG(*Z_n)) − 3 if

i. *m* and *n* are not prime .

ii. *m* is prime and *n* not prime and $\mathcal{X}$ (*P-VG(*Z_n*)*) =4

Proof :

1-i :

Let $R = Z_m \times Z_n$, *m* and *n* are a prime then $\mathcal{X}$ (*P-VG(*Z_m*)*) and $\mathcal{X}$ (*P-VG(*Z_n)) equal to 3,(by lemma 3.5)

Now, let (*a*, *b*) ∈ *R* and $a \in Z_m$, $b \in Z_n$, then (*a*, *b*) is adjacent only to (a^{-1}, b^{-1}) , $(a^{-1}, 0)$ and $(0, b^{-1})$.

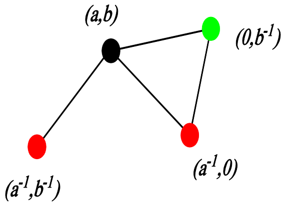
But $(a^{-1}, 0)$ and $(0, b^{-1})$are adjacent therefore we colored it by 3 colors, and since all vertices are adjacent to the vertex (0,0), then $\mathcal{X}$ (*P-VG(R)*) = 3+1=4.

$\mathcal{X}$ (*P-VG*(Z_m)) + $\mathcal{X}$ (*P-VG*(Z_n)) $- 2 = 3 + 3 - 2 = 4 = \mathcal{X}$ (*P-VG(R)*)

1-ii :

Let m is prime then $\mathcal{X}$ (*P-VG*(Z_m)) equal to 3 , and n is not prime and $\mathcal{X}$ (*P-VG*(Z_n)) equal to 3

Then we have three sets



$\{(a,b_1),(a,b_2),(a,0)\},\{(a^{-1},b_1),(a^{-1},b_2),(a^{-1},0)\}$ and $\{(0,b_1),(0,b_2)\}$ where $a \in Z_m$ and $b_1,b_2 \in Z_n$ such that $\overline{b_1b_2} \in E(P - VG(Z_n))$

such that the vertices in the same set not adjacent to each other , then we have 3- partite graph and to colored it we need 3 colors, but all vertices are adjacent to vertex (0,0) then $\mathcal{X}$ (*P-VG(R)*) = 3+1=4

$\mathcal{X}$ (*P-VG*(Z_m)) + $\mathcal{X}$ (*P-VG*(Z_n)) $- 2 = 3 + 3 - 2 = 4 = \mathcal{X}$ (*P-VG(R)*) .

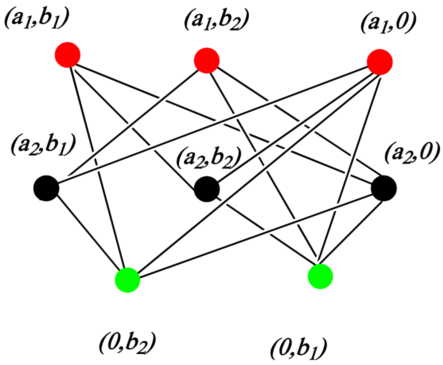
1-iii:

Let m and n are not prime and $\mathcal{X}$ (*P-VG*(Z_m)) and $\mathcal{X}$ (*P-VG*(Z_n)) are equal to 3

Let $a_1,a_2 \in Z_m$ such that $\overline{a_1a_2} \in E(P - VG(Z_m))$,$b_1,b_2 \in Z_n$ such that $\overline{b_1b_2} \in E(P - VG(Z_n))$

Now, we have three sets

$\{(a_1,b_1),(a_1,b_2),(a_1,0)\}$, $\{(a_2,b_1),(a_2,b_2),(a_2,0)\}$ and $\{(0,b_1),(0,b_2)\}$



such that the vertices in the same set not adjacent to each other, then we have 3- partite graph and to colored it we need 3 colors, but all vertices are adjacent to vertex (0,0) then $\mathcal{X}$ (*P-VG(R)*) = 3+1=4

$\mathcal{X}$ (*P-VG*(Z_m)) + $\mathcal{X}$ (*P-VG*(Z_n)) $- 2 = 3 + 3 - 2 = 4 = \mathcal{X}$ (*P-VG(R)*).

2-i :

Let m and n are not prime and n,m $\neq$ 4,8 then $\mathcal{X}$ (*P-VG*(Z_m)) and $\mathcal{X}$ (*P-VG*(Z_n)) equal to 3 or 4

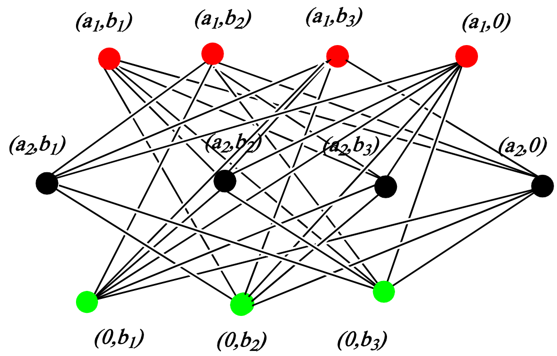
Then either $\mathcal{X}$ (*P-VG*(Z_m)) = 3 and $\mathcal{X}$ (*P-VG*(Z_n)) = 4 or $\mathcal{X}$ (*P-VG*(Z_m)) and $\mathcal{X}$ (*P-VG*(Z_n)) equal to 4

case 1:

$\mathcal{X}$ (*P-VG*(Z_m)) = 3 and $\mathcal{X}$ (*P-VG*(Z_n)) = 4 then we have three sets

$\{(a_1,b_1),(a_1,b_2),(a_1,b_3),(a_1,0)\}$,
$\{(a_2,b_1),(a_2,b_2),(a_2,b_3),(a_2,0)\}$ and $\{(0,b_1),(0,b_2),(0,b_3)\}$ such that

where $a_1,a_2 \in Z_m$such that $\overline{a_1a_2} \in E(P - VG(Z_m))$, $b_1,b_2,b_3 \in Z_n$such that $\overline{b_1b_2}$, $\overline{b_1b_3}$, $\overline{b_2b_3} \in E\left(P - VG(Z_n)\right)$.



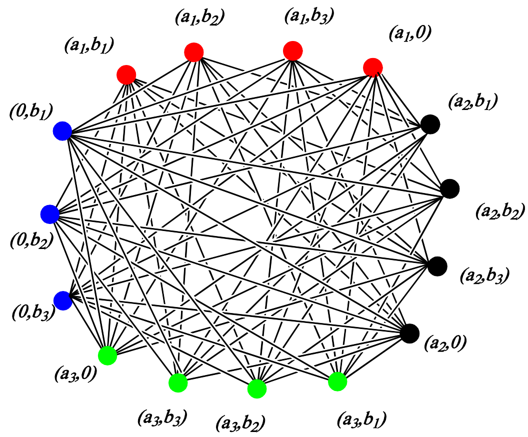
The vertices in the same set not adjacent to each other , then we have 3- partite graph and to colored it we need 3 colors , but all vertices are adjacent to vertex (0,0) then $\mathcal{X}$ (*P-VG(R)*) = 3+1=4

$\mathcal{X}$ (*P-VG*(Z_m)) + $\mathcal{X}$ (*P-VG*(Z_n)) $- 3 = 3 + 4 - 3 = 4 = \mathcal{X}$ (*P-VG(R)*).

Case2:

$\mathcal{X}$ (*P-VG*(Z_m)) and $\mathcal{X}$ (*P-VG*(Z_n)) are equal to 4 then we have four sets

$\{(a_1,b_1),(a_1,b_2),(a_1,b_3),(a_1,0)\}$,
$\{(a_2,b_1),(a_2,b_2),(a_2,b_3),(a_2,0)\}$,
$\{(a_3,b_1),(a_3,b_2),(a_3,b_3),(a_3,0)\}$,
$\{(0,b_1),(0,b_2),(0,b_3)\}$.



where $a_1,a_2,a_3 \in Z_m$ such that $\overline{a_1a_2},\overline{a_1a_3},\overline{a_2a_3} \in E(P - VG(Z_m))$, $b_1,b_2,b_3 \in Z_n$ such that $\overline{b_1b_2}$, $\overline{b_1b_3}$, $\overline{b_2b_2} \in E\left(P - VG(Z_n)\right)$.

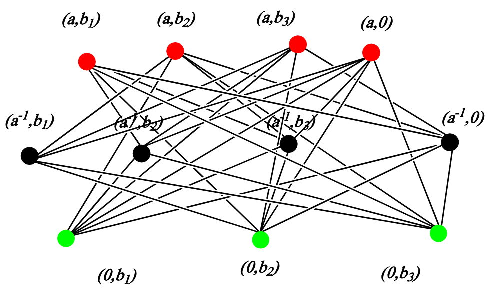
The vertices in the same set not adjacent to each other, then we have 4- partite graph and to colored it we need 4 colors , but all vertices are adjacent to vertex (0,0) then $\mathcal{X}$ (*P-VG(R)*) = 4+1=5.

$\mathcal{X}$ (*P-VG*(Z_m)) + $\mathcal{X}$ (*P-VG*(Z_n)) $- 3 = 4 + 4 - 3 = 5 = \mathcal{X}$ (*P-VG(R)*) .

2-ii:

Let m is prime and n not prime and $\mathcal{X}$ (*P-VG*(Z_n)) =4 then we have three sets

$\{(a,b_1),(a,b_2),(a,b_3),(a,0)\}$,
$\{(a^{-1},b_1),(a^{-1},b_2),(a^{-1},b_3),(a^{-1},0)\}$ and
$\{(0,b_1),(0,b_2),(0,b_3)\}$



where $a \in Z_m$ and $b_1, b_2, b_3 \in Z_n$ such that $\overline{b_1b_2}, \overline{b_1b_3}, \overline{b_2b_3} \in E(P - VG(Z_n))$, the vertices in the same set are not adjacent to each other , then we have 3- partite graph and to colored it we need 3 colors , but all vertices are adjacent to vertex (0,0) then $\mathcal{X}$ (*P-VG(R)*) = 3+1=4.

$\mathcal{X}$ (*P-VG*(Z_m)) + $\mathcal{X}$ (*P-VG*(Z_n)) $- 3 = 3 + 4 -$ 3=4 $=$ $\mathcal{X}$ (*P-VG(R)*) .

Example 3.10:

$R = Z_3 \times Z_5$

$\mathcal{X}$ (*P-VG(R)*) = $\mathcal{X}$ (*P-VG*(Z_3)) + $\mathcal{X}$ (*P-VG*(Z_5)) $- 2 = 2 + 3 - 2 = 3$

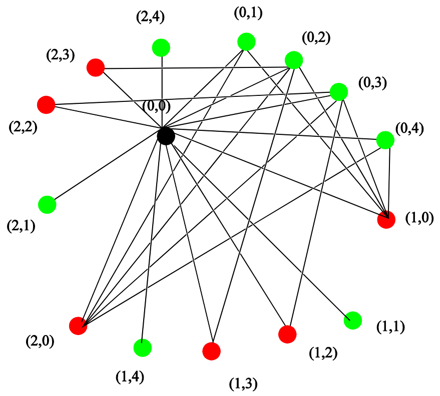


Fig. 7 : $\mathcal{X}$ **(** ***P-VG***$(Z_3 \times Z_5)$ **)**

Example 3.11:

$R = Z_6 \times Z_9$

$\mathcal{X}$ (*P-VG(R)*) = $\mathcal{X}$ (*P-VG*(Z_6)) + $\mathcal{X}$ (*P-VG*(Z_9)) $- 2 = 3 + 3 - 2 = 4$

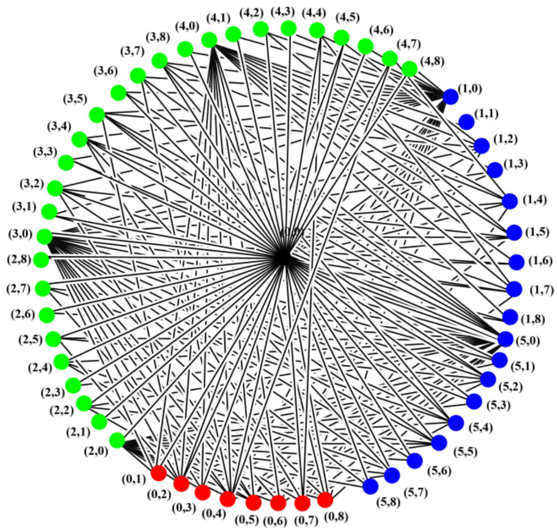


Fig. 8: $\mathcal{X}$ **(** ***P-VG***$(Z_6 \times Z_9)$ **)**

4- Conclusion

In this paper we gave a definition of Pseudo-Von Neumann regular graph of Cartesian product of commutative rings and found the relation between it and Cartesian product of Pseudo-Von Neumann regular graph of commutative ring. Also found the chromatic number $\mathcal{X}$ (*P-VG*(R)) where R is commutative ring, and R is Cartesian product of commutative ring.

References

[1] Beck.I, (1988).Coloring of commutative rings, *Journal of Algebra*.**116(1)** :208-226.

[2] Bhavanari .S, Prasad .Syam and Dasari .N, (2010).Prime Graph of a Ring, *Journal of Combinatorics, Information and System Sciences*, **35(1-2)**, 27-42.

[3] Bhavanari S., and Devanaboina S., (2015). Cartesian Product of Graphs VS Prime Graphs of Rings, *Global Journal of Pure and Applied Mathematics* , **11(2) ,** 199-205.

[4] Kalita.S, (2014). Some Aspects of Prime Graph of Some Rings, Phd , Univrsity of Gauhati, India.

[5] Patra .K, Kalita. S , (2014). Prime Graph of the Commutative Rings Z_n, *UTM center for Industrial and Applied Mathematics*, **30(1)** : 59-67.

[6] Ramos .R.E., (2015).Colorings of Zero-Divisor Graphs of Commutative Rings, M.Sc, North Dakota state University,

[7] AL-Hisso Sh.S , (2004). Types of Strongly Regular Rings, M.Sc, Mosul Univrsity, Mosul.

[8] Rahman .Md..S, (2017).Basic Graph Theoey, Springer, .

[9] Bhavanari. S, Syam. P, (2014). Discrete Mathematics and Graph Theory, PHL Learning Private Limited, Delhi.

[10] Gross, J.L. and Yellen J. (2006). Graph Theory and its Applications, second edition, Chapman & Francis Group, Boca Raton FL33487-2742,.

[11] R.M.R. Lewis, (2016). A Guide to Graph Colouring Algorthms and Applications, Springer.

[12] Aurenhammer, F.; Haganer, J.; Imrich, W.(1992), "Cartesian graph factorization at logarithmic cost per edge", *Computational complexity*, **2(4):**331-349

[13] Arif. N, Khalel. N, (2019). Chromatic number of pseudo-Von Neumann Regular , *Ibn AL-Haitham Journal for pure and Applied Science*. Accepted .

العدد اللوني و بعض خصائص البيان المنتظم الزائف للضرب الديكارتي للحلقات الابدالية

نرمين جمال خليل ، نبيل عز الدين عارف

قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

الملخص

لتكن R حلقة ابدالية, فأن البيان المنتظم الزائف للحلقة R يعرف على انه البيان الذي مجموعة رؤوسه هي جميع عناصر الحلقة R واي رأسين مختلفين a و b في Rيكونان متجاوران اذا وفقط اذا $b = b^2a$ او $a = a^2b$, نرمز لهذا البيان بالرمز *P-VG(R)* , في بحثنا حصلنا على بعض النتائج حول العدد اللوني للبيان المنتظم الزائف بالنسبة للضرب الديكارتي لبعض الحلقات الابدالية .