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SIS MODEL WITH HARVESTING IN FOOD CHAIN MODEL

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ABSTRACT

The aim of this study the mathematical model of the type SIS, healthy prey is infected by disease and the study proved that solution and restrictive in which the molecular system do not have periodic boundaries, then it discussed the stability of those points. the study also showed how to control the disease using the harvest so as not to become an epidemic.

1. Introduction

Mathematical modeling is an abstract model that uses mathematical language to describe the behavior of a system. Mathematical models are particularly used in computational theory in computer science, natural sciences, engineering of fields (such as physics, biology, electrical engineering) and also in social sciences (such as economics, sociology and politics). Physicists, engineers, computer scientists, and economists use mathematical models very widely [1]. in several years the study of predatory- prey systems was an important research in the 1920s, new horizons were opened by Lutka and Volterra [2,3]. For biological species, many researchers later made many achievements in this area in 1927, Kermack and Mckendrick were proposed the classical model affected and infectious and redux that attracted more scientists attention and references in it. SI Epidemic model: In which there is no removal that means the infected individuals is still infected for all the time, so whenever susceptible becomes infected it will be still infected, while SIS Epidemic model: In which the infection does not lead to immunity so that infective become susceptible again after recovery.

The simplest example of using a mathematical model in the field of biology, this, in turn, is divided into two completely different parts within the biology, namely ecology (which is one of the most important branches of biology, which reflects the emerging relationships between the organism and the environment in which it lives), and the second branch, epidemiology (a branch of medicine), which can The epidemic is transmitted through a long series of neighborhoods, causing epidemic diseases [4].

Predation is, according to the definition of ecologists, a biological interaction between two objects, one of which is a predator (pirate, fracture, raptor, or hunting object) feeding on an organism or a number of other organisms known as prey (prey or hunted organism) [5].

Mathematical models have become critical tools in understanding and analyzing the spread and control of infectious diseases by studying different types of disease such as SI, SIS. Some infectious diseases in the ecosystem are transmitted through direct contact [6]. In addition to disease, harvesting can in turn greatly affect the dynamics of the prey-predator system, and harvesting can reduce the numbers of prey or predators [7]. It can also be considered as a stabilizing factor [8] ,Previous studies have indicated that Bairagi et al. indicates that harvesting can control the spread of disease in a particular branch of the population[9], where the effects of harvesting disease in prey were studied in the prey-predator model, while there was another study by cheve et al. Included harvest and disease this time the predator in the predator model and prey concluded that the predator has harvested Prevent the spread of infectious diseases[10], thus ensuring the resilience and stability of ecosystems. This paper is divided as follows: in section two, the study described mathematical model, in the third section the study the natural solution. In the fourth section, the study discussed positive solutions and periodic to subsystems in addition discussed equilibrium points with its conditions and its stability. Stability points of the main model is in the fifth section. Finally, we discussed discuss some results by using Mathematica Programin.

2. Mathematical Model

$$\left.\begin{aligned} \frac{dx}{dt} &= rx(1-x)-\alpha\frac{xy}{1+\omega x}-c\frac{xx_1}{x+x_1}+\delta x_1 \\ \frac{dx_1}{dt} &= c\frac{xx_1}{x+x_1}-\delta x_1-\omega_0 x_1 y-qx_1 \\ \frac{dy}{dt} &= \omega_1 x_1 y+\beta\frac{xy}{1+\omega x}-\gamma\frac{yz}{1+\omega y}-d_1 y \\ \frac{dz}{dt} &= \rho\frac{yz}{1+\omega y}-d_2 z \end{aligned}\right\} \quad (1)$$

Where $x', x_1', y', z' > 0$. x, x_1, y, z denoted susceptible prey, infected prey, intermediate predator and top predator respectively. Parameters denoted as follows, r the rate of growth of susceptible prey, rate α is the per capita rate of predation of the intermediate predator, rate β measures the efficiency of biomass conversion from prey to intermediate predator, rate γ is the per capita rate of predation of the top predator, rate ρ measures the efficiency of biomass conversion from intermediate predator to top predator, , rate ω_0 is the per capita rate of predation of the intermediate predator, rate ω_1 measures the efficiency of biomass conversion from infected prey to intermediate predator. Rate c is the contact between susceptible prey (S. Prey) and infected prey (I. Prey) while rate δ denoted the transformation from I. Prey to S, Prey, d_1, d_2 are natural death of intermediate and top predator respectively. Rate q is harvesting of I. Prey.

3. Nature of Solution

Lemma 1: All solutions of system (1) which initiate in R_+^4 are positive and bounded.

Proof:

Let $M = x + x_1 + y + z$ and $\mu > 0$

$$\frac{dM}{dt} + \mu x = dx + dx_1 + dy + dz + \mu x$$

$$= rx(1-x) - \alpha\frac{xy}{1+\omega x} - c\frac{xx_1}{x+x_1} + \delta x_1$$

$$+c\frac{xx_1}{x+x_1} - \delta x_1 - w_0 x_1 y - qx_1$$

$$+w_1 x_1 y + \beta\frac{xy}{1+\omega x} - \gamma\frac{yz}{1+\omega y}$$

$$-d_1 y + \rho\frac{yz}{1+\omega y} - d_2 z + \mu x$$

$$= rx(1-x) - (\alpha-\beta)\frac{xy}{1+\omega x} - (w_0 - w_1)x_1 y$$

$$-qx_1 - (\gamma-\rho)\frac{yz}{1+\omega y} - d_1 y - d_2 z + \mu x$$

$$\le rx(1-x) + \mu x$$

$$\le rx + \mu x - rx^2$$

$$\le -rx^2 + (r+\mu)x$$

$$\le -r\left(x^2 - \frac{(r+\mu)x}{r}\right)$$

$$\le -r\left(x^2 - \frac{(r+\mu)x}{r} + \frac{(r+\mu)^2}{4r^2}\right) + \frac{(r+\mu)^2}{4r}$$

$$\le -r\left(x - \left(\frac{r+\mu}{2r}\right)\right)^2 + \frac{1}{r}\left(\frac{r+\mu}{2}\right)^2$$

$$dM + \mu x \le \frac{1}{r}\left(\frac{r+\mu}{2}\right)^2 \ \text{say } v$$

$$\frac{d\mu}{dt} \le \frac{1}{4r}(r+\mu)^2, \ \text{say } v.$$

Then, by using differential inequality [11], we get

$$0 < \mu(x(t), x_1(t), y(t), z(t)) \le \frac{v}{\mu}(1 - e^{-\mu t})$$

$$+(x(t), x_1(t), y(t), z(t))e^{-\mu t}$$

4. Subsystems

In absence one or two of population, system (1) reduce to subsystems. For the purpose of studying dynamics of system (1), we study all the possibilities that we mean by subsystems. There are several subsystems as follows:

4.1 In Case of Absence predator

In absence of all predator system (1) became as the subsystem content susceptible and infected prey as follows:

$$\left.\begin{aligned} \frac{dx}{dt} &= rx(1-x) - c\frac{xx_1}{x+x_1} + \delta x_1 \\ \frac{dx_1}{dt} &= c\frac{xx_1}{x+x_1} - \delta x_1 - qx_1 \end{aligned}\right\} (2)$$

4.1.1 Nature of Solution

Lemma 2: All solutions of subsystem (2) are positive and bounded.

Proof: As lemma 1, see figure 1.

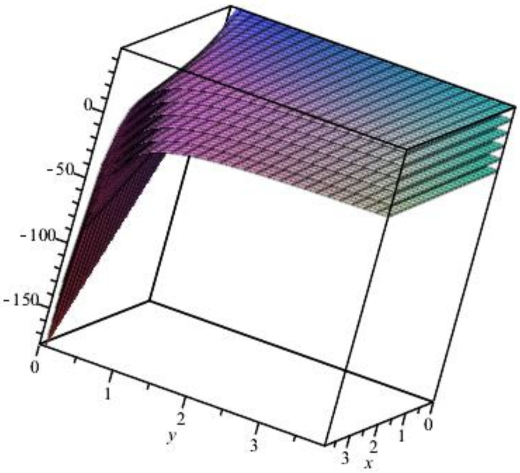

Fig. 1: shows the path of the subsystem (2) in the absence of the predator

Lemma 3: The subsystem (2) has no periodic orbit in R^2.

Proof:

Let $H=\frac{1}{xx_1}$, $h_1=rx(1-x)-c\frac{xx_1}{x+x_1}+\delta x_1$ and $h_2=c\frac{xx_1}{x+x_1}-\delta x_1-qx_1$

$H.h_1=\frac{r}{x_1}-\frac{rx}{x_1}-\frac{c}{x+x_1}+\frac{\delta}{x}$ and

$H.h_2=\frac{c}{x+x_1}-\frac{\delta}{x}-\frac{q}{x}$, then

$\Delta(x,x_1)=\frac{\partial(h_1,H)}{\partial x}+\frac{\partial(h_2,H)}{\partial x_1}=-\frac{r}{x_1}-\frac{\delta}{x^2}$. Now, we note that $\Delta(x,x_1)$ does not change sign also is not identically zero and is not identically zero in R^2 in its plane. According to Bendixson - Dulic criterion there is no periodic solution. [12].

Lemma 4: In subsystem 2, $c>\delta+q$

Proof: if $c\le\delta+q$ and since the carrying capacity of prey in one, then $c\frac{x}{x+x_1}\le c$ implies $c\frac{x}{x+x_1}\le\delta+q$, therefore $\frac{dx_1}{dt}\le 0$ which contradiction, then $c>\delta+q$.

4.1.2. Equilibrium Points and Stability.

In subsystem (2) there are three equilibrium points as following

1. The trivial point and always exists $\bar{p}_0(0,0)$
2. This point is a border point and always exists $\bar{p}_1(1,0)$
3. $\bar{p}_2(\bar{x},\bar{x}_1)$ where $\bar{x}=\frac{(\delta+q)\bar{x}_1}{c-(\delta+q)}$ and Jacobean matrix of system (2) is

$$J_2=\begin{bmatrix} r-2rx-c\frac{x_1^2}{(x+x_1)^2} & -c\frac{x^2}{(x+x_1)^2}+\delta \\ c\frac{x_1^2}{(x+x_1)^2} & c\frac{x^2}{(x+x_1)^2}-\delta-q \end{bmatrix}$$

We study the stability of positive equilibrium point $\bar{p}_2(\bar{x},\bar{x}_1)$ and remind the other later. Jacobian matrix near this point is

$$J_2=\begin{bmatrix} r-2r\bar{x}-c\frac{\bar{x}_1^2}{(\bar{x}+\bar{x}_1)^2} & -c\frac{\bar{x}^2}{(\bar{x}+\bar{x}_1)^2}+\delta \\ c\frac{\bar{x}_1^2}{(\bar{x}+\bar{x}_1)^2} & (\delta+q)\left(\frac{(\delta+q)-c}{c}\right) \end{bmatrix}$$

Hence it's stability with condition $\bar{x}\ge\frac{1}{2}$ this condition reduce to the main diameter will be positive and in other hand the secondary diameter is negative.

Lemma 5: The equilibrium $\bar{p}_2$ is global stability in the first positive cone.

Proof: The unique positive equilibrium point $\bar{p}_2(\bar{x},\bar{x}_1)$ is locally asymptotically stable and subsystem (2) has no periodic solution in R_+^2 then by using Poincare-Bendixon theorem, $\bar{p}_2(\bar{x},\bar{x}_1)$ is globally asymptotically stable, see Figure (2-a) and Figure (2-b).

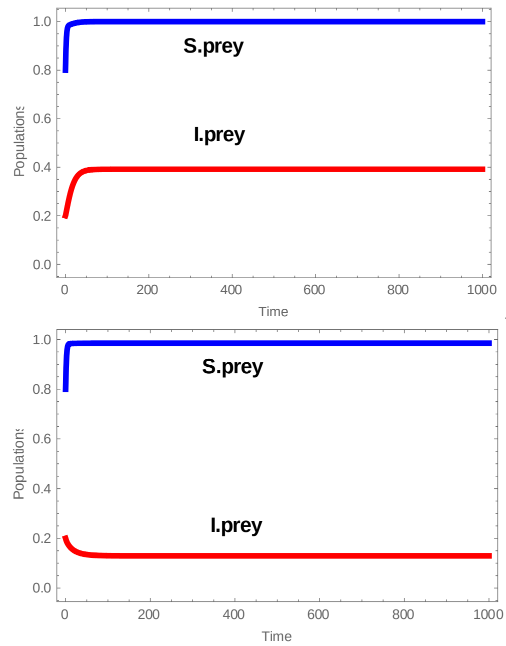

Fig. 2: stability (a) without harvesting (b) with harvesting

4.2. Prey Predator Subsystem

This subsystem has a healthy prey and predator. As this system known prey predator model or Lotka Volterra equations. We describe interaction between them as:

$$\left.\begin{aligned} \frac{dx}{dt}&=rx(1-x)-\alpha\frac{xy}{1+\omega x}\\ \frac{dy}{dt}&=\beta\frac{xy}{1+\omega x}-d_1y \end{aligned}\right\}(3)$$

4.2.1. Nature of Solution

Lemma 6: All solutions of subsystem (3) are positive and bounded.

Proof: As lemma 1, see figure 3.

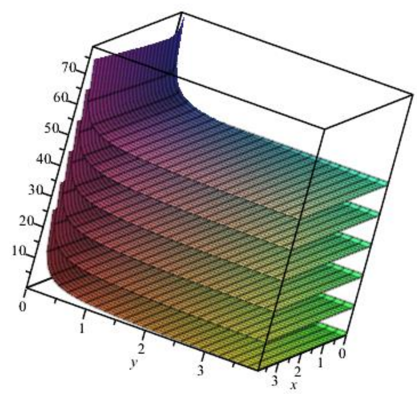

Fig. 3: The solutions of system (3) is bounded.

Lemma 7: In subsystem (3) $\beta > \omega d_1$.

Proof:

Assume $\beta \le \omega d_1$ since caring capacity of prey is one, then $\beta x \le \omega d_1$ hence $\frac{\beta x}{1+\omega x} \le \omega d_1$ therefore $\frac{dy}{dt} \le 0$.

Its contradiction, then $\beta > \omega d_1$

Lemma 8: subsystem (3) has no periodic orbit in R^2.

Proof: As lemma 3.

4.2.2. Equilibrium Points and Stability

This subsystem also contains three equilibrium points

1. $\hat{p}_0 = (0,0)$
2. $\hat{p}_1 = (1,0)$
3. $\hat{p}_2 = (\hat{x}, \hat{y})$ where

$\hat{x} = \frac{d_1}{\beta - d_1\omega}$ *and* $\hat{y} = \frac{1+\omega\hat{x}}{\alpha}(r - r\hat{x})$, the jacobean matrix of system (3) is

$$J_3 = \begin{bmatrix} r - 2r\hat{x} - \frac{\hat{y}}{(1+\omega\hat{x})^2} & -\alpha\frac{\hat{x}}{1+\omega\hat{x}} \\ \frac{\beta\hat{y}}{(1+\omega\hat{x})^2} & \beta\frac{\hat{x}}{1+\omega\hat{x}} \end{bmatrix}$$

And near $\hat{p}_2 = (\hat{x}, \hat{y})$ is

$$J_3 = \begin{bmatrix} r - 2r\hat{x} - \frac{\hat{y}}{(1+\omega\hat{x})^2} - \lambda & -\alpha\frac{\hat{x}}{1+\omega\hat{x}} \\ \frac{\beta\hat{y}}{(1+\omega\hat{x})^2} & -\lambda \end{bmatrix}$$

Then the characteristic equation is

$$\lambda^2 + \left(r(-1+2\hat{x}) + \frac{\hat{y}}{(1+\omega\hat{x})^2} \right)\lambda + \frac{\alpha\beta\hat{x}\hat{y}}{(1+\omega\hat{x})^3} = 0,$$ its

stability if $\hat{x} \ge \frac{1}{2}$.

Lemma 9: The equilibrium $\hat{p}_2 = (\hat{x}, \hat{y})$ is global stability

Proof: As in lemma 5, see figure (4).

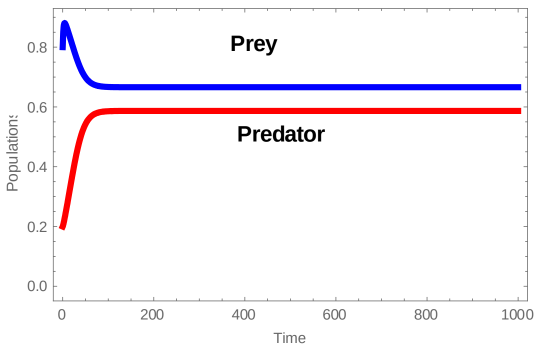

Fig. 4: The oscillation of solution of system (4)

4.3. Classical Subsystem with Disease

In absence of top predator, system (1) become classical model with disease. This subsystem known SIS model because susceptible population become infected by rate c and transform to susceptible by rate δ again, hence we describe that as:

$$\left.\begin{aligned} \frac{dx}{dt} &= rx(1-x) - \alpha\frac{xy}{1+\omega x} - c\frac{xx_1}{x+x_1} + \delta x_1 \\ \frac{dx_1}{dt} &= c\frac{xx_1}{x+x_1} - \delta x_1 - \omega_0 x_1 y - qx_1 \\ \frac{dy}{dt} &= \omega_1 x_1 y + \beta\frac{xy}{1+\omega x} - d_1 y \end{aligned}\right\} (4)$$

4.3.1 Nature of Solution

Lemma 10: All solutions of system (4) which initiate in R^3_+ are positive and bounded.

Proof: As lemma (1)

4.3.2. Equilibrium Points and Stability

And then subsystem also contains three equilibrium points

1. $\bar{\bar{p}}_0 = (0,0,0)$ The point is trivial means society is nil
2. $\bar{\bar{p}}_1 = (1,0,0)$ Here only the sound prey exists
3. $\bar{\bar{p}}_2 = (\bar{\bar{x}}, \bar{\bar{x}}_1, \bar{\bar{y}})$ The equilibrium point when the system does not contain the top predator where

$\bar{\bar{x}} = \frac{\bar{\bar{x}}_1(\delta + \omega_0\bar{\bar{y}} + q)}{c - (\delta + \omega_0\bar{\bar{y}} + q)}$,

$\bar{\bar{x}}_1 = \frac{1}{\omega_1}\left(d_1 - \beta\frac{\bar{\bar{x}}}{1+\omega\bar{\bar{x}}} \right)$,

$\bar{\bar{y}} = \frac{1+\omega\bar{\bar{x}}}{\alpha\bar{\bar{x}}}\left(r\bar{\bar{x}}(1-\bar{\bar{x}}) - c\frac{\bar{\bar{x}}\bar{\bar{x}}_1}{\bar{\bar{x}}+\bar{\bar{x}}_1} + \delta\bar{\bar{x}}_1 \right)$

Jacobean matrix of system as

$\frac{\partial f_1}{\partial x} = r - 2rx - \frac{\alpha y}{(1+\omega x)^2} - \frac{cx_1^2}{(x+x_1)^2}$,

$\frac{\partial f_1}{\partial x_1} = -\frac{cx^2}{(x+x_1)^2} + \delta$, $\frac{\partial f_1}{\partial y} = -\frac{\alpha x}{1+\omega x}$

$\frac{\partial f_2}{\partial x} = \frac{cx_1^2}{(x+x_1)^2}$, $\frac{\partial f_2}{\partial x_1} = \frac{cx^2}{(x+x_1)^2} - \delta - \omega_0 y - q$

, $\frac{\partial f_2}{\partial y} = -\omega_0 x_1$

$\frac{\partial f_3}{\partial x} = \frac{\beta y}{(1+\omega x)^2}$, $\frac{\partial f_3}{\partial x_1} = \omega_1 y$,

$\frac{\partial f_3}{\partial y} = \omega_1 x_1 + \frac{\beta x}{1+\omega x} - d_1$

Jacobian matrix near a positive equilibrium point is:

$\frac{\partial f_1}{\partial \bar{\bar{x}}} = r - 2r\bar{\bar{x}} - \frac{\alpha\bar{\bar{y}}}{(1+\omega\bar{\bar{x}})^2} - \frac{c\bar{\bar{x}}_1^2}{(\bar{\bar{x}}+\bar{\bar{x}}_1)^2}$,

$\frac{\partial f_1}{\partial \bar{\bar{x}}_1} = -\frac{c\bar{\bar{x}}^2}{(\bar{\bar{x}}+\bar{\bar{x}}_1)^2} + \delta$, $\frac{\partial f_1}{\partial \bar{\bar{y}}} = -\frac{\alpha\bar{\bar{x}}}{1+\omega\bar{\bar{x}}}$

$\frac{\partial f_2}{\partial \bar{\bar{x}}} = \frac{c\bar{\bar{x}}_1^2}{(\bar{\bar{x}}+\bar{\bar{x}}_1)^2}$, $\frac{\partial f_2}{\partial \bar{\bar{y}}} = -\omega_0\bar{\bar{x}}_1$

$\frac{\partial f_2}{\partial \bar{\bar{x}}_1} = \frac{-(\delta + \omega_0\bar{\bar{y}} + q)(c - (\delta + \omega_0\bar{\bar{y}} + q))}{c}$,

$\frac{\partial f_3}{\partial \bar{\bar{x}}} = \frac{\beta\bar{\bar{y}}}{(1+\omega\bar{\bar{x}})^2}$, $\frac{\partial f_3}{\partial \bar{\bar{x}}_1} = \omega_1\bar{\bar{y}}$, $\frac{\partial f_3}{\partial \bar{\bar{y}}} = 0$ let

$k_1 = r - 2r\bar{\bar{x}} - \frac{\alpha\bar{\bar{y}}}{(1+\omega\bar{\bar{x}})^2} - \frac{c\bar{\bar{x}}_1^2}{(\bar{\bar{x}}+\bar{\bar{x}}_1)^2}$,

$k_2 = -\frac{c\bar{\bar{x}}^2}{(\bar{\bar{x}}+\bar{\bar{x}}_1)^2} + \delta$, $k_3 = -\frac{\alpha\bar{\bar{x}}}{1+\omega\bar{\bar{x}}}$

$k_4 = \frac{c\bar{\bar{x}}_1^2}{(\bar{\bar{x}}+\bar{\bar{x}}_1)^2}$, $k_5 = -\frac{(\delta+\omega_0\bar{\bar{y}}+q)(c-(\delta+\omega_0\bar{\bar{y}}+q))}{c}$

, $k_6 = -\omega_0\bar{\bar{x}}_1$

$k_7 = \frac{\beta\bar{\bar{y}}}{(1+\omega\bar{\bar{x}})^2}$, $k_8 = \omega_1\bar{\bar{y}}$, $k_9 = 0$

Equilibrium Point $\bar{\bar{p}}_3 = (\bar{\bar{x}}, \bar{\bar{x}}_1, \bar{\bar{y}})$

$\lambda^3 + A\lambda^2 + B\lambda + C = 0$, where

$A = -(Arc\ J(p_3)) > 0$ if $\bar{\bar{x}} \ge \frac{1}{2}$

$B = (k_1k_5 - k_6k_8 - k_2k_4 - k_3k_7)$

$C = (k_1k_6 - k_3k_4)k_8 - (k_2k_6 - k_3k_5)k_7 > 0$

The Equilibrium Point $\bar{\bar{p}}_3$ is stable if $AB - C > 0$.

Lemma 11: The equilibrium $\bar{\bar{p}}_3 = (\bar{\bar{x}}, \bar{\bar{x}}_1, \bar{\bar{y}})$ is global stability with conditions $y\bar{\bar{x}} < \bar{\bar{y}}x$, $\bar{\bar{x}}x_1 < \bar{\bar{x}}_1 x$ and $\frac{c}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)} < \frac{\delta}{\bar{\bar{x}}x}$

Proof:

$$W(x,x_1,y) = C_1\left(x - \bar{\bar{x}} - \bar{\bar{x}}\ln\frac{x}{\bar{\bar{x}}}\right) + C_2\left(x_1 - \bar{\bar{x}}_1 - \bar{\bar{x}}_1\ln\frac{x_1}{\bar{\bar{x}}_1}\right) + C_3\left(y - \bar{\bar{y}} - \bar{\bar{y}}\ln\frac{y}{\bar{\bar{y}}}\right)$$

$$\frac{dW}{dt} = C_1\left(\frac{x-\bar{\bar{x}}}{x}\right)dx + C_2\left(\frac{x_1-\bar{\bar{x}}_1}{x_1}\right)dx_1 + C_3\left(\frac{y-\bar{\bar{y}}}{y}\right)dy$$

$$\frac{dW}{dt} = (x-\bar{\bar{x}})C_1\left(r - rx - \frac{\alpha y}{1+\omega x} - \frac{cx_1}{x+x_1} + \delta\frac{x_1}{x}\right) +$$
$$(x_1-\bar{\bar{x}}_1)C_2\left(\frac{cx}{x+x_1} - \delta - \omega_0 y - q\right) + (y-\bar{\bar{y}})C_3\left(\omega_1 x_1 + \frac{\beta x}{1+\omega x} - d_1\right)$$

$$\frac{dW}{dt} = (x-\bar{\bar{x}})C_1\left(-r(x-\bar{\bar{x}}) - \alpha\left(\frac{y}{1+\omega x} - \frac{\bar{\bar{y}}}{1+\omega\bar{\bar{x}}}\right) - c\left(\frac{x_1}{x+x_1} - \frac{\bar{\bar{x}}_1}{\bar{\bar{x}}+\bar{\bar{x}}_1}\right) + \delta\left(\frac{x_1}{x} - \frac{\bar{\bar{x}}_1}{\bar{\bar{x}}}\right)\right)$$
$$+(x_1-\bar{\bar{x}}_1)C_2\left(c\left(\frac{x}{x+x_1} - \frac{\bar{\bar{x}}}{\bar{\bar{x}}+\bar{\bar{x}}_1}\right) - \omega_0(y-\bar{\bar{y}})\right)$$
$$+(y-\bar{\bar{y}})C_3\left(\omega_1(x_1-\bar{\bar{x}}_1) + \beta\left(\frac{x}{1+\omega x} - \frac{\bar{\bar{x}}}{1+\omega\bar{\bar{x}}}\right)\right)$$

Let $C_3 = C_2\frac{\omega_0}{\omega_1}$

$$\frac{dW}{dt} = (x-\bar{\bar{x}})C_1\left(-r(x-\bar{\bar{x}}) - \alpha\left(\frac{y}{1+\omega x} - \frac{\bar{\bar{y}}}{1+\omega\bar{\bar{x}}}\right) - c\left(\frac{x_1}{x+x_1} - \frac{\bar{\bar{x}}_1}{\bar{\bar{x}}+\bar{\bar{x}}_1}\right) + \delta\left(\frac{x_1}{x} - \frac{\bar{\bar{x}}_1}{\bar{\bar{x}}}\right)\right)$$
$$+c(x_1-\bar{\bar{x}}_1)C_2\left(\frac{x}{x+x_1} - \frac{\bar{\bar{x}}}{\bar{\bar{x}}+\bar{\bar{x}}_1}\right) + (y-\bar{\bar{y}})C_2\beta\frac{\omega_0}{\omega_1}\left(\frac{x}{1+\omega x} - \frac{\bar{\bar{x}}}{1+\omega\bar{\bar{x}}}\right)$$

$$\frac{dW}{dt} = -rC_1(x-\bar{\bar{x}})^2 - \alpha C_1(x-\bar{\bar{x}})\left(\frac{y}{1+\omega x} - \frac{\bar{\bar{y}}}{1+\omega\bar{\bar{x}}}\right) - cC_1(x-\bar{\bar{x}})\left(\frac{\bar{\bar{x}}x_1 - \bar{\bar{x}}_1 x}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)}\right)$$
$$+\delta C_1(x-\bar{\bar{x}})\left(\frac{x_1}{x} - \frac{\bar{\bar{x}}_1}{\bar{\bar{x}}}\right) - c(x_1-\bar{\bar{x}}_1)C_2\left(\frac{\bar{\bar{x}}x_1 - \bar{\bar{x}}_1 x}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)}\right)$$
$$+(y-\bar{\bar{y}})C_2\beta\frac{\omega_0}{\omega_1}\left(\frac{x-\bar{\bar{x}}}{(1+\omega x)(1+\omega\bar{\bar{x}})}\right)$$

$$\frac{dW}{dt} = -rC_1(x-\bar{\bar{x}})^2 - \alpha C_1(x-\bar{\bar{x}})\left(\frac{y + \omega y\bar{\bar{x}} - \bar{\bar{y}} - \omega\bar{\bar{y}}x}{(1+\omega x)(1+\omega\bar{\bar{x}})}\right) - cC_1(x-\bar{\bar{x}})\left(\frac{\bar{\bar{x}}x_1 - \bar{\bar{x}}_1 x}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)}\right)$$
$$+\delta C_1(x-\bar{\bar{x}})\left(\frac{x_1}{x} - \frac{\bar{\bar{x}}_1}{\bar{\bar{x}}}\right) - c(x_1-\bar{\bar{x}}_1)C_2\left(\frac{\bar{\bar{x}}x_1 - \bar{\bar{x}}_1 x}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)}\right) + (y-\bar{\bar{y}})C_2\beta\frac{\omega_0}{\omega_1}\left(\frac{x-\bar{\bar{x}}}{(1+\omega x)(1+\omega\bar{\bar{x}})}\right)$$

$$\frac{dW}{dt} = -rC_1(x-\bar{\bar{x}})^2 - \alpha C_1\left(\frac{xy - \bar{\bar{x}}y + \omega y\bar{\bar{x}}x - \omega y\bar{\bar{x}}\bar{\bar{x}} - x\bar{\bar{y}} + \bar{\bar{x}}\bar{\bar{y}} - \omega\bar{\bar{y}}xx + \omega\bar{\bar{y}}x\bar{\bar{x}}}{(1+\omega x)(1+\omega\bar{\bar{x}})}\right) - cC_1(x-\bar{\bar{x}})\left(\frac{\bar{\bar{x}}x_1 - \bar{\bar{x}}_1 x}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)}\right)$$
$$+\delta C_1(x-\bar{\bar{x}})\left(\frac{x_1}{x} - \frac{\bar{\bar{x}}_1}{\bar{\bar{x}}}\right) - c(x_1-\bar{\bar{x}}_1)C_2\left(\frac{\bar{\bar{x}}x_1 - \bar{\bar{x}}_1 x}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)}\right) + C_2\beta\frac{\omega_0}{\omega_1}\left(\frac{xy - x\bar{\bar{y}} - \bar{\bar{x}}y + \bar{\bar{x}}\bar{\bar{y}}}{(1+\omega x)(1+\omega\bar{\bar{x}})}\right)$$

Let $C_2\beta\frac{\omega_0}{\omega_1} = C_1\alpha$

$$\frac{dW}{dt}=-rC_1\left(x-\bar{\bar{x}}\right)^2-\alpha C_1\left(\frac{+\omega y\bar{\bar{x}}x-\omega y\bar{\bar{x}}\bar{\bar{x}}-\omega\bar{\bar{y}}xx+\omega\bar{\bar{y}}x\bar{\bar{x}}}{(1+\omega x)(1+\omega\bar{\bar{x}})}\right)-cC_1\left(x-\bar{\bar{x}}\right)\left(\frac{\bar{\bar{x}}x_1-\bar{\bar{x}}_1x}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)}\right)$$
$$+\delta C_1\left(x-\bar{\bar{x}}\right)\left(\frac{x_1}{x}-\frac{\bar{\bar{x}}_1}{\bar{\bar{x}}}\right)-c\left(x_1-\bar{\bar{x}}_1\right)C_2\left(\frac{\bar{\bar{x}}x_1-\bar{\bar{x}}_1x}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)}\right)$$

$$\frac{dW}{dt}=-rC_1\left(x-\bar{\bar{x}}\right)^2-\alpha\omega C_1\left(\frac{(y\bar{\bar{x}}-\bar{\bar{y}}x)(x-\bar{\bar{x}})}{(1+\omega x)(1+\omega\bar{\bar{x}})}\right)-cC_1\left(x-\bar{\bar{x}}\right)\left(\frac{\bar{\bar{x}}x_1-\bar{\bar{x}}_1x}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)}\right)$$
$$+\delta C_1\left(x-\bar{\bar{x}}\right)\left(\frac{x_1}{x}-\frac{\bar{\bar{x}}_1}{\bar{\bar{x}}}\right)-c\left(x_1-\bar{\bar{x}}_1\right)C_2\left(\frac{\bar{\bar{x}}x_1-\bar{\bar{x}}_1x}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)}\right)$$

$$\frac{dW}{dt}=-rC_1\left(x-\bar{\bar{x}}\right)^2-\alpha\omega C_1\left(\frac{(y\bar{\bar{x}}-\bar{\bar{y}}x)(x-\bar{\bar{x}})}{(1+\omega x)(1+\omega\bar{\bar{x}})}\right)-cC_1\left(x-\bar{\bar{x}}\right)\left(\frac{\bar{\bar{x}}x_1-\bar{\bar{x}}_1x}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)}\right)$$
$$+\delta C_1\left(x-\bar{\bar{x}}\right)\left(\frac{x_1}{x}-\frac{\bar{\bar{x}}_1}{\bar{\bar{x}}}\right)-c\left(x_1-\bar{\bar{x}}_1\right)C_2\left(\frac{\bar{\bar{x}}x_1-\bar{\bar{x}}_1x}{(x+x_1)(\bar{\bar{x}}+\bar{\bar{x}}_1)}\right)<0$$

Figure (5-a) and (5-b) Oscillation of system (4) when we fixed the parameters as: $r=0.856,\alpha=0.482,\beta=0.247,$ $\delta=0.008,c=0.5,$ $\omega=0.503,q=0.022,d_1=0.057,\omega_0=0.181,\omega_1=0.109$

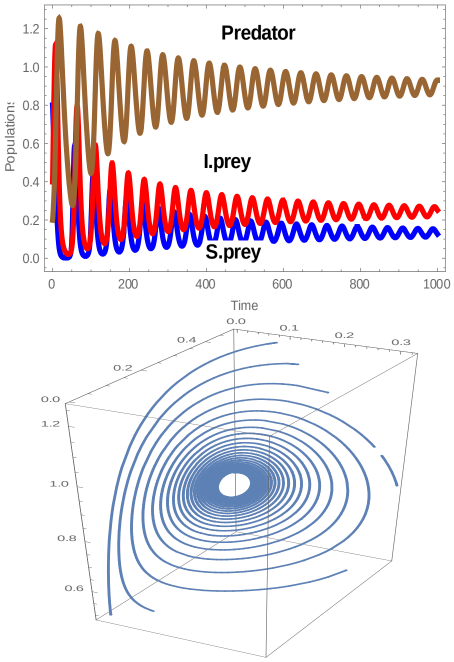

Fig. (5-a) and (5-b):Solution of subsystem (4).

4.4. Subsystem without Disease.

In the case of absence disease, subsystem known as food chain model. Food Chain Model consist three populations, prey, intermediate predator and top predator. In such model, intermediate predator depends entirely on prey in its food, in other word, no resource in his food except prey. Also, no source for top predator except intermediate predator. Describe this subsystem as follows:

$$\frac{dx}{dt}=rx(1-x)-\alpha\frac{xy}{1+\omega x}$$
$$\frac{dy}{dt}=\beta\frac{xy}{1+\omega x}-\gamma\frac{yz}{1+\omega y}-d_1y \quad (5)$$
$$\frac{dz}{dt}=\rho\frac{yz}{1+\omega y}-d_2z$$

4.4.1. Nature of Solution

lemma 12:

All the solutions of the subsystem (5) in R^3 are positive and bounded.

Proof: As lemma (1).

Lemma 13: In subsystem (5) $\rho>\omega d_2$.

Proof: As lemma 7.

4.4.2. Equilibrium Points and Stability

And then subsystem also contains three equilibrium points

1. $\tilde{p}_0=(0,0,0)$
2. $\tilde{p}_1=(1,0,0)$
3. $\tilde{p}_3=(x_*,y_*,z_*)$

$$x_*=\frac{r(\omega-1)\pm\sqrt{r^2(\omega-1)^2+4\omega r(\mathrm{r}-\alpha y_*)}}{2\omega r},$$

$$y_*=\frac{d_2}{\rho-\omega d_2},z_*=\frac{1+\omega y_*}{\gamma}\left(\frac{\beta x_*}{1+\omega x_*}-d_1\right)$$

The Jacobean matrix of subsystem (5) as:

$$M_1=r-2rx-\frac{\alpha y}{(1+\omega x)^2},\ M_2=-\frac{\alpha x}{1+\omega x},$$

$$M_3=\frac{\beta y}{(1+\omega x)^2},M_4=\frac{\beta x}{1+\omega x}-\frac{\gamma z}{(1+\omega y)^2}-d_1$$ The

$$,M_5=\frac{-\gamma y}{1+\omega y},M_6=\frac{\rho z}{(1+\omega y)^2}$$

$$,M_7=\frac{\rho y}{1+\omega y}-d_2.$$

characteristic equation near the interior equilibrium point is

$\lambda^3+A\lambda^2+B\lambda+C=0$ where

$$A=(-M_1-M_4-M_7)$$
$$B=(M_1M_4+M_1M_7+M_4M_7-M_5M_6-M_2M_3)$$
$$C=(-M_1M_4M_7+M_1M_5M_6+M_2M_3M_7)$$

Then from Routh Hurwitz criteria, this point is stable if $A>0$, $C>0$ and $AB-C>0$

Lemma 14: The equilibrium $\tilde{p}_3=(x_*,y_*,z_*)$ is global stability

Proof : as in lemma 11

5. Existence of Equilibrium Point and Stability of General System .

And then system also contains three equilibrium points

1. The vanishing equilibrium point $E_0=(0,0,0,0)$ always exist
2. The axial equilibrium point $E_1=(1,0,0,0)$ always exist
3. The positive equilibrium point $E_6=(x^*,x_1^*,y^*,z^*)$

$$x^*=\frac{x_1^*\left(\delta+\omega_0 y^*+q\right)}{c-\left(\delta+\omega_0 y^*+q\right)}$$

$$x_1^*=\frac{1}{\omega_1}\left(\frac{\gamma z^*}{1+\omega y^*}+d_1-\frac{\beta x^*}{1+\omega x^*}\right)$$

$$y^*=\frac{d_2}{(\rho-\omega d_2)}$$

$$z^*=\frac{1+\omega y^*}{\gamma}\left(\omega_1 x_1^*+\beta\frac{x^*}{1+\omega x^*}-d_1\right)$$ with condition $c>\left(\delta+\omega_0 y^*+q\right)$

The Jacobean matrix of system is

$$J_6=\begin{bmatrix}\frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z}\\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z}\\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z}\\ \frac{\partial f_4}{\partial x} & \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial y} & \frac{\partial f_4}{\partial z}\end{bmatrix}$$, where

$$\frac{\partial f_1}{\partial x}=r-2rx-\frac{\alpha y}{(1+\omega x)^2}-\frac{cx_1^2}{(x+x_1)^2},$$

$$\frac{\partial f_1}{\partial x_1}=\frac{cx^2}{(x+x_1)^2}+\delta,\ \frac{\partial f_1}{\partial y}=\frac{-\alpha x}{b+x^2}$$

$$\frac{\partial f_2}{\partial x}=\frac{cx_1^2}{(x+x_1)^2},\ \frac{\partial f_2}{\partial x_1}=\frac{cx^2}{(x+x_1)^2}-\delta-\omega_0 y-q$$

$$,\frac{\partial f_2}{\partial y}=-\omega_0 x_1$$

$$\frac{\partial f_3}{\partial x}=\frac{\beta y}{(1+\omega x)^2},\ \frac{\partial f_3}{\partial x_1}=\omega_1 y,$$

$$\frac{\partial f_3}{\partial y}=\omega_1 x_1+\frac{\beta x}{1+\omega x}-\frac{\gamma z}{(1+\omega y)^2}-d_1,\ \frac{\partial f_3}{\partial z}=\frac{-\gamma y}{1+\omega y}$$

$$\frac{\partial f_4}{\partial y}=\frac{\rho z}{(1+\omega y)^2},\ \frac{\partial f_4}{\partial z}=\frac{\rho y}{1+\omega y}-d_2,$$

$$\frac{\partial f_4}{\partial x}=\frac{\partial f_2}{\partial z}=\frac{\partial f_1}{\partial z}=\frac{\partial f_4}{\partial x_1}=0$$

1. The eigenvalues near $E_0=(0,0,0,0)$ is

$\lambda_1=r$, $\lambda_2=-(\delta+q)$, $\lambda_3=-d_1$, $\lambda_4=-d_2$

Saddle point.

All societies are extinct except for prey because intrinsic value is positive.

2. Equilibrium Point $E_1=(1,0,0,0)$

$$\begin{bmatrix} r-2rx-\frac{\alpha y}{(1+\omega x)}-\frac{cx_1^2}{(x+x_1)^2} & \frac{cx^2}{(x+x_1)^2}+\delta & -\frac{\alpha x}{1+\omega x} & 0\\ \frac{cx^2}{(x+x_1)^2} & \frac{cx^2}{(x+x_1)^2}-\delta-\omega_0 y-q & \omega_0 x_1 & 0\\ \frac{\beta y}{(1+\omega x)^2} & \omega_1 y & \frac{\beta x}{1+\omega x}-\frac{\gamma z}{(1+\omega y)^2}-d_1 & -\frac{\gamma y}{1+\omega y}\\ 0 & 0 & \frac{\rho z}{(1+\omega y)^2} & \frac{\rho y}{1+\omega y}-d_2 \end{bmatrix}$$

$$|J(E_1)-\lambda I|=0$$

$$\begin{vmatrix} -r-\lambda & c+\delta & -\frac{\alpha}{1+\omega} & 0\\ c & c-\delta-q-\lambda & 0 & 0\\ 0 & 0 & \frac{\beta}{1+\omega}-d_1-\lambda & 0\\ 0 & 0 & 0 & -d_2-\lambda \end{vmatrix}=0$$

$$\lambda^4\oplus(r+\delta+q-\frac{\beta}{1+\omega}+d_1+d_2)\lambda^3\oplus$$
$$(\delta r+qr-\frac{\beta r}{1+\omega}+rd_1+rd_2+\frac{\beta c}{1+\omega}-cd_1$$
$$-cd_2-c-\frac{\beta\delta}{1+\omega}+\delta d_2+\delta d_1-\frac{\beta q}{1+\omega}+$$
$$qd_1+qd_2-\frac{\beta d_2}{1+\omega}+d_1d_2-c^2-c\delta)\lambda^2$$
$$\oplus(\frac{c\beta r}{1+\omega}-crd_1-crd_2-cr-\frac{\beta r\delta}{1+\omega}+r\delta d_1$$
$$+r\delta d_2-\frac{\beta rq}{1+\omega}+qrd_1+qrd_2-\frac{\beta rd_2}{1+\omega}+rd_1d_2$$
$$+\frac{c\beta d_2}{1+\omega}-cd_1d_2-\frac{\beta\delta d_2}{1+\omega}+\delta d_1d_2-\frac{\beta qd_2}{1+\omega}$$
$$+qd_1d_2+\frac{\beta c^2}{1+\omega}-c^2d_1-c^2d_2-\frac{c\beta\delta}{1+\omega}+$$
$$\delta cd_1+\delta cd_2)\lambda\oplus(\frac{c\beta rd_2}{1+\omega}-\frac{\beta r\delta d_2}{1+\omega}-$$
$$\frac{\beta rqd_2}{1+\omega}+\frac{\beta c^2d_2}{1+\omega}+\frac{\beta\delta cd_2}{1+\omega}-crd_1d_2+\delta rd_1d_2$$
$$+qrd_1d_2-c^2d_1d_2+\delta cd_1d_2)=0$$

$\lambda^4+A\lambda^3+B\lambda^2+C\lambda+D=0$ where

$$A=r+\delta+q-\frac{\beta}{1+\omega}+d_1+d_2$$

$$B=\delta r+qr-\frac{\beta r}{1+\omega}+rd_1+rd_2+\frac{\beta c}{1+\omega}-cd_1-cd_2$$
$$-c-\frac{\beta\delta}{1+\omega}+\delta d_2+\delta d_1-\frac{\beta q}{1+\omega}+qd_1+qd_2-\frac{\beta d_2}{1+\omega}$$
$$+d_1d_2-c^2-c\delta$$

$$C=\frac{c\beta r}{1+\omega}-crd_1-crd_2-cr-\frac{\beta r\delta}{1+\omega}+r\delta d_1+r\delta d_2$$
$$-\frac{\beta rq}{1+\omega}+qrd_1+qrd_2-\frac{\beta rd_2}{1+\omega}+rd_1d_2+\frac{c\beta d_2}{1+\omega}-cd_1d_2$$
$$-\frac{\beta\delta d_2}{1+\omega}+\delta d_1d_2-\frac{\beta qd_2}{1+\omega}+qd_1d_2+\frac{\beta c^2}{1+\omega}-c^2d_1-c^2d_2$$
$$-\frac{c\beta\delta}{1+\omega}+\delta cd_1+\delta cd_2$$
$$D=\Big(\frac{c\beta rd_2}{1+\omega}-\frac{\beta r\delta d_2}{1+\omega}-\frac{\beta rqd_2}{1+\omega}+\frac{\beta c^2d_2}{1+\omega}+\frac{\beta\delta cd_2}{1+\omega}$$
$$-crd_1d_2+\delta rd_1d_2+qrd_1d_2-c^2d_1d_2+\delta cd_1d_2\Big)$$
$$\therefore \Delta=ABC-C^2-A^2D$$

By Routh Hurwitz theorem this point is stable if $A>0\ and\ \Delta>0\ and\ D>0.$.

3. The positive equilibrium point $E_6=(x^*,x_1^*,y^*,z^*)$

$$\begin{vmatrix} k_1-\lambda & k_2 & k_3 & 0 \\ k_4 & k_5-\lambda & k_6 & 0 \\ k_7 & k_8 & k_9-\lambda & k_{10} \\ 0 & 0 & k_{11} & k_{12}-\lambda \end{vmatrix}=0$$

$$\lambda^4+(-k_1-k_5-k_{12}-k_9)\lambda^3+(k_1k_5+k_1k_9+k_1k_{12}+k_5k_9+k_5k_{12}+k_9k_{12}-k_{10}k_{11}$$
$$-k_6k_{12}-k_3k_7-k_2k_4)\lambda^2+(-k_1k_5k_9-k_1k_5k_{12}-k_1k_{12}+k_1k_{10}k_{11}+k_1k_6k_8-k_5k_9k_{12}$$
$$+k_5k_9k_{10}+k_6k_8k_{12}+k_2k_4k_{12}+k_2k_4k_9-k_2k_6k_7-k_3k_4k_8+k_3k_5k_7+k_3k_7k_{12})\lambda+$$
$$k_1k_5k_9k_{12}+k_1k_5k_9k_{10}-k_6k_8k_{12}-k_5k_9k_{12}-k_2k_4k_9k_{12}+k_2k_4k_{10}k_{11}+k_2k_7k_{12}$$
$$+k_3k_4k_8k_{12}-k_3k_5k_7k_{12}=0$$

$\lambda^4+A\lambda^3+B\lambda^2+C\lambda+D=0$ where

$\therefore \Delta=ABC-C^2-A^2D$

By Routh Hurwitz theorem this point is stable if $A>0\ with\ condition\ M_9-M_{12}<-M_1-M_5$ $and\ \ D>0\ ,\ \Delta>0.$

Lemma 15: The equilibrium $E_6=(x^*,x_1^*,y^*,z^*)$ is global stability

Proof: as in lemma 11·

6. Numerical Simulation

In this section, the study employs Mathematica Programing to illustration some results. the study shows that the effect of harvesting and cure rate from the disease on the behavior of the solution. the study deals with two cases: First when the model is the kind SI. In such model, susceptible prey become infected prey and not able to become susceptible again. Then it employs the harvest to see its impact on behavior. Figure (6) the behavior of solution of system (1) as SI model without harvesting, while, figure (7) the behavior of solution with harvesting. the study reveals the note in these two cases how employ the harvesting to disease control.

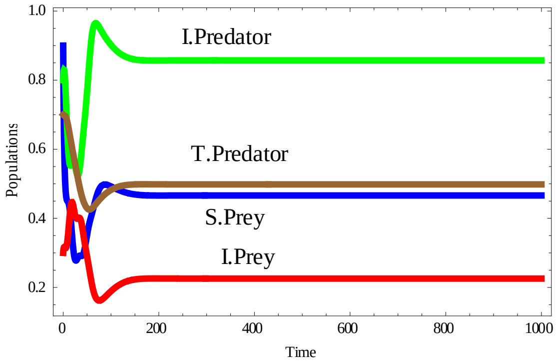

Fig. 6: SI model without harvesting.

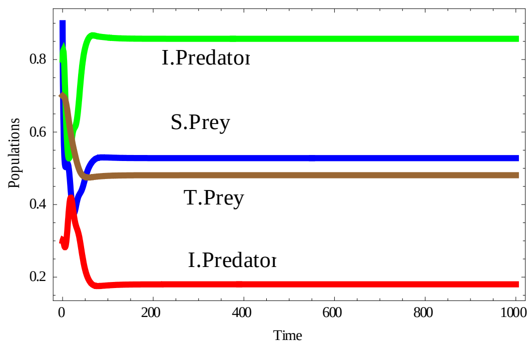

Fig. 7: SI model with harvesting.

The second case, the model is the kind SIS. In such model susceptible prey become infected prey and become susceptible again. Figure (8) behavior of solution of system (4) as SIS model without harvesting, while figure (9) employ the harvesting to disease control. Also, we note the effect of harvesting on disease to not become as epidemic.

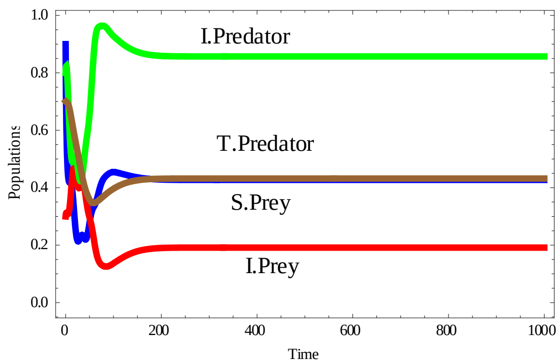

Fig. 8: SIS model without harvesting.

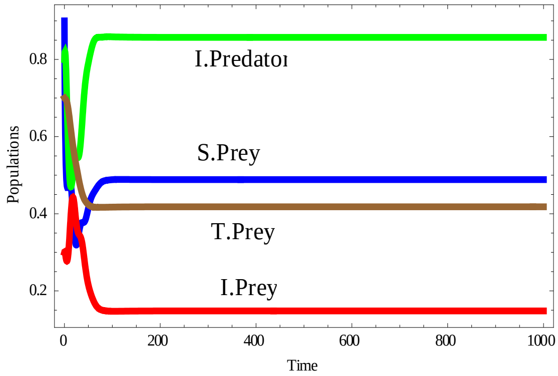

Fig. 9: SIS model with harvesting.

7. Conclusion

1- SIS and SI models were studied as well as partial models as the study proven that solutions are constrained and positive .

2- It was concluded that there are no periodic solutions for bilateral models.

3- Stability points were found and conditions were established, and the conditions that make the stability points stable local stability, where the benefit of the lack of periodic solutions to prove the overall stability, as was the benefit of the Lebanov function to prove the overall stability of non-bilateral models.

4- the study also showed how to control the sick prey and prevent the disease from turning into a pandemic and it proved that (harvest / vaccine) does not affect the stability of the system , and we discussed some of the results by the numerical simulations whether the model is SIS or SI.

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نموذج SIS مع الحصاد في نموذج السلسلة الغذائية

سفيان عباس وهيب ، بلال عزاوي ياسين

قسم الرياضيات ,كلية علوم الحاسوب والرياضيات, جامعة تكريت ، تكريت ، العراق

الملخص

في هذه الورقة، سوف ندرس النموذج الرياضي لنوع SIS، الفريسة السليمة مصابة بالمرض، وقد أثبتنا هذا الحل وتقييده حيث لا يوجد للنظام الجزيئي حدود دورية ، ثم ناقشنا استقرار تلك النقاط. كما أظهرنا كيفية السيطرة على المرض باستخدام الحصاد حتى لا يصبح وباء.