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Geometry of concircular curvature tensor of Kahler manifolds

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ABSTRACT

The study deals with the necessary condition where a nearly Kahler manifold of flat concircular tensor has been found. And the relationship between these invariants and additional properties of symmetry concircular tensor, as well as geometrical meaning of the reference in zero of these invariants .

Introduction

Let $\mathcal{M}$ - smooth manifold of dimension $2n$; $C^\infty(\mathcal{M})$-algebra of smooth functions on $\mathcal{M}$; $\mathbb{X}(\mathcal{M})$the module of smooth vector fields on manifold of $\mathcal{M}$; $g = <.,.>$ - Riemannian metrics; ∇ – Riemannian connection of the metrics g on $\mathcal{M}$; d - the operator of exterior differentiation. In the further all manifold, Tensor fields, etc. objects are assumed smooth a class C^∞.

So Almost Hermition (is shorter, AH) structure on a manifold $\mathcal{M}$ the pair (J, g) ,where J-almost complex structure

$(J^2 = id)$ on $\mathcal{M}, g =<.,.>$

$-(pseudo)$ $Riemannian$ $metric$ on $\mathcal{M}$. In $this$ $case$ $< J\mathbb{X}, J\mathbb{Y} > =< \mathbb{X}, \mathbb{Y} >; \mathbb{X}, \mathbb{Y} \in X(\mathcal{M})$.

Endomorphism J is called structural endomorphism .Manifold which is fixed almost manifold $\mathcal{M}$ equivalently to the task of G-structure above $\mathcal{M}$ with structural U(n) ,the elements of space which[1].

Definition 1: **[2]**

"Almost Hermition structure (J, g) on manifold of $\mathcal{M}$ is called Kahler (more shortly ,K-) structure if $\mathcal{M}$ the identity is carried out (if $\mathcal{M}$ satisfies the identity) $\nabla_{\mathbb{X}}(J)\mathbb{Y} + \nabla_{Y}(J)\mathbb{X} = 0 ; \mathbb{X}, \mathbb{Y} \in \mathbb{X}(\mathcal{M})$".

Proposition2:[3]

Let $(\mathcal{M}\ J, g)$ - AH- manifold.

Invariant concircular transformations metric is tensor C type (3, 1) determined by the formula

C($\mathbb{X}$,$\mathbb{Y}$)$\mathbb{Z}$ =R($\mathbb{X}$,$\mathbb{Y}$)$\mathbb{Z}$-$\frac{K}{n(n-1)}\{\langle\mathbb{X},\mathbb{Z}\rangle\mathbb{Y} - \langle\mathbb{Y},\mathbb{Z}\rangle\mathbb{X}\}$.... (1)

And called concircular curvature tensor was introduced will be Reminded Yano [8]on n-dimensional Riemannian manifold

Where R–the Riemann curvature tensor, g $=\langle.,.\rangle$ is the Riemannian metric and k-is the scalar curvature $\mathbb{X}, \mathbb{Y},\mathbb{Z},\mathbb{W} \in X(\mathcal{M})$)

where $\mathbb{X}(\mathcal{M})$ is the Lie algebra of C^∞vector fields on $\mathcal{M}$.

This tensor is invariant under concircular transformations,

i.e. with conformal transformations of space keeping a harmony of functions.

Definition 3:[4]

"A concircular curvature tensor on AH-manifold $\mathcal{M}$ is a tensor of type (4,0) and satisfied the relation $e^{-2f}\bar{C}(\mathbb{X},\mathbb{Y},\mathbb{Z},\mathbb{W})$= C($\mathbb{X}$,$\mathbb{Y}$,$\mathbb{Z}$,$\mathbb{W}$),which is defined as the form: C($\mathbb{X}$,$\mathbb{Y}$,$\mathbb{Z}$,$\mathbb{W}$) = R($\mathbb{X}$,$\mathbb{Y}$,$\mathbb{W}$,$\mathbb{Z}$) - $\frac{k}{n(n-1)}\{g(\mathbb{X},\mathbb{W})g(\mathbb{Y},\mathbb{Z}) - g(\mathbb{X},\mathbb{Z})g(\mathbb{Y},\mathbb{W})\}$ ...(2)

Where R is the Riemannian curvature tensor ,g is the Riemannian metric and k-is the scalar curvature $\mathbb{X},\mathbb{Y},\mathbb{Z},\mathbb{W} \in \mathbb{X}(\mathcal{M})$ .Where $\mathbb{X}(\mathcal{M})$ is the Lie algebra of C^∞vector fields on $\mathcal{M}$.

Let's consider properties tensor concircular curvature".

Remark 4: **[3]**

Thus concircular curvature tensor satisfies all the properties of algebraic curvature tensor:

1) C ($\mathbb{X}$,$\mathbb{Y}$,$\mathbb{Z}$,$\mathbb{W}$)= – C($\mathbb{Y}$, $\mathbb{X}$, $\mathbb{Z}$,$\mathbb{W}$) ;

2) C($\mathbb{X}$,$\mathbb{Y}$,$\mathbb{Z}$,$\mathbb{W}$)= - C($\mathbb{X}$,$\mathbb{Y}$,$\mathbb{Z}$,$\mathbb{W}$) ;

3) C ($\mathbb{X}$,$\mathbb{Y}$,$\mathbb{Z}$,$\mathbb{W}$) + C($\mathbb{Y}$, $\mathbb{Z}$, $\mathbb{X}$, $\mathbb{W}$) + C($\mathbb{Z}$ $\mathbb{X}$, $\mathbb{Y}$, $\mathbb{W}$) = 0 ;

4) C ($\mathbb{X}$,$\mathbb{Y}$,$\mathbb{Z}$,$\mathbb{W}$)=C($\mathbb{Z}$,$\mathbb{W}$,$\mathbb{X}$,$\mathbb{Y}$) ; $\mathbb{X}$,$\mathbb{Y}$,$\mathbb{Z}$,$\mathbb{W}$ $\in \mathbb{X}(\mathcal{M})$. ..... *(4)*

1) C $(\mathbb{X},\mathbb{Y},\mathbb{Z},\mathbb{W})$ =R $(\mathbb{X},\mathbb{Y},\mathbb{Z},\mathbb{W})$-$\frac{1}{2(n-1)}$ $[g(\mathbb{X},\mathbb{W})S(\mathbb{Y},\mathbb{Z})-g(\mathbb{X},\mathbb{Z})S(\mathbb{Y},\mathbb{W})]$- $-\frac{1}{2(n-1)}[g(\mathbb{Y},\mathbb{Z})S(\mathbb{X},\mathbb{W})-g(\mathbb{Y},\mathbb{W})S(\mathbb{X},\mathbb{Z})]$=R$(\mathbb{X},\mathbb{Y},\mathbb{Z},\mathbb{W})$+ $+\frac{1}{2(n-1)}[-g(\mathbb{X},\mathbb{W})S(\mathbb{Y},\mathbb{Z})+g(\mathbb{X},\mathbb{Z})+S(\mathbb{Y},\mathbb{W})]$+ $+\frac{1}{2(n-1)}[-g(\mathbb{Y},\mathbb{Z})S(\mathbb{X},\mathbb{W})+g(\mathbb{Y},\mathbb{W})S](\mathbb{X},\mathbb{Z})=-C(\mathbb{Y},\mathbb{X},\mathbb{Z},\mathbb{W})$

Properties are similarly proved :

2) C$(\mathbb{X},\mathbb{Y},\mathbb{Z},\mathbb{W})$ =-C$(\mathbb{X},\mathbb{Y},\mathbb{W},\mathbb{Z})$;

3) C$(\mathbb{X},\mathbb{Y},\mathbb{Z},\mathbb{W})$ =-C$(\mathbb{Z},\mathbb{W},\mathbb{X}.\mathbb{Y})$;

4) C$(\mathbb{X},\mathbb{Y},\mathbb{Z},\mathbb{W})$ +C$(\mathbb{Y},\mathbb{Z},\mathbb{X},\mathbb{W})$+C$(\mathbb{Z},\mathbb{X},\mathbb{Y},\mathbb{W})$ = 0.

Covarient –tensor concircular curvatare C type (3,1) have form

C$(\mathbb{X},\mathbb{X})\mathbb{Z}$ =R$(\mathbb{X},\mathbb{Y})\mathbb{Z}$-$\frac{K}{n(n-1)}\{<\mathbb{X},\mathbb{Z}>\mathbb{Y}-<\mathbb{Y},\mathbb{Z}>\mathbb{X}\}$ ………..(5)

Where R-is the Riemannian curvature tensor and k-is the scalar curvature, $\mathbb{X},\mathbb{Y},\mathbb{Z}\in\mathbb{X}(\mathcal{M})$

By definition of a spectrum tensor.

C$(\mathbb{X},\mathbb{Y})\mathbb{Z}$=$C_0(\mathbb{X},\mathbb{Y})\mathbb{Z}$+$C_1(\mathbb{X},\mathbb{Y})\mathbb{Z}$+ $C_2(\mathbb{X},\mathbb{Y})\mathbb{Z}$+ $C_3(\mathbb{X},\mathbb{Y})\mathbb{Z}$+$C_4(\mathbb{X},\mathbb{Y})\mathbb{Z}$+$C_5(\mathbb{X},\mathbb{Y})\mathbb{Z}$+$C_6(\mathbb{X},\mathbb{Y})\mathbb{Z}$+ $C_7(\mathbb{X},\mathbb{Y})\mathbb{Z}$; $\mathbb{X},\mathbb{Y},\mathbb{Z}\in\mathbb{X}(\mathcal{M})$

tensor $C_0(\mathbb{X},\mathbb{Y})\mathbb{Z}$ as nonzero. The component can have only components of the form $\{C_0^{a}{}_{bcd},C_0^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}}\}=\{C^{a}{}_{bcd},C^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}}\}$;

tensor $C_1(\mathbb{X},\mathbb{Y})\mathbb{Z}$ - components of the form $\{C_1^{a}{}_{bc\hat{d}},C_1^{\hat{a}}{}_{\hat{b}\hat{c}d}\}=\{C^{a}{}_{bc\hat{d}},C^{\hat{a}}{}_{\hat{b}\hat{c}d}\}$;

tensor $C_2(\mathbb{X},\mathbb{Y})\mathbb{Z}$ - components of the form $\{C_2^{a}{}_{b\hat{c}d},C_2^{\hat{a}}{}_{\hat{b}c\hat{d}}\}=\{C^{a}{}_{b\hat{c}d},C^{\hat{a}}{}_{\hat{b}c\hat{d}}\}$;

tensor $C_3(\mathbb{X},\mathbb{Y})\mathbb{Z}$ - components of the form $\{C_3^{a}{}_{b\hat{c}\hat{d}},C_3^{\hat{a}}{}_{\hat{b}cd}\}=\{C^{a}{}_{b\hat{c}\hat{d}},C^{\hat{a}}{}_{\hat{b}cd}\}$;

tensor $C_4(\mathbb{X},\mathbb{Y})\mathbb{Z}$ - components of the form$\{C_4^{a}{}_{\hat{b}cd},C_4^{\hat{a}}{}_{b\hat{c}\hat{d}}\}=\{C^{a}{}_{\hat{b}cd},C^{\hat{a}}{}_{b\hat{c}\hat{d}}\}$;

tensor $C_5(\mathbb{X},\mathbb{Y})\mathbb{Z}$ - components of the form $\{C_5^{a}{}_{\hat{b}c\hat{d}},C_5^{\hat{a}}{}_{b\hat{c}d}\}=\{C^{a}{}_{\hat{b}c\hat{d}},C^{\hat{a}}{}_{b\hat{c}d}\}$;

tensor $C_6(\mathbb{X},\mathbb{Y})\mathbb{Z}$- components of the form $\{C_6^{a}{}_{\hat{b}\hat{c}d},C_6^{\hat{a}}{}_{bc\hat{d}}\}=\{C_6^{a}{}_{\hat{b}\hat{c}d},C_6^{\hat{a}}{}_{bc\hat{d}}\}$;

tensor $C_7(\mathbb{X},\mathbb{Y})\mathbb{Z}$- components of the form$\{C_7^{a}{}_{\hat{b}\hat{c}\hat{d}},C_7^{\hat{a}}{}_{bcd}\}=\{C^{a}{}_{\hat{b}\hat{b}\hat{c}\hat{d}},C^{\hat{a}}{}_{bcd}\}$.

Tensors $C_0=C_0(\mathbb{X},\mathbb{Y})\mathbb{Z}$, $C_1=C_1(\mathbb{X},\mathbb{Y})\mathbb{Z}$, …, $C_7=C_7(\mathbb{X},\mathbb{Y})\mathbb{Z}$.

The basic invariants concircular *AH*-manifold will be named.

Definition 5:

AH - manifold for which C_i=0, *AH*- manifold of class C_i, i = 0, 1, …, 7.

Theorem6:

1) *AH*- manifold of class C_0 characterized by identity

$C(\mathbb{X},\mathbb{Y})\mathbb{Z}$ $-C(\mathbb{X},J\mathbb{Y})J\mathbb{Z}$$-C(J\mathbb{X},\mathbb{Y})J\mathbb{Z}$$-C(J\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $-JC(\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $-JC(\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $-JC(J\mathbb{X},\mathbb{Y})\mathbb{Z}$ $+JC(J\mathbb{X},J\mathbb{Y})J\mathbb{Z}=0$, $\mathbb{X},\mathbb{Y},\mathbb{Z}\in\mathbb{X}(\mathcal{M})$. ………. (7)

2) *AH* - manifold of class C_1 characterized by identity

3) $C(\mathbb{X},\mathbb{Y})\mathbb{Z}$ $+C(\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ $-C(J\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $+C(J\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $+JC(\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $-JC(\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $-JC(J\mathbb{X},\mathbb{Y})\mathbb{Z}-JC(J\mathbb{X},J\mathbb{Y})J\mathbb{Z}=0$, $\mathbb{X},\mathbb{Y},\mathbb{Z}\in\mathbb{X}(\mathcal{M})$. ………. (8)

4) *AH* - manifold of class C_2 characterized by identity

$C(\mathbb{X},\mathbb{Y})\mathbb{Z}$ $-C(\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ $+C(J\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $+C(J\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $-JC(\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $-JC(\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $+JC(J\mathbb{X},\mathbb{Y})\mathbb{Z}$ $-JC(J\mathbb{X},J\mathbb{Y})J\mathbb{Z}=0$, $\mathbb{X},\mathbb{Y},\mathbb{Z}\in\mathbb{X}(\mathcal{M})$…. (9)

5) AH - manifold of class C_3 characterized by identity

$C(\mathbb{X},\mathbb{Y})\mathbb{Z}$ $+C(\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ $+C(J\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $-C(J\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $-JC(\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $+JC(\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $+JC(J\mathbb{X},\mathbb{Y})\mathbb{Z}$ $+JC(J\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ =0; $\mathbb{X},\mathbb{Y},\mathbb{Z}\in\mathbb{X}(\mathcal{M})$……. (10)

6) *AH*- manifold of class C_4 characterized by identity

$C(\mathbb{X},\mathbb{Y})\mathbb{Z}$ $+C(\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ $+C(J\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $-C(J\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $+JC(\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $-JC(\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $-JC(J\mathbb{X},\mathbb{Y})\mathbb{Z}$ $-JC(J\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ =0; $\mathbb{X},\mathbb{Y},\mathbb{Z}\in\mathbb{X}(\mathcal{M})$. ……. (11)

7) *AH* - manifold of class C_5 characterized by identity

C$(\mathbb{X},\mathbb{Y})\mathbb{Z}$ $-C(\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ $+C(J\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $+C(J\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $+JC(\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $+JC(\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $-JC(J\mathbb{X},\mathbb{Y})\mathbb{Z}$ $+JC(J\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ =0; $\mathbb{X},\mathbb{Y},\mathbb{Z}\in\mathbb{X}(\mathcal{M})$. …. (12)

8) *AH*- manifold of class C_6 characterized by identity

$C(\mathbb{X},\mathbb{Y})\mathbb{Z}$ $+C(\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ $-CK(J\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $+C(J\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $+JC(\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $-JC(\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $+JC(J\mathbb{X},\mathbb{Y})\mathbb{Z}$ $+JC(J\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ = 0; $\mathbb{X},\mathbb{Y},\mathbb{Z}\in\mathbb{X}(\mathcal{M})$. ……. (13)

9) *AH* - manifold of class C_7 characterized by identity

$C(\mathbb{X},\mathbb{Y})\mathbb{Z}$ $-C(\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ $-C(J\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $-C(J\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $+JC(\mathbb{X},\mathbb{Y})J\mathbb{Z}$ $+JC(\mathbb{X},J\mathbb{Y})\mathbb{Z}$ $+JC(J\mathbb{X},\mathbb{Y})\mathbb{Z}$ $-JC(J\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ = 0; $\mathbb{X},\mathbb{Y},\mathbb{Z}\in\mathbb{X}(\mathcal{M})$. ……. (14)

Proof:-

1) Let *AH*- manifold of class C_0 , the manifold of class C_0 characterized by a condition $C_0^{a}{}_{bcd}=0$, or $C^{a}_{bcd}=0$

i.e. $[C(\varepsilon_{c,\varepsilon_d})\varepsilon_b]^a\varepsilon_a$.

As σ - a projector on $D_J^{\sqrt{-1}}$, that $\sigma\circ\{C(\sigma x,\sigma y)\sigma z\}=0$;

i.e (id-$\sqrt{-1}$ J)$\{C(\mathbb{X}-\sqrt{-1}J\mathbb{X},\mathbb{Y}-\sqrt{-1}J\mathbb{Y})(\mathbb{Z}-\sqrt{-1}J\mathbb{Z})\}$ = 0.

Removing the brackets can be received: i.e.

C$(\mathbb{X},\mathbb{Y})\mathbb{Z}$ -C$(\mathbb{X},J\mathbb{Y})$-C$(J\mathbb{X},\mathbb{Y})J\mathbb{Z}$ -C$(J\mathbb{X},J\mathbb{Y})\mathbb{Z}$ -JC$(\mathbb{X},\mathbb{Y})J\mathbb{Z}$ -JC$(\mathbb{X},J\mathbb{Y})\mathbb{Z}$ -JC$(J\mathbb{X},\mathbb{Y})$+JC$(J\mathbb{X}.J\mathbb{Y})J\mathbb{Z}$ $-\!\!-1\{C(\mathbb{X},\mathbb{Y})J\mathbb{Z}+C(\mathbb{X},J\mathbb{Y})\mathbb{Z}+C(J\mathbb{X},\mathbb{Y})\mathbb{Z}-C(J\mathbb{X},J\mathbb{Y})J\mathbb{Z}\}$

$\{JC(\mathbb{X},Y)\mathbb{Z}-JC(\mathbb{X},JY)J\mathbb{Z}-JC(J\mathbb{X},Y)J\mathbb{Z}-JC(J\mathbb{X},,JY)\mathbb{Z}\}$=0

i.e

1) C$(\mathbb{X},\mathbb{Y})\mathbb{Z}$ - C$(\mathbb{X},J\mathbb{Y})J\mathbb{Z}$ - C$(J\mathbb{X},\mathbb{Y})J\mathbb{Z}$ - C$(J\mathbb{X},J\mathbb{Y})\mathbb{Z}$ - JC$(\mathbb{X},\mathbb{Y})J\mathbb{Z}$ - JC$(\mathbb{X},J\mathbb{Y})\mathbb{Y}$ -JC$(J\mathbb{X},\mathbb{Y})\mathbb{Z}$ +JC$(J\mathbb{X}.J\mathbb{Y})J\mathbb{Z}$ = 0 ………… (15)

2) $C(\mathbb{X},\mathbb{Y})J\mathbb{Z} + C(\mathbb{X},J\mathbb{Y})\mathbb{Z} + C(J\mathbb{X},\mathbb{Y})\mathbb{Z} - C(J\mathbb{X},J\mathbb{Y})J\mathbb{Z} + JC(\mathbb{X},\mathbb{Y})\mathbb{Z} - JC(\mathbb{X},J\mathbb{Y})J\mathbb{Z} - JC(J\mathbb{X},\mathbb{Y})J\mathbb{Z} - JC(J\mathbb{X},J\mathbb{Y})J\mathbb{Z} = 0$ .... (16)

These equalities (15) and (16) are equivalent. The second equality turns out from the first replacement $\mathbb{Z}$ on $J\mathbb{Z}$.

Thus AH - manifold of class C_0 characterized by identity

$C(\mathbb{X},\mathbb{Y})\mathbb{Z} - C(\mathbb{X},J\mathbb{Y})J\mathbb{Z} - C(J\mathbb{X},\mathbb{Y})J\mathbb{Z} - C(J\mathbb{X},J\mathbb{Y})\mathbb{Z} - JC(\mathbb{X},\mathbb{Y})J\mathbb{Z} - JC(J\mathbb{X},J\mathbb{Y})\ Z - JC(J\mathbb{X},\mathbb{Y})\mathbb{Z} + JC(J\mathbb{X},J\mathbb{Y})J\mathbb{Z} = 0,\quad \mathbb{X},\mathbb{Y},\mathbb{Z} \in \mathbb{X}(\mathcal{M})$. ......(17)

Similarly considering AH - manifold of classes C_1-C_7 can be received the 2,3,4,5,6,7 and 8.

Theorem 7:

The study shows that following inclusion relations

1) C_1 =- C_2 +

2) $C_0 = C_3 = C_5 = C_6$.

Proof : "For an example the study prove equality $C_5 = C_6$ Let $(\mathcal{M}, J, g)$ - AH- manifold of class C_5, i.e $C^{\hat{a}}_{\hat{b}c\hat{d}} = 0$. Then according to (4) we have $C^{a}_{\hat{b}\hat{c}d}$=0, i.e. The AH- manifold is manifold of class C_6 Back, let $\mathcal{M}$ – AH- manifold of class C_6, $C^{\mathfrak{a}}_{\hat{\mathfrak{b}}\hat{c}d}$=0 then, so, according to (4) and. $C^{\mathfrak{a}}_{\hat{\mathfrak{b}}c\hat{d}}$=0 Thus, classes C_5 and C_6 AH- manifold coincide."

Other equality are similarly proved. The study prove inclusion $C_4 \supset C_1 = C_2$. Let (*the* $\mathcal{M}$, J, g) - AH- manifold of class C_2, i.e. take place equality$C^{\mathfrak{a}}_{\mathfrak{b}c\hat{d}} = C^{\mathfrak{a}}_{\mathfrak{b}dc}$. According to property (4) we have:

$C^{\mathfrak{a}}_{\hat{\mathfrak{b}}cd} + C^{\mathfrak{a}}_{cd\hat{\mathfrak{b}}} + C^{\mathfrak{a}}_{d\hat{\mathfrak{b}}c}$, i.e $\mathcal{K}^{\mathfrak{a}}_{\hat{\mathfrak{b}}cd}$. Thus, the AH- manifold of a class $C_1 = C_2$ is AH- manifold of class C_4 i.e. $C_1 = C_2 \subset C_4$

Putting(Folding) equality (8) and (9) The study receive identity describing *AH- manifold* of class $C_1 = C_2$

$C(\mathbb{X},\mathbb{Y})\,\mathbb{Z}JK(J\mathbb{X},J\mathbb{Y})Z + JC(\mathbb{X},\mathbb{Y})J\mathbb{Z} - JC(J\mathbb{X},J\mathbb{Y})J\mathbb{Z} = 0,\ \mathbb{X},\mathbb{Y},\mathbb{Z} \in \mathbb{X}(\mathcal{M})$ .... (18)

From equality (7), (10), (12), (13) The study receive the identity describing *AH- manifold* of classes $C_0 = C_3 = C_5 = C_6$:

C$(\mathbb{X},\mathbb{Y})\,\mathbb{Z}$+JK(JK,J $\mathbb{Y}$)JZ=0, $\mathbb{X},\mathbb{Y},\mathbb{Z} \in X(\mathcal{M})$ .... (19)

Definition8

Let $(\mathcal{M},J,g)$ is K- manifold of dimension $2n$, K - tensor conharmonic curvature. that components tensor Riemann- Christoffel on space of the adjoint, G-structure will be Reminded [**5**] look like:

$$1)\ R^{a}_{bcd} = R^{a}_{b\hat{c}\hat{d}} = 0;\ 2)\ R^{a}_{bc\hat{d}} = -R^{a}_{b\hat{d}c} = A^{ad}_{bc} - B^{adh}B_{hbc};$$

$$3)\ R^{\hat{a}}_{bcd} = R^{\hat{a}}_{b\hat{c}\hat{d}} = 0;\ 4)\ R^{\hat{a}}_{\hat{b}c\hat{d}} = -R^{\hat{a}}_{\hat{b}\hat{d}c} = -A^{bd}_{ac} + B^{bdh}B_{hac};\cdots$$

$$5)\ R^{a}_{\hat{b}c\hat{d}} = R^{a}_{\hat{b}\hat{c}d} = 0;\ 6)\ R^{a}_{\hat{b}cd} = 2B^{abh}B_{hcd};\ 7)\ R^{\hat{a}}_{bcd} = 0;$$

$$8)\ R^{\hat{a}}_{bc\hat{d}} = R^{\hat{a}}_{\hat{b}\hat{c}d} = 0;\ 9)\ R^{\hat{a}}_{b\hat{c}\hat{d}} = 2B^{cdh}B_{hab};\ 10)\ R^{\hat{a}}_{\hat{b}cd} = 0.$$

(20)

and the components of Ricci tensor S on space of the adjoint G-structure look like:

$$1)\ S_{ab} = 0;\ 2)\ S_{\hat{a}\hat{b}} = 0;\ 3)\ S_{\hat{a}b} = S_{b\hat{a}} = A^{ac}_{bc} + 3B^{ach}B_{bch}.\cdots\cdots \quad (21)$$

At last scalar curvature χ Kahler manifolds in the space of the adjoint G-structure is calculated under the formula

$\chi = 2\mathcal{A}^{\mathfrak{ab}}_{\mathfrak{ab}} + 6\mathcal{B}^{\mathfrak{abc}}\mathcal{B}_{\mathfrak{abc}}.$ .... (22)

Theorem 9:-

The components of the concircular tensor of K-manifold in the adjoined G-structure space are given as the following forms

1) $C_{\hat{a}b\hat{c}d} = \mathcal{B}^{adc}B_{bdh} - \mathcal{A}^{ac}_{bd} - \frac{\chi}{n(n-1)}\delta^{a}_{d}\delta^{c}_{b};$

2) $C_{\hat{a}bc\hat{d}} = \mathcal{B}^{adh}\mathcal{B}_{hbc} + AV\mathcal{A}^{ad}_{bc} - \frac{\chi}{n(n-1)}\delta^{a}_{c}\delta^{d}_{b}.$

And the others conjugate to the above components or equal to zero.

Proof.

By using Theorems (3.2.5) we compute the components of concircular tensor as the following :

1) Put $\mathfrak{i} = \mathfrak{a}$, $\mathfrak{j} = \mathfrak{b}$, $k = \mathfrak{c}$, $\mathfrak{l} = d$

$C_{\mathfrak{a}bcd} = R_{\mathfrak{a}bcd} - \frac{\chi}{n(n-1)}\{g_{\mathfrak{a}d}g_{\mathfrak{b}c} - g_{\mathfrak{a}c}g_{\mathfrak{b}d}\}$

$C_{abcd} = 0$

2) Put $\mathfrak{i} = \hat{\mathfrak{a}}$, $\mathfrak{j} = \mathfrak{b}$, $k = \mathfrak{c}$, $\mathfrak{l} = d$

$C_{\hat{a}bcd} = R_{\hat{a}bcd} - \frac{\chi}{n(n-1)}\{g_{\hat{a}d}g_{bc} - g_{\hat{a}c}g_{bd}\}$

$C_{\hat{a}bcd} = 0$

3) Put $\mathfrak{i} = \mathfrak{a}$, $\mathfrak{j} = \hat{\mathfrak{b}}$, $k = \mathfrak{c}$ and $\mathfrak{l} = d$

$C_{a\hat{b}cd} = R_{a\hat{b}cd} - \frac{\chi}{n(n-1)}\{g_{ad}g_{\hat{b}c} - g_{ac}g_{\hat{b}d}\}$

$C_{a\hat{b}cd} = 0$

4) Put $\mathfrak{i} = \mathfrak{a}$, $j = \mathfrak{b}$, $k = \mathfrak{c}$, and $\mathfrak{l} = d$

$C_{ab\hat{c}d} = R_{ab\hat{c}d} - \frac{\chi}{(n-1)}\{g_{ad}g_{b\hat{c}} - g_{a\hat{c}}g_{bd}\}$

$C_{ab\hat{c}d} = 0$

5) Put $\mathfrak{i} = \mathfrak{a}, \mathfrak{j} = \mathfrak{v}$, $k = c$ and $\mathfrak{l} = \hat{d}$

$C_{abc\hat{d}} = R_{abc\hat{d}} - \frac{\chi}{n(n-1)}\{g_{a\hat{d}}g_{bc} - g_{ac}g_{b\hat{d}}\}$

$C_{abc\hat{d}} = 0$

6) Put $\mathfrak{i} = \hat{\mathfrak{a}}$, $j = \hat{\mathfrak{v}}$, $k = \mathfrak{c}$, $\mathfrak{l} = d$

$C_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} - \frac{\chi}{n(n-1)}\{g_{\hat{a}d}g_{\hat{b}c} - g_{\hat{a}c}g_{\hat{b}d}\}$

$C_{\hat{a}\hat{b}cd} = 0$

7) Put $\mathfrak{i} = \hat{\mathfrak{a}}$, $\mathfrak{j} = \mathfrak{b}$, $k = \hat{c}$ and $\mathfrak{l} = d$

$C_{\hat{a}b\hat{c}d} = R_{\hat{a}b\hat{c}d} - \frac{\chi}{n(n-1)}\{g_{\hat{a}d}g_{b\hat{c}} - g_{\hat{a}\hat{c}}g_{bd}\}$

$C_{\hat{a}b\hat{c}d} = \mathcal{B}^{adc}B_{bdh} - \mathcal{A}^{ac}_{bd} - \frac{\chi}{n(n-1)}\delta^{a}_{d}\delta^{c}_{b}$

8) Put $\mathfrak{i} = \hat{\mathfrak{a}}, \mathfrak{j} = \mathfrak{b}$, $k = \mathfrak{c}$ and $\mathfrak{l} = \hat{d}$

$C_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} - \frac{\chi}{n(n-1)}\{g_{\hat{a}\hat{d}}g_{bc} - g_{\hat{a}c}g_{b\hat{d}}\}$

$C_{\hat{a}bc\hat{d}} = \mathcal{B}^{adh}\mathcal{B}_{hbc} + \mathcal{A}^{ad}_{bc} + \frac{\chi}{n(n-1)}\delta^{a}_{c}\delta^{d}_{b}$

$C_{\hat{a}bc\hat{d}} = \mathcal{A}^{ad}_{bc} + \frac{K}{4n(n+1)}\delta^{a}_{c}\delta^{d}_{b} + \frac{K}{4n(n+1)}\{\delta^{a}_{c}\delta^{d}_{b} + 2\delta^{a}_{b}\delta^{d}_{c}\}$

$C_{\hat{a}bc\hat{d}} = \mathcal{A}^{ad}_{bc} + \frac{K}{2n(n+1)}\tilde{\delta}^{ad}_{bc}$

By using the properties of concircular tensor we obtained :

$C_{\hat{a}bc\hat{d}} = C_{\hat{a}b\hat{c}d}$ as follows

$C_{\hat{a}b\hat{d}c} = -C_{\hat{a}bc\hat{d}}$

$C_{\hat{a}b\hat{d}c} = -A^{ad}_{bc} - \frac{K}{2n(n+1)}\tilde{\delta}^{ad}_{bc}$

Therefore,

$C_{\hat{a}b\hat{c}d} = -\mathcal{A}^{ac}_{bd} - \frac{K}{2n(n+1)}\tilde{\delta}^{ac}_{bd}.$

In above theorem me calculated components concircular tensor curvature on space of the adjointe G-structure for K-manifolds and we $C_1 and C_2$ have only other components concircular curvature tensor are equal to zero .

i.e for K-manifold only two concircular curvature tensor donts equal zero $C_1$with component $\{C^a_{bc\hat{d}}, C^a_{b\hat{c}d}\}$and C_2 with component $\{C^a_{b\hat{c}d}, C^{\hat{a}}_{\hat{b}c\hat{d}}\}$.

In the theory of *almost Hermitian* structures, there is a principle of classification of such structures on differential-geometric invariants of the second order (symmetry properties of Riemann-Christoffel tensor). The principle put forward by A. Gray and generated in a number of their works are put depented on the basis ([1], [5], [6],.), according to which key to understanding of differential -geometrical properties Kahler manifolds identities with which satisfies them Riemann curvature tensor are:

$R_1: \langle R(\mathbb{X}, \mathbb{Y}))\mathbb{Z}, \mathbb{W}\rangle = \langle R(J\mathbb{X}, J\mathbb{Y})Z, \mathbb{W}\rangle$;

$R_2: \langle R(\mathbb{X}, \mathbb{Y})\mathbb{Z}, \mathbb{W}\rangle = \langle R(J\mathbb{X}, J\mathbb{Y})Z, \mathbb{W}\rangle + \langle R(J\mathbb{X}, \mathbb{Y})J\mathbb{Z}, \mathbb{W}\rangle + \langle R(J\mathbb{X}, \mathbb{Y})\mathbb{Z}, J\mathbb{W}\rangle$;

$R_3: \langle R(\mathbb{X}, \mathbb{Y})\mathbb{Z}, \mathbb{W}\rangle = \langle R(J\mathbb{X}, J\,\mathbb{Y})J\mathbb{Z}, J\mathbb{W}\rangle$;

Definition 10

AH -structures, tensor R which satisfies to identity Ri, are called the structures of class R_i. If $\theta \subset AH$ - any subclass of AH-structures designation s $\cap R_i =$ s where $i = 1, 2, 3$.

well - known that $C \subset R_1 \subset R_2 \subset R_3$ [12]. So it is natural to expect that among AH-manifolds for differential - geometrical and topological properties closest to the kahler manifold class, manifold class, R_1manifold class R_2 and at last manifold of class R_3. The manifold of class R_2 while having no special items were introdused into consideration by A.Gray's in connection with studying Kahler manifold whereas $K \subset R_2$, and were considered Gray and Vanhecke [6], and other authors.

Let $(\mathcal{M}, J, g)$ - Kahler manifold of dimension $2n$,C concircular curvature tensor.

Definition 11:-

The manifold $(\mathcal{M}, J, g)$ refers to as manifold of a class:

1\. $\bar{C}_1$ if $< C(\mathbb{X}, \mathbb{Y})\mathbb{Z}, \mathbb{W} > = < C(\mathbb{X}, Y)J\mathbb{Z}, J\mathbb{W} >$;

2\.$\bar{C}_1$ if $< C(\mathbb{X}, \mathbb{Y})\mathbb{Z}, \mathbb{W} > = < C(J\mathbb{X}, JY)\mathbb{Z}, \mathbb{W} > + < C(J\mathbb{X}, Y)J\mathbb{Z}, \mathbb{W} > + < C(J\mathbb{X}, \mathbb{Y})\mathbb{Z}, J\mathbb{W} >$;

3\. $\bar{C}$ if $< C(\mathbb{X}, \mathbb{Y})\mathbb{Z}, \mathbb{W} > = < C(J\mathbb{X}, J\mathbb{Y})J\mathbb{Z}, J\mathbb{W} >$. ……. (23)

Note 12.

From equation (19) follows that K - manifold of class $C_0 = C_3 = C_5 = C_6$ are also manifold of a class $\bar{C}_3$.

Sense of the specified identities of curvature it is most transparent it is shown in terms of a spectrum concircular curvature tensor.

Theorem13:-

Let $\theta = (J, g = <\times, \times>)$ is Kahler structure. Then the following statements are equivalent:

(1) θ- Structure of a class $\bar{C}_3$;

(2) $C_{(0)} = 0$; and

(3) On space of the adjoint G-structure identities $C^a_{bcd\,=0}$ are fair.

proof.

Let θ - structure of a class $\bar{C}_3$. Obviously, it is equivalent to identity $C(\mathbb{X}, \mathbb{Y})\,\mathbb{Z} + JC(J\,\mathbb{X}, J\,\mathbb{Y})J\,\mathbb{Z} = 0$; $\mathbb{X}, \mathbb{Y}, \mathbb{Z} \in X(\mathcal{M})$. By definition of a spectrum tensor

$C(\mathbb{X}, \mathbb{Y})\mathbb{Z}$
$= C_{(0)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(1)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(2)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(3)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(4)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(5)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(6)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(7)}(\mathbb{X}, \mathbb{Y})\mathbb{Z}$; $\mathbb{X}, \mathbb{Y}, \mathbb{Z} \in X(\mathcal{M})$

$J \circ C(J\mathbb{X}, J\mathbb{Y})J\mathbb{Z} = J \circ C_{(0)}(J\mathbb{X}, J\mathbb{Y})J\mathbb{Z} + J \circ C_{(1)}(J\mathbb{X}, J\mathbb{Y})J\mathbb{Z} + J \circ C_{(2)}(J\mathbb{X}, J\mathbb{Y})J\mathbb{Z} + J \circ C_{(3)}(J\mathbb{X}, J\mathbb{Y})J\mathbb{Z} + J \circ C_{(4)}(J\mathbb{X}, J\mathbb{Y})J\mathbb{Z} + J \circ C_{(5)}(J\mathbb{X}, J\mathbb{Y})J\mathbb{Z} + J \circ C_{(6)}(J\mathbb{X}, J\mathbb{Y})J\mathbb{Z} + J \circ C_{(7)}(J\mathbb{X}, J\mathbb{Y})J\mathbb{Z} = C_{(0)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} - C_{(1)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} - C_{(2)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(3)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} - C_{(4)}(\mathbb{X}, \mathbb{Y})Z + C_{(5)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(6)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} - C_{(7)}(\mathbb{X}, \mathbb{Y})\mathbb{Z}$; $\mathbb{X}, \mathbb{Y}, \mathbb{Z} \in X(\mathcal{M})$.

Putting term by these identities, will be received:

$C(\mathbb{X}, \mathbb{Y})\,\mathbb{Z} + JC(J\mathbb{X}, J\mathbb{Y})J\mathbb{Z} = \{C_{(0)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(3)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(5)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(6)}(\mathbb{X}, \mathbb{Y})\mathbb{Z}\}$

With means, the identity $C(\mathbb{X}, \mathbb{Y})\,\mathbb{Z} + JC(J\mathbb{X}, J\mathbb{Y})JZ\mathbb{Z} = 0$ is equivalent to that

$C_{(0)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(3)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(5)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(0)}(\mathbb{X}, \mathbb{Y})\mathbb{Z}$

"and this identity is equivalent to identities $C_{(0)} = C_{(3)} = C_{(5)} = C_{(6)} = 0$. According to properties (4), the received identities on space of the adjoint G-structure are equivalent to relations. $C^a_{bcd} = C^a_{b\hat{c}\hat{d}} = C^{\,a}_{\hat{b}c\hat{d}} = 0$"

"By virtue of materiality tensor C and its properties(4) received relations which are equivalent to relations $C^a_{bcd} = 0$,i.e. identity $C_{(0)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} = 0$. The opposite, according to (19), obviously."

Theorem 15:-

Let $\theta = (J, g = <\cdot,\cdot>)$ is Kahler structure, then the following statements are equivalent:

(1) θ- Structure of a class $\bar{C}_2$;

(2) $C_{(0)} = C_{(7)} = 0$; and

(3) On space of the attached G-structure identities $C^a_{bcd} = C^a_{\hat{b}\hat{c}\hat{d}} = 0$ are fair.

proof.

Let θ - structure of a class $\bar{C}_2$. We shall copy identity $\bar{C}_2$ in the following form.

With everyone composed this identity will be painted according to definition of a spectrum tensor:

1\) $C(\mathbb{X}, \mathbb{Y})\,\mathbb{Z} = C_{(0)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(1)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(2)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(3)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(4)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(5)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(6)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + K_{(7)}(\mathbb{X}, \mathbb{Y})\mathbb{Z}$;

2\) $C(J\,\mathbb{X}, J\,\mathbb{Y})\,\mathbb{Z} = C_{(0)}(J\mathbb{X}, J\mathbb{Y})\mathbb{Z} + C_{(1)}(J\mathbb{X}, J\mathbb{Y})\mathbb{Z} + C_{(2)}(J\mathbb{X}, J\mathbb{Y})\mathbb{Z} + C_{(3)}(J\mathbb{X}, J\mathbb{Y})\mathbb{Z} + C_{(4)}(J\mathbb{X}, J\mathbb{Y})\mathbb{Z} + C_{(5)}(J\mathbb{X}, J\mathbb{Y})\mathbb{Z} + C_{(6)}(J\mathbb{X}, J\mathbb{Y})\mathbb{Z} + C_{(7)}(J\mathbb{X}, J\mathbb{Y})\mathbb{Z} = -C_{(0)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(1)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(2)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} - C_{(3)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} - C_{(4)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(5)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(6)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} - C_{(7)}(\mathbb{X}, \mathbb{Y})\mathbb{Z}$

3\) $C(J\mathbb{X}\,\mathbb{X}, \mathbb{Y})J\,\mathbb{Z} = C_{(0)}(J\mathbb{X}, \mathbb{Y})J\mathbb{Z} + C_{(1)}(J\mathbb{X}, \mathbb{Y})J\mathbb{Z} + C_{(2)}(J\mathbb{X}, \mathbb{Y})J\mathbb{Z} + C_{(3)}(J\mathbb{X}, \mathbb{Y})J\mathbb{Z} + C_{(4)}(J\mathbb{X}, \mathbb{Y})J\mathbb{Z} + C_{(5)}(J\mathbb{X}, \mathbb{Y})J\mathbb{Z} + C_{(6)}(J\mathbb{X}, \mathbb{Y})J\mathbb{Z} + C_{(7)}(J\mathbb{X}, \mathbb{Y})J\mathbb{Z} = -C_{(0)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} - C_{(1)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(2)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(3)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(4)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} + C_{(5)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} - C_{(6)}(\mathbb{X}, \mathbb{Y})\mathbb{Z} - C_{(7)}(\mathbb{X}, \mathbb{Y})\mathbb{Z}$

4) $JC(J\mathbb{X},\mathbb{Y})\mathbb{Z} = JC_{(0)}(J\mathbb{X},\mathbb{Y})\mathbb{Z} + JC_{(1)}(J\mathbb{X},\mathbb{Y})\mathbb{Z} + J\,C_{(2)}(J\mathbb{X},\mathbb{Y})\mathbb{Z} + JC_{(3)}(J\mathbb{X},\mathbb{Y})\mathbb{Z} + JC_{(4)}(J\mathbb{X},\mathbb{Y})\mathbb{Z} + J\,C_{(5)}(J\mathbb{X},\mathbb{Y})\mathbb{Z} + JC_{(6)}(J\mathbb{X},\mathbb{Y})\mathbb{Z} + JC_{(7)}(J\mathbb{X},\mathbb{Y})\mathbb{Z} = -C_{(0)}(\mathbb{X},\mathbb{Y})\mathbb{Z} - C_{(1)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(2)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(3)}(\mathbb{X},\mathbb{Y})\mathbb{Z} - C_{(4)}(\mathbb{X},\mathbb{Y})\mathbb{Z} - C_{(5)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(6)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(7)}(\mathbb{X},\mathbb{Y})\mathbb{Z}$

Substituting these decomposition in the previous equality, we shall receive:

$C(\mathbb{X},\mathbb{Y})\mathbb{Z} - C(J\mathbb{X},J\mathbb{Y})\mathbb{Z} - C(J\mathbb{X},\mathbb{Y})J\mathbb{Z} + JC(J\mathbb{X},\mathbb{Y})\mathbb{Z} + JC(J\mathbb{X},\mathbb{Y})\mathbb{Z} =$

$2\{C_{(0)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(3)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(6)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(7)}(\mathbb{X},\mathbb{Y})\mathbb{Z}\}$

This identity is equivalent to that

$C_{(0)}(\mathbb{X},\mathbb{Y})\mathbb{Z} = C_{(3)}(\mathbb{X},\mathbb{Y})\mathbb{Z} = C_{(5)}(\mathbb{X},\mathbb{Y})\mathbb{Z} = C_{(7)}(\mathbb{X},\mathbb{Y})\mathbb{Z} = 0$

and these identities on space of the adjoint *G-structure* are equivalent to identities $C^a_{bcd} = C^a_{\hat{b}\hat{c}\hat{d}} = C^a_{\hat{b}c\hat{d}} = C^a_{\hat{b}\hat{c}d} = C^a_{\hat{b}\hat{c}\hat{d}}$.

By virtue of materiality tensor C and his properties (4), the received relations are equivalent to relations: $C^a_{\hat{b}cd} = C^a_{\hat{b}\hat{c}\hat{d}}$, i.e. to identities $C_{(0)}(\mathbb{X},\mathbb{Y})\mathbb{Z} = C_{(0)}(\mathbb{X},\mathbb{Y})\mathbb{Z}$.

Back, let for *K*- manifold identities $C_{(0)}(\mathbb{X},\mathbb{Y})\mathbb{Z} = C_{(7)}(\mathbb{X},\mathbb{Y})\mathbb{Z} = 0$ are executed. Then from (10) and (17) have:

$C(\mathbb{X},\mathbb{Y})\mathbb{Z} - C(\mathbb{X},J\mathbb{Y})J\mathbb{Z} - C(J\mathbb{X},\mathbb{Y})J\mathbb{Z} - C(J\mathbb{X},J\mathbb{Y})\mathbb{Z} = 0$

i.e.

$C(\mathbb{X},\mathbb{Y})\mathbb{Z} = C(J\mathbb{X},\mathbb{Y})J\mathbb{Z} = C(J\mathbb{X},J\mathbb{Y})\mathbb{Z} = C(\mathbb{X},JY)\mathbb{Z}$

In the received identity instead of $C(\mathbb{X}.J\mathbb{Y})\mathbb{Z}$ we shall put the value received from (17) replacement $\mathbb{Y} \to J\mathbb{Y}$ and $\mathbb{Z} \to J\mathbb{Z}$, i.e $C(\mathbb{X},J\mathbb{Y})J\mathbb{Z} = -JC(J\mathbb{X},\mathbb{Y})\mathbb{Z}$. Then

$C(\mathbb{X},\mathbb{Y})\mathbb{Z} = C(J\mathbb{X},J\mathbb{Y}) + C(\mathbb{X},\mathbb{Y})J\mathbb{Z} - JC(J\mathbb{X},J\mathbb{Y})\mathbb{Z}$

i.e.

$<C(\mathbb{X},\mathbb{Y})\mathbb{Z},W> = <C(J\mathbb{X},J\mathbb{Y})\mathbb{Z},W> + <C(J\mathbb{X},\mathbb{Y})J\mathbb{Z},W> - <C(J\mathbb{X},J\mathbb{Y})\mathbb{Z},JW>, \mathbb{X},\mathbb{Y},\mathbb{Z},W \in \mathbb{X}(M)$

Thus, the manifold satisfies to identity $\bar{C}_2$.

The following theorem is similarly proved.

Theorem 16:-

Let $\theta = (J, g = <\times, \times>)$ is Kahler structure. Then the following statements are equivalent:

(1) θ - structure of a class $\bar{C}_1$;

(2) $C_{(0)} = C_{(4)} = C_{(7)} = 0$;

(3) On space of the attached *G*-structure identities $C^a_{bcd} = C^a_{\hat{b}cd} = C^a_{\hat{b}\hat{c}\hat{d}}$ are fair.

proof :

Let S - structure of a class $\bar{C}_1$. Obviously, it is equivalent to identity

$<C(\mathbb{X},\mathbb{Y})\mathbb{Z}, W> = <C(\mathbb{X},\mathbb{Y})J\mathbb{Z}, JW>$

and we get $C(\mathbb{X},\mathbb{Y})\mathbb{Z} + JC(\mathbb{X},\mathbb{Y})J\mathbb{Z} = 0$; $\mathbb{X},\mathbb{Y},\mathbb{Z} \in \mathbb{X}(\mathcal{M})$

By definition of a spectrum tensor

1) $C(\mathbb{X},\mathbb{Y})\mathbb{Z} = C_{(0)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(1)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(2)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(3)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(4)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(5)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(6)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(7)}(\mathbb{X},\mathbb{Y})\mathbb{Z}$; $\mathbb{X},\mathbb{Y},\mathbb{Z} \in \mathbb{X}(\mathcal{M})$

2) $J \circ C(\mathbb{X},\mathbb{Y})J\mathbb{Z} = J \circ C_{(0)}(\mathbb{X},\mathbb{Y})J\mathbb{Z} + J \circ C_{(1)}(\mathbb{X},\mathbb{Y})J\mathbb{Z} + J \circ C_{(2)}(\mathbb{X},\mathbb{Y})J\mathbb{Z} + J \circ C_{(3)}(\mathbb{X},\mathbb{Y})J\mathbb{Z} + J \circ C_{(4)}(\mathbb{X},\mathbb{Y})J\mathbb{Z} + J \circ C_{(5)}(\mathbb{X},\mathbb{Y})J\mathbb{Z} + J \circ C_{(6)}(\mathbb{X},\mathbb{Y})J\mathbb{Z} + J \circ C_{(7)}(\mathbb{X},\mathbb{Y})J\mathbb{Z} = -C_{(0)}(\mathbb{X},\mathbb{Y})\mathbb{Z} - C_{(1)}(\mathbb{X},\mathbb{Y})\mathbb{Z} - C_{(2)}(\mathbb{X},\mathbb{Y})\mathbb{Z} - C_{(3)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(4)}(\mathbb{X},\mathbb{Y})\mathbb{Z} - C_{(5)}(\mathbb{X},\mathbb{Y})\mathbb{Z} - C_{(6)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(7)}(\mathbb{X},\mathbb{Y})\mathbb{Z}$; $\mathbb{X},\mathbb{Y},\mathbb{Z} \in X(\mathcal{M})$

Putting (1) and (2) in

$C(\mathbb{X},\mathbb{Y})\mathbb{Z} + JC(\mathbb{X},\mathbb{Y})J\mathbb{Z}$ means, this identity is equivalent to that $C_{(0)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(4)}(\mathbb{X},\mathbb{Y})\mathbb{Z} + C_{(7)}(\mathbb{X},\mathbb{Y})\mathbb{Z} = 0$

And this identity is equivalent to identities $C_{(0)} = C_{(4)} = C_{(7)} = 0$

According to properties (4), The received identities in space of the adjoint *G*- structure are equivalent to relation $C^a_{bcd} = C^a_{\hat{b}cd} = C^a_{\hat{b}\hat{c}\hat{d}} = 0$.

Corollary 17*:-*

Let $\theta = (J, g = <\times, \times>)$ is Kahler structure. Then the following inclusions of classes $\bar{C}_1 \subset \bar{C}_2 \subset \bar{C}_3$ are fair.

Conclusion

-The notion of tensor concircular curvature tensor of Kahler manifolds has been studied and analyzing

The study of non- zero compounds of the concircular curvature tensor of Kahler manifolds in the adjoint G-structure space .

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تنزر الانحناء الدائري لمنطوي كوهلر هندسة

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الملخص

في هذا البحث تم دراسة الشروط الاساسية لايجاد منطوي كوهلر التقريبي على تنزر الانحناء الدائري. والعلاقة بين الخواص والثوابت لتنزر التماثل الدائري, مع دراسة المعنى الهندسي لمركبات التنزر عندما يساوي صفر.