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Bi- Supra Hereditary and Bi – Supra Topological Properties for some Supra – Separation Axioms

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ABSTRACT

A new definitions of some Supra hereditary and Supra topological property was introduced namely (bi - Supra hereditary and bi – Supra Topological) properties , the relation between them was studied for some Supra – Separation Axioms

At last many characterization and theorem was proved .

1. Introduction

In 1963 Kelly [1] introduced the concept of bi – topological space which was called a set X equipped with two topologies a bi - topological space and is denoted by (X, τ_1, τ_2) where τ_1, τ_2 are two topologies defined on X.

In 1983 Mashhour ,M. S. Allam. A. A. Mohamoud f.s. and Khedr, F.H.[2] introduced the supra topological spaces

In 2011 Ravi, O., Pious, M.S and Salai, P.T. [3], introduced the concept A new type of homeomorphism in a bi -topological space .

In 2017 B. K. Mahmoud [4], introduced the concept supra – separation axioms for supra topological spaces.

In this paper some new definitions are introduced with some properties.via this definitions we generalization bi - Supra hereditary and bi – Supra Topological properties .

2. bi – Supra topological properties

In this section bi-[resp (i,j)] supra topological properties are studied and it`s characterization are presented and some applications.

Definition 2.1 :[5]

Let (X, μ_1, μ_2) be a bi- supra topological space, and Let A be a subset of X. Then A is said to be $(i, j) -$ supra open set if $A = G_1 \, U \, G_2$ where $G_1 \in \mu_1$ and $G_2 \in \mu_2$. The complement of (i, j)- supra open set is called $(i, j) -$ supra closed set, where $(i, j) = 1{,}2$ and i≠j.

Definition 2.2:[5]

Let (X, μ_1, μ_2) be a bi-supra topological space and let A be a subset of X. Then A is said to be bi-supra open set if $A = G_1 \cap G_2$ where $G_1 \in \mu_1$ and $G_2 \in \mu_2$. The complement of bi-supra open set is called bi-supra closed.

Proposition 2.3 :

Every bi- supra open set is $(i, j) -$ supra open sets and every $(i, j) -$supra closed set is bi-supra closed sets.

The proof of proposition directly from the definitions.

Definitions 2.4 :[5]

A function $f: (X, \mu_{1x}, \mu_{2x}) \rightarrow (Y, \mu_{1y}, \mu_{2y})$ is called:

1. $(i, j) -$ supra homeomorphism if f is a bijective and f, f^{-1} are $(i, j) -$ supra continuous functions.

2. $bi-$ supra homeomorphism if f is a bijective and f, f^{-1} are $bi-$ supra continuous functions.[5]

Definitions 2.5 :

1. If f is a bi- supra homeomorphism function (X,μ_{x1},μ_{x2}) in to (Y,μ_{y1},μ_{y2}), and p be any property in (X,μ_{x1},μ_{x2}), Then p is a $bi-$ supra topological property if $f(p)$ is the some property of p .

2. If f is a(i,j)- supra homeomorphism function (X,μ_{x1},μ_{x2}) in to (Y,μ_{y1},μ_{y2}), and p be any property in (X,μ_{x1},μ_{x2}), Then p is a (i,j)- supra topological property if $f(p)$ is the some property of p .

Note :That $bi-$ supra topological property is (i,j)- supra topological property.

The proof of note directly from the definitions.

Theorem 2.6:

A bi-supra topological space is a $(i,j)[resp\ bi]-$ supra-T_n –space, n =0,1,2

If f it satisfies the following axiom:

1-When n =0

$\forall\, x,y \in X, x \neq y, \exists\ (i,j)[resp\ bi]$ $-$supra open set G $\ni x \in G$ and $y \notin G$ or $x \notin G, y \in G$

2- When n =1

$\forall\, x,y \in X\ x \neq y, \exists\ two\ (i,j)[resp\ bi]$ $-$supra open sets G, H

$\ni\ x \in G, y \notin G\ and\ x \notin H, y \in H$

3-When n =2

$\forall\, x,y \in X, x \neq y, \exists$ two distinct $(i,j)[resp\ bi]$-supra open sets G, H

$\ni\ x \in G, y \in H\ \ G \cap H \neq \varphi$.

Theorem 2.7 :

$(i,j)[resp\ bi]$- supra $-T_0$ property is a supra topological property.

Proof:

Let $f:(X,\mu_1,\mu_2) \to (Y,\sigma_1,\sigma_2)$ be $(i,j)[resp\ bi]-$ supra homeomorphism

Let (X,μ_1,μ_2) be a $(i,j)[resp\ bi]$- supra $-T_0$,to prove that (Y,σ_1,σ_2) is $(i,j)[resp\ bi] - supra - T_0$

Let $x^*, y^* \in Y, x^* \neq y^*$ since f is on to

$\exists\, x,y \in X, x^* = f(x), y^* = f(y)$

Since f one to one and $f(x) \neq f(y) \ \to x \neq y$

(X,μ_1,μ_2) be a $(i,j)[resp\ bi]$- supra $-T_0$

$x,y \in X, x \neq y$ then

There exist $(i,j)[resp\ bi]$- supra open set G

Such that $x \in G\ and\ y \notin G$

Since $f\ is\ (i,j)[resp\ bi]$- supra open function $\to f(G) = G^*\ is\ a\ (i,j)[resp\ bi]$- supra open in (Y,σ_1,σ_2)

$x \in G \to f(x) \in f(G) \to x^* \in G^*$

$y \notin G \to f(y) \notin f(G) \to y^* \notin G^*$

There exist $(i,j)[resp\ bi]$- supra open G^*

Such that $x^* \in G^*\ and\ y^* \notin G^*$

so (Y,σ_1,σ_2) is $(i,j)[resp\ bi]$- supra-T_0- space .

Theorem 2.8 :

$(i,j)[resp\ bi] -$ supra-T_1 property is a supra topological property.

Proof:

Let $f:(X,\mu_1,\mu_2) \to (Y,\sigma_1,\sigma_2)$ be $(i,j)[resp\ bi]$-supra homeomorphism

Let (X,μ_1,μ_2) be a $(i,j)[resp\ bi] -$ supra T_1 – space

To prove that $(Y,\sigma_1,\sigma_2)\ is\ (i,j)[resp\ bi] -$ supra-T_1– space

Since $x^*, y^* \in Y,\ x^* \neq y^*$ since f is on to

$x^* = f(x), y^* = f(y),$

$\exists\, x,y \in X,\ f(x) \neq f(y) weget\ x \neq y$

However $f\ is\ one\ to\ one\ and$ Since (X,μ_1,μ_2) is $(i,j)[resp\ bi] -$ supra- T_1 -space

then there exist $(i,j)[resp\ bi] -$ supra open set G, H for every $x,y \in X, x \neq y$

Such that $x \in G\ and\ y \notin G\ and\ x \notin H, y \in H$

Since $f\ is\ (i,j)[resp\ bi] -$ supra open function $\to f(G) = G^*, f(H) = H^*$ is a $(i,j)[resp\ bi]$-supra open in (Y,σ_1,σ_2)

$x \in G \to f(x) \in f(G) \to x^* \in G^*$

$y \notin G \to f(y) \notin f(G) \to y^* \notin G^*$

$x \notin H \to f(x) \notin f(H) \to x^* \notin H^*$

$y \in H \to f(y) \in f(H) \to y^* \in H*$

$\therefore\ \exists\, G^*, H^*\ are\ (i,j)[resp\ bi] -$ supra open sets

$x^* \in G^*, y^* \notin G^*\ and\ x^* \notin H^*, y^* \in H^*\ so\ (Y,\sigma_1,\sigma_2)\ is\ (i,j)[resp\ bi] -$ supra- T_1 –space.

Theorem 2.9 :

$(i,j)[resp\ bi] -$ supra-T_2 property is a supra topological property

Proof:

(X,μ_1,μ_2) is a $(i,j)[resp\ bi]$- supra T_2 – space

Let $f:(X,\mu_1,\mu_2) \to (Y,\sigma_1,\sigma_2)$ is a $(i,j)[resp\ bi] -$ supra homeomorphism

to prove that (Y,σ_1,σ_2) is $(i,j)[resp\ bi]$- supra-T_2–space .

Let $x^*, y^* \in Y, x^* \neq y^*$

since f is on to

$\exists x,y \in X$ such that $x^* = f(x), y^* = f(y)$.Since f is one to one

$\to f(x) \neq f(y) \ \ \to x \neq y$

Since $(X,\mu_1,\mu_2)\ is\ (i,j)[resp\ bi] -$ supra- T_2 -space $\forall x,y \in X, x \neq y$

$\exists$ two disjoint $(i,j)[resp\ bi] -$ supra open set G, H

Such that $x \in G, y \in H, G \cap H = \varphi$

Since $f\ is\ (i,j)[resp\ bi] -$ supra open function$\to$ $f(G) = G^*,\ f(H) = H^*$

are $(i,j)[resp.bi]$- supra open sets

$G \cap H = \varphi \to f(G) \cap f(H) = \varphi$

$x \in G \to x^* \in f(G) \quad x^* \in G^*$

$y \in H \to y^* \in f(H) \quad y^* \in H^* \quad (Y,\sigma_1,\sigma_2)$ is $(i,j)[resp\ bi]$- supra- T_2 –space.

3. *bi* – supra hereditary property

In this section bi [resp (i,j)] - supra hereditary properties are studied and

It`s characterization are presented and some applications .

Definitions 3.1 :

Let (X,μ_1,μ_2) be a bi-supra topological space, and let A be a subset of X. Then:

1. The subspace (i,j) $-$supra topology on A determined by $(i,j)_A -$ supra open is intersection

(i,j) - supra open with A as $\{A \cap u : u \in (i,j) - supra\ open\}$.

2. The subspace bi- supra topology on A determined by bi_A –supra open is intersection $bi - supra\ open$ with $A\ as$ $\{A \cap u : u \in bi - supra\ open\}$.

Example 3.2 :

Let $X = \{a,b,c,d\}$

$\mu_1 = \{\varphi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$

$\mu_2 = \{\varphi, X, \{a\}, \{b\}, \{a,b\}, \{a,b,d\}, \{a,b,c\}\}$

$(i,j) -$ supra open set

$= \{\varphi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$

$bi -$ supra open set

$= \{\varphi, X, \{a\}, \{a,b\}, \{b\}, \{a,b,d\}, \{a,b,c\}\}$

Let $A = \{a,c\}$

$(i,j)_A$-supra topology $= \{\varphi, A, \{a\}, \{c\}\}$

$bi_A -$ supra topology $= \{\varphi, A, \{a\}\}$.

Definitions 3.3 :

1.If (Y, μ_{y1}, μ_{y2}) sub space of (X, μ_{x1}, μ_{x2}), and p any property in (X, μ_{x1}, μ_{x2}), then p is called bi [resp (i,j)] – supra hereditary property where p is the same property in (Y, μ_{y1}, μ_{y2}).

Theorem 3.4 :

$(i,j)[resp\ bi] -$ supra-T_0 property is a supra hereditary property

Proof:

Let (X, μ_1, μ_2) be a bi-supra topological space and (X, μ_1, μ_2) is a $(i,j)[resp\ bi] -$ supra -T_0- space

Let (Y, σ_1, σ_2) be a bi-supra topological subspace of (X, μ_1, μ_2)

To prove that (Y, σ_1, σ_2) is $(i,j)[resp\ bi]$- supra-T_0 –space

Let $x, y \in Y, x \neq y$

Since (X, μ_1, μ_2) is a $(i,j)[resp\ bi]$- supra-T_0- space

Then there exist $(i,j)[resp\ bi]$- supra open set G . Such that $x \in G\ and\ y \notin G$,

$(i,j)_Y[resp\ bi_Y]$- supra topology $=\{G^* = Y \cap G : G \in (i,j)[resp\ bi] - supra\ open\}$

$x \in G, x \in Y \to x \in G \cap Y \to x \in G^*$

$is\ (i,j)_Y[resp\ bi_Y] - supra\ open$

$y \notin G, y \notin Y \to y \notin G \cap Y \to y \notin G^*$

there exist $((i,j)_Y[resp\ bi_Y]$- supra open , Such that $x \in G^*, y \notin G^*$

$\forall x, y \in Y, x \neq y$

$so\ (Y, \sigma_1, \sigma_2)\ is\ (i,j)[resp\ bi]$- supra-$T_0$ –space

Theorem 3.5 :

$(i,j)[resp\ bi]$ –supra-T_1 property is a supra hereditary property

Proof:

Let (X, μ_1, μ_2) be a $(i,j)[resp\ bi]$- supra-T_1- space

Therefore (Y, σ_1, σ_2) be a $(i,j)_A[resp\ bi_A]$- supra topology sub space of (X, μ_1, μ_2)

To prove that (Y, σ_1, σ_2) is $(i,j)[resp\ bi]$- supra-T_1 -space

Let $x, y \in Y, x \neq y$

Since (X, μ_1, μ_2) is a $(i,j)[resp\ bi]$- supra-T_1- space

Then there exist two $(i,j)[resp\ bi] -$ supra open set G, H

Such that $x \in G\ and\ y \notin G, x \notin H, y \in H$,

Since $Y \subseteq X$

$(i,j)_Y[resp\ bi_Y] - supra\ open = \{G \cap Y : G\ is (i,j)[resp\ bi] - supra\ open\}$

Let $G^*, H^*\ are\ (i,j)_Y[resp\ bi_Y]$- supra open set

$\therefore x \in G, x \in Y \to x \in G \cap Y \to x \in G^*$

$y \notin G, x \notin Y \to x \notin G \cap Y \to x \notin G^*$

$x \notin H, x \notin Y \to x \notin H \cap Y \to x \notin H^*$

$y \in H, y \in Y \to y \in H \cap Y \to y \in H^*$

There exist two $(i,j)_Y[resp\ bi_Y]$- supra open set G^*, H^*

Such that $x \in G^*, y \notin G^* and\ x \notin H^*, y \in H^*, \forall x, y \in Y, x \neq y$

(Y, σ_1, σ_2) is $(i,j)[resp\ bi]$- supra-T_1 –space.

Theorem 3.6 :

$(i,j)[resp\ bi]$ –supra-T_2 property is a supra hereditary property

Proof:

(X, μ_1, μ_2) be a $(i,j)[resp\ bi]$- supra-T_2 space and (Y, σ_1, σ_2) be a $(i,j)_Y[resp\ bi_Y]$- supra topology sub space of (X, μ_1, μ_2)

to prove that (Y, σ_1, σ_2) is $(i,j)[resp\ bi] -$ supra-T_2 -space

Let $x, y \in Y, x \neq y$, but $Y \subseteq X$

Implies that $x, y \in X, x \neq y$

Since (X, μ_1, μ_2) is a $(i,j)[resp\ bi]$- supra-T_2–space

There exist two $disjoint\ (i,j)[resp\ bi] -$ supra open sets

G, H such that $x \in G, y \in H, G \cap H = \emptyset$

Since $Y \subseteq X$

$(i,j)_Y[resp\ bi_Y] -$ supra open

$=\{\mu \cap Y : \mu\ is\ (i,j)[resp\ bi]$- supra open$\}$

We have $G \subseteq X \to G^* = G \cap Y$ is $(i,j)_Y\ [resp\ bi_Y]$ - supra open

$H \subseteq X \to H^* = H \cap Y$ $is\ (i,j)_Y\ [resp\ bi_Y]$ -supra open

Since $x \in G \cap Y \to x \in G^*$

$y \in H \cap Y \to y \in H^*$

$G^* \cap Y^* = (G \cap Y) \cap (H \cap Y)$

$(G \cap H) \cap Y = \varphi \cap Y = \varphi$

$\therefore G^* \cap Y^* = \varphi$

Hence for all $x, y \in X, x \neq y$

$\exists$ two disjoint $(i,j)[resp\ bi]$ - supra open sets G^*, H^* such that $x \in G^*, y \in H^*, G^* \cap H^* = \varphi$

$(Y, \sigma_1, \sigma_2)\ is\ a\ (i,j)[resp\ bi] -$ supra T_2 – space.

4. Some Proposition and Example

Proposition 4.1 :

1-Every $bi -$ supra T_0- space is $an\ (i,j)$ –supra T_0– space

Proof : Let (X, μ_1, μ_2) be a bi- supra topological spaces

And let (X, μ_1, μ_2) be a bi- supra T_0- space And let $x, y \in X, x \neq y$

Then there exist bi- supra open set $G \subseteq X$

Such that $x \in G\ and\ y \notin G$

Since every $bi-$ supra open set is an $(i,j)-$ supra open set
Then $G \subseteq X$ is $(i,j)-$ supra open set such that $x \in G$ and $y \notin G$
Hence (X,μ_1,μ_2) is (i,j)- supra- T_0-space.
2-Every bi-supra T_1- space is an (i,j)- supraT_1–space
Proof :
Let $(X,\mu_1,\mu_2$) be a $bi-$ supra T_1- space
and let $x,y \in X$, $x \neq y$
Then there exist two $bi-$ supra open set G,H
Such that $x \in G$ and $y \notin G$ and $x \notin H, y \in H$
Since every $bi-$Supra open set is a $(i,j)-$ supra open set
Then $G,H \subseteq X$ are two $(i,j)-$ supra open sets such that $x \in G$ and $y \notin G$ and $x \notin H, y \in H$
Hence (X,μ_1,μ_2) is $(i,j)-$ supra- T_1-space.
3-Every bi-supra T_2- space is a (i,j)- supra T_2–space
Proof :
Let (X,μ_1,μ_2) be a $bi-$ supra T_2- space
And let $x,y \in X, x \neq y$
Then there exist two bi- supra open set $G,H \subseteq X$
Such that $x \in G, y \in H, G \cap H = \varphi$
Since every $bi-$ supra open set is a $(i,j)-$ supra open set
Then $G,H \subseteq X$ are two $(i,j)-$ supra open sets such that $x \in G, y \in H$, $G \cap H = \varphi$
Hence (X,μ_1,μ_2) is (i,j)- supra T_2-space.

Proposition 4.2:
1- Every $(i,j)[resp\ bi]$ $-$supra T_2 $-$space is (i,j) $[resp\ bi]$- supra T_1-space.
Proof : Let $(X,\mu_1,\mu_2$) be a bi- supra topological spaces.
And let $(X,\mu_1,\mu_2$) be a $(i,j)[resp\ bi]$- supraT_2–space
Assume that $x,y \in X, x \neq y$
Then there exist two $(i,j)[resp\ bi]$ $-$ supra open sets $G,H \subseteq X$
Such that $x \in G, y \in H, G \cap H = \varphi$.
Since $G \cap H = \varphi$ that is means $x \in G$ and $x \notin H, y \notin G, y \in H$.
Hence (X,μ_1,μ_2) is $(i,j)[resp\ bi]$ - supra T_1-space.
2-Every $(i,j)[resp\ bi]$- supraT_2–space is a (i,j) $[resp\ bi]$ $-$ supra T_0-space.
Proof :
Let (X,μ_1,μ_2) be a $(i,j)[resp\ bi]$ $-$ supraT_2 –space
and let $x,y \in X, x \neq y$
Then there exist two $(i,j)[resp\ bi]$- supra open sets $G,H \subseteq X$
Such that $x \in G, y \in H, G \cap H = \varphi$.
Since $G \cap H = \varphi$ that is means $x \in G$ and $x \notin H, y \notin G, y \in H$.
Then there exist a $(i,j)[resp\ bi]$ $-$ supra open set $G \subseteq X$
Such that $x \in G, y \notin G$
Hence $(X,\mu_1,\mu_2$) is $(i,j)[resp\ bi]$ - supra T_0-space.
3-Every $(i,j)[resp\ bi]$ $-$ supraT_1–space is a (i,j) $[resp\ bi]$- supra T_0-space.
Proof :
Let (X,μ_1,μ_2) be a $(i,j)[resp\ bi]$ $-$ supra T_1 –space
Assume that $x,y \in X, x \neq y$
Then there exist two $(i,j)[resp\ bi]$- supra open sets $G,H \subseteq X$
Such that $x \in G$ and $x \notin H, y \notin G, y \in H$
That is mean there exist a $(i,j)[resp\ bi]$ $-$ supra open set $G \subseteq X$
Such that $x \in G, y \notin G$
Hence $(X,\mu_1,\mu_2$) is $(i,j)[resp\ bi]$ - supra T_0-space.
The converse of the propositions
Is not necessarily true as the following example :
To show that (i,j) - supra T_0-space not implies bi-supra T_0-space

Example 4.4 :
Let $X = \{1,2\}$
μ_1=$\{\varphi,X,\{1\}\}$,
$\mu_2 = \{\varphi, X, \{1\}, \{2\}\}$
$(i,j) -$ $supra$ $open$ set $= \{\varphi, X, \{1\}, \{2\}\}$
$bi -$ $supra$ $open$ $set\{\varphi, X, \{1\}\}$
Hence $(X,\mu_1,\mu_2$) is $(i,j)-$ supra T_0-space but not bi - supra T_0-space.
To show that (i,j) - supra T_1-space not implies $bi-$ supra T_1-space

Example 4.5 :
Let$X = \{1,2,3,4\}$
$\mu_1 = \{\varphi, X, \{1\}, \{3\}, \{2\}, \{1,2\}, \{1,4\}, \{1,3\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{1,2,4\}, \{2,3,4\}\}$
μ_2=$\{\varphi, X, \{1\}, \{4\}, \{3\}, \{1,3\}, \{1,4\}, \{3,4\}, \{1,3,4\}, \{1,2,4\}, \{2,3,4\}\}$.
$(i,j)-$ supra open set $= \{\varphi, X, \{1\}, \{3\}, \{1,3\}, \{1,2\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\}$.
$bi-$ supra open set = $\{\varphi, X, \{1\}, \{3\}, \{1,3\}, \{3,4\}, \{1,4\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\}$
Hence $(X,\mu_1,\mu_2$) is $(i,j)-$ supra T_1-space but not bi - supra T_1-space.
To show that (i,j) $[resp\ bi]$ $-$ supra T_1-space not implies $(i,j)[resp\ bi]$ $-$ supra T_2-space

Example 4.6 :
Let $X = \{1,2,3,4\}$
μ_1=$\{\varphi, X, \{3\}, \{4\}, \{3,4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,3,4\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}\}$
μ_2=$\{\varphi, X, \{1\}, \{2\}, \{3\}, \{2,3\}, \{1,2\}, \{1,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}\}$.
(i,j)-supra open set $= \{\varphi, X, \{1\}, \{3\}, \{3,4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}, \{1,3,4\}\}$
bi-supra open set $= \{\varphi, X, \{1\}, \{3\}, \{1,3\}, \{2,3\}, \{1,3,4\}, \{1,2,3\}, \{1,2,4\}\}$.
Hence $(X,\mu_1,\mu_2$) is (i,j) $[resp\ bi]$ $-$ supra T_1-space but not $(i,j)[resp\ bi]$ $-$ supra T_2-space.
To show that (i,j) $[resp\ bi]$ - supra T_0space not implies bi – supra(resp. T_1) T_2-space

Example 4.7 :
Let $X = \{1,2,3\}$
$\mu_1 = \{\varphi, X, \{1\}\}$
$\mu_2 = \{\varphi, X, \{1\}\{1,2\}\}$
(i,j) - supra open set = $\{\varphi, X, \{1\}, \{1,2\}\}$
$bi-$ supra open set $= \{\varphi, X, \{1\}\}$

Hence (X, μ_1, μ_2) is (i,j) $[resp\ bi]$ - supraT_0-space but not bi – supra(resp. T_1) T_2-space

To show that (i,j) $-$supra T_2-space not implies bi - supra T_2-space

Example 4.8:

Let $X = \{1,2,3,4\}$

$\mu_1 = \{\varphi, X, \{2\}, \{3\}, \{2,3\}, \{1,4\}, \{1,2,4\}, \{1,3,4\}\}$

μ_2=$\{\varphi, X, \{1\}, \{4\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{2,3,4\}\}$.

$(i,j) -$ supra open set=$\{\varphi, X, \{2\}, \{3\}, \{2,3\}, \{1,4\}, \{1,2,4\}, \{1,3,4\}, \{1,2\}, \{2,4\}, \{1,2,3\}, \{2,4,3\}, \{1,3\}, \{3,4\}\}$.

$bi -$ supra open set= $\{\varphi, X, \{1,4\}, \{2,3\}\}$

Hence (X, μ_1, μ_2) is (i,j) $-$supra T_2-space but not bi - supra T_2-space.

Conclusions.

some new result

Proposition 4.1:

1-Every $bi -$ supra T_0- space is an (i,j) $-$supra T_0-space.

2-Every bi-supra T_1- space is an (i,j)- supraT_1-space.

3-Every bi-supra T_2- space is an (i,j)- supraT_2-space.

Proposition 4.2:

1-Every $(i,j)[resp\ bi] -$ supra T_2 $-$space is an (i,j) $[resp\ bi]$- supra T_1-space.

2-Every $(i,j)[resp\ bi]$- supraT_2-space is an (i,j) $[resp\ bi] -$ supra T_0-space.

3-Every $(i,j)[resp\ bi] -$ supra T_1-space is an (i,j) $[resp\ bi]$- supra T_0-space.

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الصفات الوراثية والتبولوجيا الثنائية الفوقية لبعض بديهيات الفصل

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الملخص

تعريفات جديدة لبعض الصفات الوراثية و التبولوجيا الفوقية قد قدمت وسميت بالصفات الوراثية والتبولوجيا الثنائية الفوقية ودرست العلاقات فيما بينها اخيرا الكثير من الخصائص والمبرهنات قد برهنت.