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Feeble ring

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ABSTRACT

An ideal A of a semiring S is called k-ideal if for any two elements r∈A and x∈S such that r+x ∈A, then x∈A″. This leads us to introduce the new concept feeble ring as generalization of k-ideal. Several basic properties, example and characterization of this concept are given. Moreover, the study investigate relationship of feeble ring with other classes.

1. Introduction

″ Semiring constiute a fairly natural generalization of ring, we first introduced by American mathematician Vandiver in 1934 [1]″, and has since then been studied by many authors. For general books and papers on semiring theory, one may refer to the resources [2,3].″A semiring is a nonempty set S together with two associative operations + and ·, such that for all a, b, c ∈ S then: a · (b + c) = a · b + a · c and (a + b) · c = a · c + b · c, a semiring is called additively [multiplicatively] commutative if (S, +) [(S, ·)] is commutative[1]″. A natural example of semiring which is not a ring, is the set of all natural numbers under usual addition and multiplication of numbers. An element x in a semiring S is called a zero if it satisfies: x+a=a=a+x and xa=x=ax, for all a∈S.″ Zeroid of a semiring (S, +, ·) is the set of all x in S such that : x + y = y or y + x = y for some y in S [4]″. For example let R^+ be the set of all nonnegative real numbers, we define: x + y = min {(x,y): x , y in R^+ }, x . y = be usual multiplication. The study suggests that (R^+ , + , .) semiring , and all elements of (R^+ , +, .) are zeroids. A ring contains no non-trivial zeroids, but a semiring containing no non-trivial zeroid need not be a ring, every a zero element in semiring is zeroid . But the converse need not be true in general, we called zero to be trivial zeroid and other zeroids be non-trivial zeroids. ″A nonempty subset A of a semiring S is said to be an ideal of S, if c + b ∈ A for all c, b ∈ A and sc ∈ A for all s ∈ S and c ∈ A. It is clear that the zero element 0 belongs to any ideal of S. An ideal A of a semiring S is called a proper ideal of the semiring S if A ≠ S [5]″. Semiring, as the basic algebraic structure is used in the areas of theoretical computer science as well as in the solution of graph theory, optimization theory, coding theory, formal languages and has many applications in other branches of mathematics. In 1958, ″Henriksen[7] defined a more restricted class of ideals in semiring, which he called this special kind of ideals a k-ideal[6]. An ideal A of a semiring S is called k-ideal if for any two elements r ∈ A and x ∈ S such that r + x ∈ A, then x ∈ A [7]″, for example in the semiring Z^+ under the operations max and min, the set A_n = {1, 2, 3, ..., n} is a k-ideal of Z^+. Since for any element r ∈ A_n and x ∈ Z^+ such that r + x = max{r, x} ∈ A_n , implies x ∈ A_n [7]. Every k- ideal in semiring is ideal , but the converse in general is need not be true, R is k-ideal of itself and {0} is also k-ideal of R if 0 ∈ R, The sum of any two k- ideals need not be k-ideal in general, for example: Let Z^+ be the semiring of all non-negative integers together with the usual addition and multiplication such that : (2) = 2Z^+ , (3) = 3Z^+ and k-ideal. But (2) + (3) is not k-ideal both 7 and 6 are in (2) + (3) , 1 + 6 = 7 but 1 not in (2) + (3), then the sum (2) and (3) is not k-ideal. A homomorphism of semirings is a map θ : S→S′ that preserves addition and multiplication. θ satisfies the following properties for all a and b in S such that: θ (a + b) = θ (a)+ θ (b)

, θ (a. b) = θ (a). θ (b) and θ $(0_S) = (0_{S'})$, If there exists a homomorphism from S onto S′, we say that S′ is a homomorphic image of S [3], let θ be a semiring homomorphism from S into S′ . ″The kernel of θ is the set kerθ={ a ∈ S : θ(a) = 0′}, let θ be a semiring homomorphism from S into S′. Then kernel of θ is a k- ideal of S. Let A be an ideal of a semiring S, define a relation ≡ on S by: c ≡ b (mod A) if only if $c + a_1 = b + a_2$ for some $a_1, a_2 \in A$, Let S be a semiring and A is ideal of semiring S″. Then their exists semiring homomorphism from S onto S/ A and A ⊆ kerθ. In particular if A is a k-ideal then kerθ =A. The current study establishes new concept feeble-ring as generalization of feeble-ring with other class are studied, and given homomorphism theorem.

2. Feeble-ring

This section tries to introduce and study the concept of a feeble ring as a generalization of k-ideals.

″**Definition 2.1.** A semiring S is feeble-ring if for any a,b∈S , a ≠ b , there exists x∈S such that either a+x=b or a=b+x ″.

Remark 2.2. It is easily seen that x≠0 and S is a feeble-ring if only if S_0 is a feeble-ring.

$$\text{Where } S_0 = \begin{cases} S & \text{if S has zero} \\ S \cup 0 & \text{if S has not zero} \end{cases}$$

″**Proposition 2.3.** If S_1 and S_2 are two feeble-rings of semiring, then $S_1 \cap S_2$ is also a feeble-ring of the S″.

Proof: Suppose that S_1 and S_2 are two feeble-ring of semiring. The study affirms that $S_1 \cap S_2$ is feeble-ring of S. Let $a,b \in S_1 \cap S_2$, and a≠b implies $a,b \in S_1$ and $a,b \in S_2$. Since S_1, S_2 are feeble-ring of semiring S, there are exists $x \in S_1$ and $x \in S_2$ such that either a+x=b or a=b+x holds, implies that $S_1 \cap S_2$ is feeble-ring of S.

Lemma 2.4. The homomorphic image of a feeble-ring is a feeble.

Proof: Assume that S feeble-ring and θ homomorphic image from S into S′. So that for any $b_1 \neq b_2 \in \theta(S)$, there are a_1, a_2 such that $\theta(a_1)= b_1$ and $\theta(a_2)=b_2$. Since S is a feeble-ring, then there is x ∈ S such that $a_1+x=a_2$ or $a_1=a_2+x$, this implies that $\theta(a_1)+\theta(x)=\theta(a_2)$ or $\theta(a_1)= \theta(a_2) + \theta(x)$. It is means that θ(S) is feeble.

Lemma 2.5. Every k-ideal in a feeble-ring is a feeble.

Lemma 2.6. Let A be an ideal of a feeble-ring S. If all zeroids of S are contained in A, then the quotient $\frac{S}{A}$ contains no nontrivial zeroids.

Proof: It is obvious true for A={0}, suppose that A≠{0} and let $\bar{x}$ be a zeroid of $\frac{S}{A}$. Then there exists some $\bar{y} \in \frac{S}{A}$ such that $\bar{x} + \bar{y} = \bar{y}$, that is $\overline{x+y} = \bar{y}$ and so x + y ≡ y (mod A).

Hence $y+x+a_1=y + a_2$ for some $a_1, a_2 \in A$, if $x + a_1 = a_2$, then $\bar{x} + \bar{a_1} = \bar{a_2}$ which implies $\bar{x}=\bar{0}$.

If $x + a_1 \neq a_2$ in feeble-ring S_0, then ∃ w ∈ S_0 such that $(x + a_1)+ w = a_2$ or $x + a_1 = a_2 + w$ which implies that $y + x + a_1= y + a_2= y + x + a_1+w$ or $y + a_2+ w = y + a_2$ .In either case, w is a zeroid of S_0 and so w∈A. Hence $\bar{x}+\bar{a_1} = \bar{a_2}$, which means that $\bar{x} = \bar{0}$.

Theorem 2 .7.

Let S be a feeble-ring and S′ be a semiring containing no non-trivial zeroids. If θ is a semiring homomorphism of S onto S′, then S′ is isomorphic to $\frac{S}{\ker\theta}$. In particular S isomorphic to S′.

Proof: Since θ is a semiring homomorphism of S onto S′, every element in S′ has the form θ(r) where r∈S. Define a mapping $\alpha : S' \longrightarrow \frac{S}{\ker\theta}$ by : $\alpha(\theta(r) = \bar{r}$, where $\bar{r}$ is a coset of modulo kerθ. The mapping α is well defined. For if we suppose that θ(r) = θ(s)

If r ≠ s, then ∃ 0 ≠ x∈S such that x + r = s or r = x + s since S is a feeble-ring, consequently, we have : θ(x) + θ(r) = θ(s) = θ(r) or θ(r) = θ(x) + θ(s) = θ(s). This means that θ(x) is a zeroid of S′. Since S′ has no zeroid, then the contradiction leads to the fact that r = s and hence θ is an isomorphic. Also it implies $\bar{r} = \bar{s}$ (mod ker θ).

On the other hand, if S′ has zero 0′, then θ(x) = 0′. Hence $\bar{x} = \bar{0'}$. Hence $\bar{x} = \bar{0'}$ the zero of $\frac{S}{\ker\theta}$.

As α(θ(x + r)) = α(θ(s)) or α(θ(r)) = α(θ(x + s)), so $\bar{x} + \bar{r} = \bar{s}$ or $\bar{r} = \bar{x} + \bar{s}$.

Thus we obtain $\bar{r} = \bar{s}$. To prove that α is a semiring isomorphism.

1- To prove that α is a semiring homomorphism, Let θ(r), θ(s) ∈ S′, then:

$\alpha(\theta(r) + \theta(s)) = \alpha(\theta(r + s)) = \overline{r+s} = \bar{r} + \bar{s} = \alpha(\theta(r)) + \alpha(\theta(s))$.

$\alpha(\theta(r) . \theta(s)) = \alpha(\theta(r . s)) = \overline{r.s} = \bar{r} . \bar{s} = = \alpha(\theta(r)) . \alpha(\theta(s))$, then α is a semiring homomorphism.

2-To prove that α is one to one. Let θ(r) , θ(s) ∈ S′, such that α(θ(r)) = α(θ(s)) implies $\bar{r} = \bar{s}$, then $\bar{r} - \bar{s} = \bar{0}$ implies $\overline{r-s} = \bar{0}$, then r – s ∈ kerθ and as well as have θ(r- s) = 0, so that θ(r)- θ(s) = 0, or θ(r) = θ(s) implies that α is one to one.

3-To prove that α is onto. $\forall \bar{r} \in \frac{s}{\ker\theta}$, ∃ θ(r) ∈ S′ such that $\alpha(\theta(r) = \bar{r}$.

$\alpha(\bar{r}) = \{ \alpha(\theta(r) : \theta(r) \in S', r \in S \}$,

$= \{\bar{r} , r \in S \} = \frac{s}{\ker\theta}$.

Theorem 2.8. Let S be a semiring and A is ideal of semiring S. Then there exists semiring homomorphism from S onto S/ A and A ⊆ kerθ. In particular if A is a k-ideal then kerθ =A.

Proof: It is easily to prove that $\theta: S \rightarrow \frac{S}{A}$, such that θ(r) = $\bar{r}$(mod A) is a homomorphism from S onto $\frac{S}{A}$ and hence that the first part of this theorem is ok .

It is important to prove now to prove that kerθ = A when A is a k-ideal. Let r ∈ kerθ then θ(r) = $\bar{0}$, that is $\bar{r} = \bar{0}$. Therefor ∃ a_1,a_2 ∈ A such that : $r+a_1 = 0 + a_2 = a_2 \in A$. Since A is a k-ideal, amit it obtain that r ∈ A that is kerθ ⊆ A. However, A ⊆ kerθ always holds when θ is a semiring homomorphism and thus A = kerθ .

Corollary 2.9. Let S be a feeble-ring and A be a k-ideal of S. If A contains all zeroids of S, then the Bourne quotient $\frac{S}{A}$ is a feeble-ring and is isomorphic to some homomorphic image to S .

Proof: $\frac{S}{A}$ is a feeble-ring by lemma2.4, and this refers to lemma2.5. That $\frac{S}{A}$ contain no non trivial zeroids. Applying theorem 2.7 and theorem 2.8, amit it obtain directly that $\frac{S}{A} \cong \frac{S}{\ker\theta}$. Where θ is a semiring homomorphism .

The following counter examples show that the hypothesis of theorem 2.7 cannot be weakened .

Example 2.10. Let S=[0,1], the unit interval the addition ⊕ and multiplication ⊙ operation on S are defined as follows :

For every pair of elements a , b∈ S, define it is a weak phrase :

a ⊕ b = ab ,the usual multiplication and a ⊙ b = 0, the zero multiplication, then it is easy to see that: (S , ⊕ , ⊙) is a feeble-ring. Let S′=({0,1}, +, .) with

+	0	1
0	0	1
1	1	1

.	0	1
0	0	0
1	0	0

Clearly, (S′, +, .) is also a semiring with a non-trivial zeroid {1} and the zero {0}.

Define a mapping $\theta : S \to S'$ as follows, $\theta(a) = \begin{cases} 1 & \text{if } a \neq 1 \\ 0 & \text{if } a = 1 \end{cases}$

Obviously θ is a semiring homomorphism with kerθ = {1}. However $\frac{S}{\ker\theta}$ = [0,1] and S′={ 0,1}. Hence, there is nothing to refer to does not exists a semiring isomorphic between $\frac{S}{\ker\theta}$ and S′.

This example remarks that that the fundamental theorem of homomorphism fails to be true if S′ contains some non- trivial zeroids .

Example 2.11.

Let A = { (x , 0 , 0) , x ∈ Z^+ }, B = {(0 , x , 0) , x ∈ Z^+ }, C = {(0 , 0 , x) , x ∈ Z^+ }, where Z^+ is set of all non-negative integers.

Denote (x , 0 , 0) by $x^{(1)}$, (0 , x , 0) by $x^{(2)}$, (0 , 0 , x) by $x^{(3)}$.

Define ⊕ on D = A ∪ B ∪ C as follows :

$X^{(i)} \oplus y^{(i)} = (x + y)^{(i)}$, i = 1,2,3, $X^{(i)} \oplus y^{(j)} = (x + y)^{(3)}$, i ≠ j.

Define ⊙ on D to be the zero multiplication. Then it can be easily to proof that D is a semiring but not feeble.

Let (Z_O^+ ,+ , .) be the non-negative integers under usual addition and zero multiplication.

(Z_O^+ ,+ , .) is clearly a semiring with out non-trivial zeroids.

Define a mapping θ: D → Z_O^+ such that θ($x^{(i)}$) = x.

Then θ is a semiring homomorphism and kerθ = { (0 , 0 , 0) }.

As $\frac{D}{\ker\theta}$ =D, so it is signifieant to argue that claim that there does not exists isomorphism from D on to Z_O^+.

For if exists such an isomorphism Ψ, then:

$\Psi(1^{(1)}) + \Psi(1^{(3)}) = \Psi(2^{(3)}) = \Psi(1^{(2)}) + \Psi(1^{(3)})$,

$\Psi(1^{(1)}) = \Psi(1^{(2)})$.

So Ψ is not one to one. This example show that the fundamental theorem of homomorphism fails to be true if the semiring is not feeble.

Now it is important to prove begin to prove theorems for semiring which are analogues those isomorphism thermos for semigroup, group, rings, modules and vector spaces.

Lemma 2.12.[3] Let S′ be a homomorphic image of a semiring S and A′ be an ideal in S′, then the inverse image of A′ is also an ideal in S. In particular, if A′ is a k-ideal in S′, then the inverse image of A′ is also a k-ideal.

Theorem 2.13.

Let S be a feeble-ring with no non-trivial zeroids and let S′ be a semiring homomorphic image of S. If A′ is a k-ideal of S′, then $\frac{S}{A} \cong \frac{S'}{A'}$, where A is the inverse of A′ in S.

Proof. Let τ be the homomorphic from S onto S′, and φ be the homomorphic from S′ onto $\frac{S'}{A'}$. Then $\tau\varphi$ is a homomorphic from S onto $\frac{S'}{A'}$.

Since ker $\varphi\tau$ = { r ∈ S : $\varphi\tau$(r) = A′ } = r ∈ S: $\varphi(r') = A'\}$ = r ∈ S : r′ ∈ A′} =A. There for, applying the Fundamental Theorem of homomorphism. Have $\frac{S}{A} \cong \frac{S'}{A'}$.

Corollary 2.14. Let S be a feeble-ring with no non-trivial zeroids. If A , B are k-ideals of S_O such that A ⊆ B, then $\frac{S_O}{B} \cong \frac{S_O/A}{B/A}$. The proof of this corollary is clear. It is preferablc to focus on the following need the following lemma to prove Second Isomorphism Theorem.

Lemma 2.15. A feeble-ring S with no non-trivial zeroids is cancellable. (cancellable means that if r+x =r+y, then x=y for all r,x,y∈S).

Proof: Let x,y,r be element of S such that r+x = r+y . Suppose x≠y. Then since S is a feeble- ring, there exists 0 ≠ t ∈ S such that x+t = y or t+y = x.

Consequently either: r+y = r+(x+t) or r+(t+y) = r+x, that t is a non-trivial zeroid which contradicts our assumption. Hence x = y.

Theorem 2. 16.

Let S be a feeble-ring with no non-trivial zeroids. If A_1 , A_2 are k-ideals of S, then

$$\frac{A_1 + A_2}{A_1} \cong \frac{A_2}{A_1 \cap A_2} .$$

Proof: Define a mapping φ : $A_1 + A_2 \to \frac{A_2}{A_1 \cap A_2}$ by $\varphi(r) = \overline{a_2}(\bmod A_1 \cap A_2)$

where r = $a_1 + a_2 \in A_1 + A_2$ and $a_1 \in A_1$, $a_1 \in A_2$. We first show φ that is well defined.

Suppose that $r = a_1 + a_2 = a'_1 + a'_2 \in A_1 + A_2$. Then for $a_1, a'_1 \in A_1$, there exists $a \in A$ such that $a_1 + a = a'_1$ or $a_1 = a'_1 + a$.
Therefore we have : $a_1 + a_2 = a_1 + a + a'_2$ or $a + a'_1 + a'_2 = a'_1 + a'_2$ and $a_2 = a + a'_2$ or
$a + a_2 = a'_2$. This implies that $a \in A_2$ since A_2 is a k-ideal. Hence $\overline{a_1} = \overline{a_2}$ (mod $A_1 \cap A_2$).
To prove that φ is a semiring homomorphism.
Let $r_1, r_2 \in A_1 + A_2$ such that $r_1 = a_1 + a_2$ and $r_2 = b_1 + b_2$ where $a_1, b_1 \in A_1$ and $a_2, b_2 \in A_2$ such that $\theta(r) = \theta(a_1 + a_2) = \overline{a_2}$
1- $\theta(r_1 + r_2) = \theta(a_1 + a_2 + b_1 + b_2) = \theta((a_1 + b_1) + (a_2 + b_2))$
$= \overline{a_2 + b_2} = \overline{a_2} + \overline{b_2}$.
$= \theta(r_1) + \theta(r_2)$.
2- $\theta(r_1 . r_2) = \theta((a_1 + a_2).(b_1 + b_2)) = \theta(a_1b_1 + a_1b_2 + a_2b_1 + a_2b_2)$
$= \overline{a_2 . b_2} = \overline{a_2} . \overline{b_2} = \theta(r_1) . \theta(r_2)$.
Hence φ is a semiring homomorphism. We still to prove that ker φ = A. It is obvious that $A_1 \subseteq$ ker φ . For converse, let $r = a_1 + a_2 \in$ ker φ. Then $\varphi(r) = \overline{a_2} = \overline{0}$ (mod $A_1 \cap A_2$).
Hence there exists $a \in A_1 \cap A_2$ such that $a_1 + a_2 \in A_1 \cap A_2$.
Thus $a_2 \in A_1 \cap A_2$ since $A_1 \cap A_2$ is a k-ideal of S. Hence $r = a_1 + a_2 \in$ A, so ker φ = A.
It is better to apply the the fundamental homomorphism theorem and the result is : $\frac{A_1 + A_2}{A_1} \cong \frac{A_2}{A_1 \cap A_2}$.

Theorem 2.17.
Let S be a feeble-ring with no non-trivial zeroids . If A, A′, B, B′ are k-ideals of S such that $A' \subset A$, $B' \subset B$ and $A' + B'$ is also k-ideals of S. Then $A' + (A \cap B')$ and $B' + (B \cap A')$ are k-ideals and there is an isomorphism
$\frac{A' + (A \cap B)}{A' + (A \cap B')} \cong \frac{B' + (B \cap A)}{B' + (B \cap A')}$.
Proof: Let H=A + B , K = (A ∩ B′) + (B ∩ A′). H is a k-ideals and K⊂H and $H \cap B' = A \cap B \cap B' = A \cap B'$. Similary, $H \cap A' = A' \cap B$.
Thererfor : $K = (H \cap B') + (H \cap A') = H \cap (A' + B')$ is k-ideal.
Now it is easy to define a mapping φ from A′ + H onto $\frac{H}{K}$ such that :
$\varphi(a' + h) = \overline{h}$ (mod K) where $a' \in A'$, $h \in H$.
i) φ is well-define .
Suppose that $a'_1 + h_1 = a'_2 + h_2$ for $a'_1, a'_2 \in A'$, there exists $a' \in A'$ such that
$a + a'_1 = a'_2$ or $a'_1 = a + a'_2$. Therefore $a'_1 + h_1 = a + a'_1 + h_2$ or $a + a'_2 + h_1 = a'_2 + h_2$.
Since k is cancellable, we have : $h_1 = a + h_2$ or $a + h_1 = h_2$.
Hence $a \in H$ since H is a k-ideal. And $a \in H \cap A = A' \cap B \subseteq k$. Thus $\bar{a} = 0$(mod k) and we have $\overline{a_1} = \overline{a_2}$ (mod K).
ii) To prove φ is a homomorphism
Let $a'_1 + h_1$, $a'_2 + h_2 \in A' + H$, where $a'_1, a'_2 \in A'$ and $h_1, h_2 \in H$.
1) $\varphi(a'_1 + h_1 + a'_2 + h_2) = \overline{h_1 + h_2} = \overline{h_1} + \overline{h_2}$.
$= \varphi(a'_1 + h_1) + \varphi(a'_2 + h_2)$.
2) $\varphi((a'_1 + h_1) . (a'_2 + h_2)) = \varphi(a'_1a'_2 + a'_1h_2 + h_1a'_2 + h_1h_2)$
$= \overline{h_1 . h_2} = \overline{h_1} . \overline{h_2} = \varphi(a'_1 + h_1) . \varphi(a'_2 + h_2)$.
iii) ker φ = A′ + (A ∩ B′) is a k-ideal.
Obvious A′ + (A + B′) ⊆ ker φ since A ∩ B′ ⊆ K.
Let $a' + h \in$ ker φ, i. e.
$0 = \varphi(a' + h) = \overline{h}$ (mod K). Hence $h \in K$ since K is k-ideal .
Therefore $a' + h \in A' + K = A' + (A \cap B') + (A' \cap B) \subseteq A' + (A \cap B')$.
By the fundamental homomorphism theorem we have : $\frac{A' + (A \cap B)}{A' + (A \cap B')} \cong \frac{H}{K}$.
Similary, we can define a homomorphism from B′ + (B ∩ A) on to $\frac{H}{K}$ with kernel B′ + (A′ ∩ B).
Therefore B′ + (A′ ∩ B) is a k-ideal of S and
$\frac{A' + (A \cap B)}{A' + (A \cap B')} \cong \frac{H}{K} \cong \frac{B' + (B \cap A)}{B' + (B \cap A')}$.

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الحلقة الضعيفة

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الملخص

يطلق على المثالي A لشبه الزمرة S أسم المثالي-k اذا كان لأي عنصرين $r\in A$ و $x\in S$ بحيث أن $r+x \in A$ فأن $x\in A$ هذا قادنا الى تقديم مفهوم الحلقة الضعيفة كتعميم للمثالي-k. وتم اعطاء العديد من الخصائص الاساسية, وأمثلة لتوصيف هذا المفهوم. علاوة على ذلك, نحن نتحرى العلاقة بين الحلقة الضعيفة و الفئات الاخرى.