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Identification Functions in Hexa Toplogical Spaces

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ABSTRACT

The fundamental point of the current work is to concentrate on new definitions of (α_h, $\mathfrak{p}_h$, $\mathfrak{b}_h$, β_h) open sets to generalize identification functions in Hexa topological spaces called (α_h, $\mathfrak{p}_h$, $\mathfrak{b}_h$, β_h) identification functions and may relationship between theme were discussed and proved with examples.

1. Introduction and Preliminaries

An α_ (pre_ ,b_ , β_)open sets have been introduced and investigated by O.Njasted who knows the α_ open set and studied α_continuous and α_ irresolute[10,3,13], Mashhour [2,3] introduced and studied the concept pre_ open set, pre_continuous and pre_irresolute in topological space.While Andrjevic [4] presented b_ open set and studied its characteristics b_continuous and b_ irresolute. El-Monsef [8,12] introduced the ideal of β_ open set and β_continuous, so studied its characteristics β_ irresolute by Maheshwair and Thakur [13].

The single topology is extended to bi-topological space , tri-topological space, quad-topological space, penta-topological space [5,6,7,9] and Hexa -topological space by R.V.Chandra, et.al ,introduced and investigated the notion of h _open sets in h_topological spaces[11] .A lso studied some types of functions of Hexa topological spaces[1]. AL-kutabi [12] introduced and studied some weak identification functions. Present work we study the concepts of the different types of identifications functions and discuss their relation. Moreover we investigate the relationship between these identification functions types and other types .In this paper, we will use the expressions, $\mathcal{N}$ and Y to denote topological spaces$(M,\mathfrak{I}_{h1})$, $(\mathcal{N},\mathfrak{I}_{h2})$ and$(Y,\mathfrak{I}_{h3})$ respectively, for subset $\mathcal{A}$ of space $(M,\mathfrak{I}_h)$ with int ($\mathcal{A}$) , and cl($\mathcal{A}$) denoting the interior and closure of set $\mathcal{A}$. Subset $\mathcal{A}$ of space M is said to be:

1. α_open set [10]if $\mathcal{A} \subseteq \text{int}\left(\text{cl}\left(\text{int}(\mathcal{A})\right)\right)$. Hence , $\mathcal{A}^c$ is called α_closed .
2. pre_open set [2,3]if $\mathcal{A} \subseteq \text{int}\left(\text{cl}(\mathcal{A})\right)$. Hence , $\mathcal{A}^c$ is called pre_closed .
3. β_open set [4]if $\mathcal{A} \subseteq \text{cl}\left(\text{int}\left(\text{cl}(\mathcal{A})\right)\right)$. Hence , $\mathcal{A}^c$ is called β_closed .
4. b_open set[8] if $\mathcal{A} \subseteq \left(\text{cl}\left(\text{int}(\mathcal{A})\right) \cup \text{int}\left(\text{cl}(\mathcal{A})\right)\right)$.Hence $\mathcal{A}^c$ is called b_closed.

Definition 1. 1 [11]

Let M be a non empty set and $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6$are general topology on M. Then a subset $\mathcal{A}$ of space $\mathbb{X}$ is said to be hexa-open(h-open) set if $\mathcal{A} \in \cup_{i=1}^{6} \tau_i$ and its complement is said to be h-closed set and the set with six topologies called (M, τ_1, τ_2, τ_3, τ_4, τ_5, τ_6) Hexa Topology and $(M,\mathfrak{I}_h)$ for Hexa Topological Space(h_topological) where $\mathfrak{I}_h = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6)$. And Hexa-open sets satisfy all the axioms of topology.

Definition1.2 [11]

If $(M,\mathfrak{I}_h)$ is a h_topological and F $\subseteq M$. Then

1. The h_ interior of F is the union of all h-open subset contained in F and is denoted by int $(\text{F})_h$. So be int $(\text{F})_h$is the largest h-open subset of F

2. The $h\_$ closure of E is the intersection of all h −closed sets containing E and is denoted by $\mathrm{cl}(E)_h$. So be $\mathrm{cl}(E)_h$ is the smallest h_closed set containing E.

2. Some Types of Hexa Open Sets

Definition 2. 1

A subset $\mathcal{A}$ of space $(M, \mathfrak{J}_h)$ is said to be:

1- Hexa α_ *open* set (α_h_open) if $\mathcal{A} \subseteq \mathrm{int}_h\left(\mathrm{cl}_h\left(\mathrm{int}_h(\mathcal{A})\right)\right)$. Hence $\mathcal{A}^c$ is called α_h_closed set.

2- Hexa pre_open set ($\mathfrak{p}_h$_*open*) if $\mathcal{A} \subseteq \mathrm{int}_h\left(\mathrm{cl}_h(\mathcal{A})\right)$. Hence $\mathcal{A}^c$ is called $\mathfrak{p}_h$_closed set.

3- Hexa β_open set (β_h_*open*) if $\mathcal{A} \subseteq \mathrm{cl}_h\left(\mathrm{int}_h\left(\mathrm{cl}_h(\mathcal{A})\right)\right)$. Hence $\mathcal{A}^c$ is called β_h_closed set.

4- Hexa b_open set ($\mathfrak{b}_h$_ open) if $\mathcal{A} \subseteq \left(\mathrm{cl}_h\left(\mathrm{int}_h(\mathcal{A})\right) \cup \mathrm{int}_h\left(\mathrm{cl}_h(\mathcal{A})\right)\right)$. Hence $\mathcal{A}^c$ called $\mathfrak{b}_h$_closed set.[11] The family of all (α_h_, $\mathfrak{p}_h$_, β_h_, $\mathfrak{b}_h$_) open sets is denoted by $\alpha hO(M)$, $\mathfrak{p}hO(M)$, $\beta hO(M)$, $\mathfrak{b}hO(M)$ respectively.

Proposition 2.2 These statements hold true:

1- Every h − open *set* is a α_h_open *set* .

2- Every α_h_open *set* is a $\mathfrak{p}_h$_ open *set*.

3- Every $\mathfrak{p}_h$_ open *set* is a $\mathfrak{b}_h$ _open *set*.

4- Every $\mathfrak{b}_h$ _open *set*is a β_h_open set.

Proof : The proof is obvious.

Remark 2.3 The converse of the proposition above is not true.

By definition 2.1, present the following diagram that illustrates the relationship between the types of h-open sets.

$$h - \text{open } set \longrightarrow \alpha_h\text{\_open } set \longrightarrow \mathfrak{p}_h\text{\_ open } set \longrightarrow \mathfrak{b}_h \text{ \_open } set \longrightarrow \beta_h\text{\_open set}$$

Diagram (1)

Example 2.4 Let $\mathfrak{J}_1 = \{M, \emptyset, \{a,b\}\}$, $\mathfrak{J}_2 = \{M, \emptyset, \{a,c\}, \{b\}, \{a,b,c\}\}$, $\mathfrak{J}_3 = \{M, \emptyset, \{a\}, \{b,c,d\}\}$, $\mathfrak{J}_4 = \{M, \emptyset, \{c\}, \{a,c,d\}\}$, $\mathfrak{J}_5 = \{M, \emptyset, \{c,d\}\}$ and $\mathfrak{J}_6 = \{M, \emptyset, \{a,b,c,d\}\}$. then $\mathfrak{J}_h = \{M, \emptyset, \{a\}, \{b\}, \{a,c\}, \{c,d\}, \{a,b\}, \{a,b,c\}, \{a,b,c,d\}\}$ on $M = \{a,b,c,d\}$.

A subset $\mathcal{A}$ of M, is called h-openset if $\mathcal{A} \in \bigcup_{i=1}^{6} \tau_i$. The family of all h-open (h_closed) sub sets of $(M, \mathfrak{J}_h)$will be denoted by ($h\mathrm{O}(M)$), ($h\mathrm{C}(M)$),then $h\mathrm{O}(M) = \{M, \emptyset, \{a\}, \{b\}, \{a,c\}, \{c,d\}, \{a,b\}, \{a,b,c\}, \{a,b,c,d\}\}$, its satisfy all the axioms of topology , $h\mathrm{C}(M) = \{\emptyset, M, \{a,b,e\}, \{b,d,e\}, \{c,d,e\}, \{a,c,d,e\}, \{e\}, \{d,e\}, \{b,c,d,e\}\}$,

And the results 1- {b, c ,d}is α_h_open set but its not h-open set

2- {c} is $\mathfrak{p}_h$_ open set but its not α_h_open set

3- {a, b, e} is $\mathfrak{b}_h$ _open set but its not $\mathfrak{p}_h$_ open set.

4- {c ,e} is β_h_open set but its not $\mathfrak{b}_h$ _open set.

Theorem 2.5 Let $\mathcal{B}$ and $\mathcal{A}$be subset of $\mathcal{M}$ such that , $\mathcal{A} \subseteq \mathcal{B} \subseteq \mathrm{int}(\mathcal{A})_h$,if $\mathcal{A}$ is α_h_($\mathfrak{p}_h$_, $\mathfrak{b}_h$_ , β_h_) open set then $\mathcal{B}$ is also α_h_($\mathfrak{p}_h$_, $\mathfrak{b}_h$_ , β_h_) open set .

Proof. suppose that $\mathcal{A}$ is $\mathfrak{p}_h$_ open set, we have$\mathcal{A} \subseteq \mathrm{int}(\mathrm{cl}(\mathcal{A})_h)_h \subseteq \mathrm{int}(\mathrm{cl}(\mathcal{B})_h)_h$, so $\mathrm{int}(\mathcal{A})_h \subseteq \mathrm{int}(\mathrm{cl}(\mathcal{B})_h)_h$. Then $\mathcal{B}$ is also $\mathfrak{p}_h$_ open set.

Theorem 2.6 The h-closed set in h_topological is α_h_($\mathfrak{p}_h$_, $\mathfrak{b}_h$_ , β_h_) closed set in h_topological.

Proof . we prove the case $\mathfrak{p}_h$_ closed set

Let $\mathcal{B}$ be a h-closed subset of $\mathcal{M}$. Thus $\mathcal{B}^c$ is h-openset ,as long $\mathcal{B}^c \subseteq \mathrm{cl}(\mathcal{B}^c)_h \rightarrow \mathrm{int}(\mathcal{B}^c)_h \subseteq \mathrm{int}(\mathrm{cl}(\mathcal{B}^c)_h)_h$, we get $\mathcal{B}^c \subseteq \mathrm{int}(\mathrm{cl}(\mathcal{B}^c)_h)_h$. Hence $\mathcal{B}^c$ is $\mathfrak{p}_h$_open set , thus $\mathcal{B}$ is $\mathfrak{p}_h$_closed set.

Definition 2.7 : Let $\mathfrak{f}$ be a function of space $\mathcal{M}$ into space $\mathcal{N}$ .Then

1- $\mathfrak{f}$ is called an h-open (h-closed) function if the image of each h-open (h-closed) set in $\mathcal{M}$ is an h _open (h _closed) set in $\mathcal{N}$.

2- $\mathfrak{f}$ is called α_h_open function if the image of each α_h_open set in $\mathcal{M}$ is α_h_open set in $\mathcal{N}$

3- $\mathfrak{f}$ is called $\mathfrak{p}_h$_*open* function if the image of each $\mathfrak{p}_h$_*open* set in $\mathcal{M}$ is $\mathfrak{p}_h$_*open* set in $\mathcal{N}$

4- $\mathfrak{f}$ is called $\mathfrak{b}_h$_ open function if the image of each $\mathfrak{b}_h$_ open set in $\mathcal{M}$ is $\mathfrak{b}_h$_ open set in $\mathcal{N}$

5- $\mathfrak{f}$ is called β_h_*open* function if the image of each β_h_*open* set in $\mathcal{M}$ is β_h_*open* set in $\mathcal{N}$

Remark 2.8 The diagram below holds for functions

$$h - \text{open fun.} \longrightarrow \alpha_h\text{\_open fun.} \longrightarrow \mathfrak{p}_h\text{\_}open\text{ fun.} \longrightarrow \mathfrak{b}_h\text{\_ open fun.} \longrightarrow \beta_h\text{\_}open\text{ fun.}$$

Diagram (2)

"the following examples ,the converse of these implications is not true in general."

Definition 2.9 A function $\mathfrak{f} : (M, \mathfrak{J}_{h1}) \longrightarrow (\mathcal{N}, \mathfrak{J}_{h2})$ is called

1. h-continuous function if $\mathfrak{f}^{-1}$ of any h-open set in $\mathcal{N}$ is an h-open set in M [11]. 2. α_h_continuous function if $\mathfrak{f}^{-1}$ of any h-open set in $\mathcal{N}$ is α_h_ open set in M

3. $\mathfrak{p}_h$_continuous function if $\mathfrak{f}^{-1}$ of any h-open set in $\mathcal{N}$ is $\mathfrak{p}_h$_ open set in M .

4. $\mathfrak{b}_h$_ continuous function if $\mathfrak{f}^{-1}$ of any h-open set in $\mathcal{N}$ is $\mathfrak{b}_h$ _open set in M

5. β_h_ continuous function if $\mathfrak{f}^{-1}$ of any h-open set in $\mathcal{N}$ is β_h _open set in M.

Remark 2.10 The diagram below shows the relationship between continuous function .

$$h - \text{continuous} \longrightarrow \alpha_h\text{\_continuous} \longrightarrow \mathfrak{p}_h\text{\_continuous} \longrightarrow \mathfrak{b}_h\text{\_continuous} \longrightarrow \beta_h\text{\_continuous}$$

Diagram (3)

"In general, the converse of these implications and the following examples. are not true .

Example 2.11 From example 2.4 became $\mathfrak{J}_h = \{M, \emptyset, \{a\}, \{b\}, \{a,c\}, \{c,d\}, \{a,b\}, \{a,b,c\}, \{a,b,c,d\}\}$ on $M = \{a,b,c,d\}$ .Then

1.$\mathfrak{f} : M \longrightarrow M$ defined by $\mathfrak{f}(a) = c$, $\mathfrak{f}(b) = b$, $\mathfrak{f}(c) = a$, $\mathfrak{f}(d) = d$, $\mathfrak{f}(e) = e$, is $\mathfrak{p}_h$_continuous function but not α_h_cont. .

2. $\mathfrak{f}: M \longrightarrow M$ defined by $\mathfrak{f}(a) = a, \mathfrak{f}(b) = b, \mathfrak{f}(c) = c, \mathfrak{f}(d) = d, \mathfrak{f}(e) = b$, is $\mathfrak{h}_h$_cont.but not $\mathfrak{p}_h$_cont. .
3. $\mathfrak{f}: M \longrightarrow M$ defined by $\mathfrak{f}(a) = c, \mathfrak{f}(b) = e, \mathfrak{f}(c) = a, \mathfrak{f}(d) = d, \mathfrak{f}(e) = b$, is β_h_cont.but not $\mathfrak{h}_h$_cont.

Definition 2.12 Let $\mathfrak{f}$ be a function of space M into space $\mathcal{N}$,Then
1. α_h_ irresolute function if $\mathfrak{f}^{-1}$ of any α_h-open set in $\mathcal{N}$ is α_h_ open set in M .
2. $\mathfrak{p}_h$_ irresolute function if $\mathfrak{f}^{-1}$ of any $\mathfrak{p}_h$-open set in $\mathcal{N}$ is $\mathfrak{p}_h$_ open set in M .
3. $\mathfrak{h}_h$ _ irresolute function if $\mathfrak{f}^{-1}$ of any $\mathfrak{h}_h$-open set in $\mathcal{N}$ is $\mathfrak{h}_h$ _open set in M .
4. β_h_ irresolute function if $\mathfrak{f}^{-1}$ of any β_h-open set in $\mathcal{N}$ is β_h _open set in M.

We get the relationship between irresolute function

α_irresolute → $\mathfrak{p}_h$_irresolute → $\mathfrak{h}_h$_irresolute → β_h_irresolute

Diagram (4)

Example 2.13 From example 2.4 .Let $\mathfrak{I}_{h1} = \{M, \emptyset, \{a\}, \{b\}, \{a,c\}, \{c,d\}, \{a,b\}, \{a,b,c\}, \{a,b,c,d\}\}$ on $M = \{a,b,c,d\}$ .Then
1. $\mathfrak{f}: M \longrightarrow M$ defined by $\mathfrak{f}(a) = a, \mathfrak{f}(b) = c, \mathfrak{f}(c) = b, \mathfrak{f}(d) = d, \mathfrak{f}(e) = e$; is $\mathfrak{p}_h$_irresol. and not α_h_irresol.
2. $\mathfrak{f}: M \longrightarrow M$ defined by $\mathfrak{f}(a) = d, \mathfrak{f}(b) = b, \mathfrak{f}(c) = e, \mathfrak{f}(d) = d, \mathfrak{f}(e) = c$; is $\mathfrak{h}_h$_irresol. and not $\mathfrak{p}_h$__irresol .
3. $\mathfrak{f}: M \longrightarrow M$ defined by $\mathfrak{f}(a) = c, \mathfrak{f}(b) = b, \mathfrak{f}(c) = a, \mathfrak{f}(d) = e, \mathfrak{f}(e) = d$ *is β_h_irresol.* and not *$\mathfrak{h}_h$_irresol.*

3. Identifications Function in Hexa Toplogical Spaces

In this section we introduce new definitions of identification function by using (α_h_, $\mathfrak{p}_h$_, $\mathfrak{h}_h$_ , β_h_) open sets . Further we study the relations between these.

Definition 3.1
A function $\mathfrak{f}: (M, \mathfrak{I}_{h1}) \longrightarrow (\mathcal{N}, \mathfrak{I}_{h2})$ is called α_identification function iff $\mathfrak{f}$ is surjective , and one of " these conditions is satisfied:"
1- W is α_h_ open set in $\mathcal{N}$ iff $\mathfrak{f}^{-1}(w)$ is α_h_open set in M.
2- W is α_h_ closed set in $\mathcal{N}$ iff $\mathfrak{f}^{-1}(w)$ is α_h_ closed set in M.

Example 3.2 Let $\mathfrak{I}_{h1} = \{M, \emptyset, \{b\}, \{b,c\}, \{a,b\}, \{a,b,c\}\}$ on $M = \{a,b,c,d\}$ and $\mathfrak{I}_{h2} = \{\mathcal{N}, \varphi, \{1,2\}, \{2\}, \{2,3\}, \{1,2,3\}\}$ on $\mathcal{N} = \{1,2,3,4\}$
If $\mathfrak{f}: M \longrightarrow \mathcal{N}$ defined by $(a) = 1, \mathfrak{f}(b) = 2, \mathfrak{f}(c) = 3, \mathfrak{f}(d) = 4$, then $\mathfrak{f}$ is under α_identification function.

Theorem 3.3
Every α_h_irresolute function and α_h_open (α_h_ closed) surjective functions is α_h_identification function.

Proof. Suppose that $\mathfrak{f}: (M, \mathfrak{I}_{h1}) \longrightarrow (\mathcal{N}, \mathfrak{I}_{h2})$ and $\mathfrak{f}(W)$ is α_h_open set
since $\mathfrak{f}$ is surjective and α_h_open set .Hence $f(\mathfrak{f}^{-1}(w)) = w$ is α_h_open set and $W \subseteq \mathcal{N}$. Since $\mathfrak{f}$ is α_h_irresolute function and $\mathfrak{f}^{-1}(w)$ is α_h_open set in M, then $\mathfrak{f}$ falls under α_h_identification function .

Definition 3.4
A surjective function $\mathfrak{f}: (M, \mathfrak{I}_{h1}) \longrightarrow (\mathcal{N}, \mathfrak{I}_{h2})$ is called $\mathfrak{p}_h$_identification function and W is $\mathfrak{p}_h$_-open set in $\mathcal{N}$ iff $\mathfrak{f}^{-1}(w)$ is $\mathfrak{p}_h$_ − open set in M .

Result 3.5
Every α_h_identification function as $\mathfrak{p}_h$_identification function but the reverse is not true.

Form example3.2L
et $\mathfrak{I}_{h1} = \{M, \emptyset, \{c,d\}, \{b\}, \{b,c,d\}, \{a,b\}, \{a,b,c,d\}\}$ on $M = \{a,b,c,d,e\}$ and $\mathfrak{I}_{h2} = \{\mathcal{N}, \varphi, \{1,2\}, \{2\}, \{2,3\}, \{1,2,3\}\}$ on $\mathcal{N} = \{1,2,3,4\}$. If $\mathfrak{f}: M \longrightarrow \mathcal{N}$ defined by $\mathfrak{f}(a) = 1, \mathfrak{f}(b) = 2, \mathfrak{f}(c) = 3, \mathfrak{f}(d) = 4$ .Then $\mathfrak{f}$ is under $\mathfrak{p}_h$_identification function.

Lemma 3.6
Let surjective function $\mathfrak{f}: (M, \mathfrak{I}_{h1}) \longrightarrow (\mathcal{N}, \mathfrak{I}_{h2})$ be called $\mathfrak{p}_h$_identification function and W is $\mathfrak{p}_h$_ closed set in $\mathcal{N}$ iff $\mathfrak{f}^{-1}(w)$ is $\mathfrak{p}_h$_ closed set in M.

Proposition 3.7
Every $\mathfrak{p}_h$_irresolute function and $\mathfrak{p}_h$_ open ($\mathfrak{p}_h$_closed) surjective functions are $\mathfrak{p}_h$_identification function.

Proof. Since every α_h_function is $\mathfrak{p}_h$_ function and every α_h_irresolute is $\mathfrak{p}_h$_irresolute.Then by theorem (3.3) obtain every α_h_identification function is $\mathfrak{p}_h$_identification function.

Definition 3.8
" A surjective function $\mathfrak{f}: (M, \mathfrak{I}_{h1}) \longrightarrow (\mathcal{N}, \mathfrak{I}_{h2})$ called $\mathfrak{h}_h$_ identification functionand one of the following conditions is satisfied"
1) W is $\mathfrak{h}_h$_ open set in, iff $\mathfrak{f}^{-1}(w)$ is $\mathfrak{h}_h$_ open set in M.
2) W is $\mathfrak{h}_h$_ closed set in $\mathcal{N}$ iff $\mathfrak{f}^{-1}(w)$ is $\mathfrak{h}_h$_ closed set in M.

"From Diagram (1), if every $\mathfrak{p}_h$_-open set is $\mathfrak{h}_h$_ open set, then each $\mathfrak{p}_h$_identification function is $\mathfrak{h}_h$_ identification function."

Example 3.9 From example 3.2
If $\mathfrak{f}: M \longrightarrow M$ defined by $\mathfrak{f}(a) = a, \mathfrak{f}(b) = b, \mathfrak{f}(c) = d, \mathfrak{f}(d) = c$, then $\mathfrak{f}$ is $\mathfrak{h}_h$_identification function but not $\mathfrak{p}_h$_identification function.

Proposition 3.10
Let $\mathfrak{f}: (M, \mathfrak{I}_{h1}) \longrightarrow (\mathcal{N}, \mathfrak{I}_{h2})$ be surjective function $\mathfrak{h}_h$_ open ($\mathfrak{h}_h$_ closed) and $\mathfrak{h}_h$_ irresolute function, then $\mathfrak{f}$ is $\mathfrak{h}_h$_ identification function.

Proposition 3.11
The composition of two α_h_identification ($\mathfrak{p}_h$_identification, $\mathfrak{h}_h$_ identification)

functions are α_h_identification($\mathfrak{p}_h$_identification, $\mathfrak{b}_h$_ identification) functions.

Proof. Suppose that $\mathfrak{f} : (M, \mathfrak{J}_{h1}) \longrightarrow (\mathcal{N}, \mathfrak{J}_{h2})$ and $g: (\mathcal{N}, \mathfrak{J}_{h2}) \longrightarrow (Y, \mathfrak{J}_{h3})$are α_h_identification function, whenever the compositions of two onto functions are surjective. If W be any α_h_open set in $\mathfrak{J}_{h3}$, by hypothesis, g and $\mathfrak{f}$ are α_h_identifications function ,then g^{-1} (W) is α_h_open set in $\mathcal{N}$ and we get $\mathfrak{f}^{-1}\left(g^{-1}(W)\right) = (gof)^{-1}(W)$ is α_h_open in M and W is α_h_open set in M .Then gof is α_h_identification function

Similarly that gof is ($\mathfrak{p}_h$_identification, $\mathfrak{b}_h$_ identification) functions.

Proposition 3.12

Let $\mathfrak{f} : (M, \mathfrak{J}_{h1}) \longrightarrow (\mathcal{N}, \mathfrak{J}_{h2})$ and $g: (\mathcal{N}, \mathfrak{J}_{h2}) \longrightarrow (Y, \mathfrak{J}_{h3})$ be functions and $\mathfrak{f}$ be $\alpha_h$identification ($\mathfrak{p}_h$_identification, $\mathfrak{b}_h$_ identification)functions

Then the following statements are valid

1- If gof is $\alpha_h$cont. ($\mathfrak{p}_h$_cont. , $\mathfrak{b}_h$_ cont.)functions , then g is $\alpha_h$cont. ($\mathfrak{p}_h$_cont. , $\mathfrak{b}_h$_ cont.) function.

2- If gof is $\alpha_h$irresolute ($\mathfrak{p}_h$__irresolute, $\mathfrak{b}_h$_irresolute) functions, theng is $\alpha_h$irresolute ($\mathfrak{p}_h$_irresolute, $\mathfrak{b}_h$_ irresolute) function.

Proof

1) Let gof: $(M, \mathfrak{J}_{h1}) \longrightarrow (Y, \mathfrak{J}_{h3})$ be $\alpha_h$cont. and assume that k is any h −open set in Y Let V = g^{-1} (k) and W= $\mathfrak{f}^{-1}$ (V) .Whenever gof^{-1} (k) = $\mathfrak{f}^{-1}$ (g^{-1} (k)) =W is $\alpha_h$open in M , then gof^{-1} (k) $\alpha_h$open set in M but $\mathfrak{f}$ is $\alpha_h$identification function. Therefore V is $\alpha_h$open set in $\mathcal{N}$. As g^{-1}(k) is $\alpha_h$open set in $\mathcal{N}$,then g is $\alpha_h$cont.

2) Assume that k is any $\alpha_h$open set in Y , Let V = g^{-1} (k) and W= $\mathfrak{f}^{-1}$ (V) ,we have gof^{-1} (k) = $\mathfrak{f}^{-1}$ (g^{-1} (k)) =W that is W is $\alpha_h$open set in M, we obtain gof^{-1} (k) $\alpha_h$open set in M ,since $\mathfrak{f}$ is $\alpha_h$identification function .Then V is $\alpha_h$open set in $\mathcal{N}$,whenever $\mathfrak{f}^{-1}$ (k) is $\alpha_h$openset in $\mathcal{N}$.we get g is $\alpha_h$irresolute function.

Definition 3.13

Let function $\mathfrak{f} : (M, \mathfrak{J}_{h1}) \longrightarrow (\mathcal{N}, \mathfrak{J}_{h2})$be surjective β_h_identification function and W is β_h_open set in $\mathcal{N}$ iff $\mathfrak{f}^{-1}$ (w) is β_h_ openset in M.

From diagram (1) we obtain every $\mathfrak{b}_h$_ identification function as β_h_identification function, but the converse is not true. For instance 3.2 let $\mathfrak{f} : M \longrightarrow M$ be defined by $\mathfrak{f}(a) = a$, $\mathfrak{f}(b) = b$, $\mathfrak{f}(c) = e$, $\mathfrak{f}(d) = d$,$\mathfrak{f}(e) = c$ then $\mathfrak{f}$ is $\mathfrak{b}_h$_ identification function but not $\mathfrak{p}_h$_identification function.

Proposition 3.14

Let function $\mathfrak{f} : M \longrightarrow \mathcal{N}$ be surjective β_h_identification function and W is β_h_open set in $\mathcal{N}$ iff $\mathfrak{f}^{-1}$ (W) is β_h_closed set in M.

Proof. Let W be β_h_ closed subset of $\mathcal{N}$,then W^c is β_h_ open in $\mathcal{N}$,since $\mathfrak{f}$ is β_h_identification function ,we get $\mathfrak{f}^{-1}$ (W) is β_h_ closed set in M (because $\mathfrak{f}$ is onto and $\left(\mathfrak{f}^{-1}(W)\right)^c = \mathfrak{f}^{-1}(W^c)$ is β_h_ openset in M) . Similarly if $\mathfrak{f}^{-1}$ (W) is β_h_ closed set in M, we obtain that $\mathfrak{f}^{-1}(W)^c = \mathfrak{f}^{-1}(W^c)$ is β_h_ openset in M and $\mathfrak{f}$ is β_h_identification function, hence W is β_h_ closed set in $\mathcal{N}$, since W be β_h_ open set in M .Then W^c is β_h_ closed set in $\mathcal{N}$, whenever $\left(\mathfrak{f}^{-1}(W)\right)^c = \mathfrak{f}^{-1}(W^c)$ is β_h_ closed set in M, we get $\mathfrak{f}^{-1}$ (W) is β_h_ open set in M .Then $W^c$is β_h_ closed set and W is β_h_ open set.

Proposition 3.15

Let surjective function $\mathfrak{f} : M \longrightarrow \mathcal{N}$ be β_h_ open(β_h_ closed) and β_h_irresolute .Then $\mathfrak{f}$ is β_h_identification function.

Proof .Suppose that W is β_h_ closed set in M and $W \subseteq \mathcal{N}$ such that $\mathfrak{f}^{-1}$ (W) is β_h_ closed set in M ,whenever $\left(\mathfrak{f}(\mathfrak{f}^{-1})(W)\right)$= W ,we obtain that W is β_h_ closed set in M(since $\mathfrak{f}^{-1}$ (W) is β_h_ closed set in M and $\mathfrak{f}$ is β_h_ closed set in M).Hence W^c is β_h_ open set in M,since $\mathfrak{f}$ is β_h_irresolute function ,then $\mathfrak{f}^{-1}$ (W)is β_h_ openset in M ,whenever $\mathfrak{f}$ is onto $\left(\mathfrak{f}^{-1}(W)\right)^c = \mathfrak{f}^{-1}(W^c)$,we get $\mathfrak{f}^{-1}$(W) is β_h_ open set in M by Proposition (3.14) .Then $\mathfrak{f}$ is β_h_ identification function.

Theorem 3.16 These statements hold true.

1- Every α_h identification functionis $\mathfrak{p}_h$_identification function

2- Every $\mathfrak{p}_h$_ identification function is $\mathfrak{b}_h$_identificationfunction

3- Every $\mathfrak{b}_h$_ identification functionis β_h_identification function

Proof . The proof is obvious.

Proposition 3.17

"The composition of two β_h_identification functions is β_h_identification function".

Proof. Let $\mathfrak{f} : M \longrightarrow \mathcal{N}$ and $g : \mathcal{N} \longrightarrow Y$ be β_h_identifications function ,whenever the composition of two onto functions is surjective ,let W be any β_h_open in Y since g ,$\mathfrak{f}$ are β_h_identifications function Then g^{-1} (W) is β_h_ open in $\mathcal{N}$,we have that $\mathfrak{f}^{-1}\left(g^{-1}(W)\right) = (gof)^{-1}(W)$ is β_h_ open in M ,we get W is β_h_ open set in M. Then gof is β_h_identification function.

Proposition 3.18

Let function $\mathfrak{f}: M \longrightarrow \mathcal{N}$and $g: \mathcal{N} \longrightarrow Y$ be β_h_identification function. These statements are valid

1- If gof is β_h_cont. then g is β_h_cont.

2- If gof is β_h_irresolute then g is β_h_irresolute .

Proof

1- Let $\mathfrak{f}$: $M \longrightarrow \mathcal{N}$ be β_h_cont. and assume that k is any open set in Y , Let V = g^{-1} (k) and W= $\mathfrak{f}^{-1}$(V) .We have gof^{-1} (k) = $\mathfrak{f}^{-1}$ (g^{-1} (k))=W is β_h_ open set in M.Moreover gof^{-1} (k)is β_h_ open in M,but $\mathfrak{f}$ is β_h_identification function then V is β_h_open.we get g^{-1}(k) is $\beta_{h\_}$ open in $\mathcal{N}$ and g is β_h_cont.

2- Let k be any $\beta_{h\_}$ open set in Y and V = $\mathfrak{f}^{-1}$(k) and W = $\mathfrak{f}^{-1}$(V).We have gof^{-1}(k) = $\mathfrak{f}^{-1}$(g^{-1} (k)) =W that is W is $\beta_{h\_}$ open set in M,we obtain gof^{-1}(k) as $\beta_{h\_}$ open in M but $\mathfrak{f}$ is β_h_identification function and V is $\beta_{h\_}$ open set in $\mathcal{N}$,hence g is β_h_irresolute .

Remark 3.19 From the above discussion and known results, we state the following implications.

α_h_identification ⟶ $\mathfrak{p}_{h\_}$ identification
⟶ $\mathfrak{b}_h$_identification
→ β_h_identification

Diagram (5)

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Conclusion

Herein, we investigated the different types of identification functions in Hexa Toplogical Spaces and discussed their relation. Also investigated the relationship of this set of identification function with other types and proved many properties of these functions. Future work, we introduce the Generalization of the separation axioms on Hexa Toplogical Spaces and use the types of identification functions to raise the characteristic of Hexa _separation $\mathfrak{I}_1 -$ spaces of the h_topological to another h_topological.

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دالة الهوية في فضاءات التبولوجية السداسية

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الملخص

الهدف الأساسي من العمل الحالي هو التركيز على تعاريف جديدة للمجموعات المفتوحة من نوع ($\alpha_{h\_}$, $\mathfrak{p}_{h\_}$, $\mathfrak{b}_{h\_}$, $\beta_{h\_}$) في فضاءات التبولوجية السداسية , لتعميم تعاريف للدالة الهوية في فضاءات التبولوجية السداسية المسماة (α_h_identification, $\mathfrak{p}_h$__identification, $\mathfrak{b}_h$_identification, β_h_identification) وقد تمت مناقشة العلاقة بين أنواع الدوال الهوية وإثباتها بالأمثلة.