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A New Hybrid of DY and CGSD Conjugate Gradient Methods for Solving Unconstrained Optimization Problems

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ABSTRACT

In this article, we present a new hybrid conjugate gradient method for solving large Scale in unconstrained optimization problems. This method is a convex combination of Dai-Yuan conjugate gradient and Andrei- sufficient descent condition, satisfies the famous D-L conjugacy condition and in the same time solidarities with the newton direction with the suitable condition. The suggestion method always yields a descent search direction at each it iteration. Under strong wolfe powell(SWP) line search condition, the direction satisfy the global convergence of the proposed method is established. Finally, the results we achieved are good and it is show that our method is forceful and effective.

1. Introduction

A conjugate gradient (CG) method is calculated to solve a nonlinear unconstrained optimization problem The unconstrained optimization problem has the following general form:

$\min\{f(x): x \in R^n\}$ (1)

where $x \in R^n$ is a real vector with $n \geq 1$ component and $f: R^n \to R$ is smooth, nonlinear function and its gradient denoted by $g(x) = \nabla f(x)$ the nonlinear CG method that starts from an initial guess $x_0 \in R^n$ will be distinct using the iterations of the sequence,Dai and Yuan[1] suggested the following nonlinear conjugate gradient algorithm:

$$x_{k+1} = x_k + \alpha_k d_k, \qquad k = 0,1,2,3,\ldots \quad (2)$$

Where x_k is current iterate point and $\alpha_k > 0$ is called a step size determined by some line Searches. The d_k is the search direction defined by:

$$d_{k+1} = \begin{cases} -g_{k+1}, & k = 0 \\ -g_{k+1} + \beta_k d_k, & k \geq 1 \end{cases} \quad (3)$$

In (3) β_k is known as the conjugate gradient parameter.

The line search in the conjugate gradient algorithms is often based on the standard Strong Wolfe Powell (SWP) conditions [2,3]

$$f(x_k + \alpha_k d_k) - f(x_k) \leq \rho \alpha_k g_k^T d_k \quad (4)$$

$$g(x_k + \alpha_k d_k)^T d_k \geq \sigma g_k^T d_k \quad (5)$$

Where d_k is a descent direction and $0 < \rho \leq \sigma < 1$

Various conjugate gradient methods have been proposed, and they mainly differ in the choice of the parameter β_k. Some well-known formulas for β_k being given below:

$$\beta_k^{HS} = \frac{g_{k+1}^T y_k}{y_k^T d_k}, \qquad \beta_k^{FR} = \frac{\|g_{k+1}\|^2}{g_k^T g_k}, \qquad \beta_k^{PRP} = \frac{g_{k+1}^T y_k}{g_k^T g_k}$$

$$\beta_k^{CD} = \frac{\|g_{k+1}\|^2}{y_k^T d_k}, \qquad \beta_k^{LS} = \frac{g_{k+1}^T y_k}{g_k^T d_k}, \qquad \beta_k^{DY} = \frac{\|g_{k+1}\|^2}{y_k^T d_k}$$

Where $\|.\|$ denotes the Euclidean norm, and the $y_k = g_{k+1} - g_k$.

The corresponding method is respectively called, (HS) Hestenes-Stiefel [4], (FR) Fletcher-Revees [5], (PRP) Polak -Ribiere-Polyak [6,7], (CD) Conjugate Descent [8], (LS) Liu-Storey [9], And (DY) Dai-Yuan [1].

For a strictly convex quadratic function $f(x)$, and the line search is exact, all these Methods are identical, meanwhile the gradients are equally orthogonal, so the parameters β_k in these methods are identical. When realized to general nonlinear function with inexact line searches, the behavior of these methods is apparent

Different. one of important group of (CG) methods is the hybrid conjugate gradient (HCG) algorithms, the hybrid computational schemes HCG work better than classical CG methods because the HCG take the compensations of two parameters β_k[10].

Many researches devoted to the hybrid or mixed conjugate gradient methods that have better computational

Have better computational performance and strong convergence properties.

In this article we focus on hybrid conjugate gradient methods as a convex combination of DY [1 ,11 ,12] and CGSD [13] CG methods for solving unconstrained optimization method with appropriate conditions the consistent conjugate gradient (CG) parameters are:

$$\beta_k^{DY} = \frac{g_{k+1}^T g_{k+1}}{y_k^T s_k} \quad (6)$$

And

$$\beta_k^{CGSD} = \frac{g_{k+1}^T g_{k+1}}{y_k^T s_k} - \frac{(y_k^T g_{k+1})(s_k^T g_{k+1})}{(y_k^T s_k)^2} \quad (7)$$

The suggested method defined by set the parameter β_k by:

$$\beta_k^{NEW} = (1-\theta_k)\beta_k^{DY} + \theta_k \beta_k^{CGSD} \quad (8)$$

In this paper, we choose the value of the parameter $\theta_k$in the convex combination the search direction $d_k$of

our algorithm not only is the Newton direction [14],so satisfies the famous DL conjugate condition proposed

by Dai and Liao [15]. Under the SWP condition, we prove the global conjunction of the proposed algorithm,

the numerical results also show the viability and activity of our algorithm.

This study is prearranged as follows in the next section we introduce the new proposed hybrid CG method

HZI, and we got the parameter θ_k and we give the algorithm of our method. We also consider the sufficient

descent property under the (SWP) condition in section 3. , section 4. the global convergence property future method is established. In section 5. , Some numerical results are described.

2. New Hybrid Conjugate Gradient Algorithm

In this unit, we will define a new proposed HCG method, in order to get the Sufficient descent direction, we will compute $\theta_k$as follows: we conglomerate $\beta_k^{DY}$and β_k^{CGSD} in (8). The direction d_{k+1} are generated by:

$$d_{k+1} = -g_{k+1} + \beta_k^{NEW} s_k \quad (9)$$

The iterates $x_k, x_k, x_k, \ldots$ of the proposed method are computed by means of the recurrence (2), where the step size α_k is definition according to the SWP condition (4) and (5).

The scale parameter θ_k in (8) satisfying $0 \le \theta_k \le 1$, which will be indomitable a specific method to be branded later. It is obvious that if $\theta_k \le 0$ then $\beta_k^{NEW} = \beta_k^{BM}$, and if $\theta_k \ge 1$,then $\beta_k^{NEW} = \beta_k^{ACGHES}$. On the other side, if $0 < \theta_k < 1$, then β_k^{NEW} is a convex combination of β_k^{BM} and β_k^{ACGHES} . from (8) and (9) it is clear that:

$$d_{k+1} = \begin{cases} -g_{k+1}, & k = 1 \\ -g_{k+1} + (1-\theta_k)\frac{g_{k+1}^T g_{k+1}}{y_k^T s_k} s_k + \theta_k \left(\frac{g_{k+1}^T g_{k+1}}{y_k^T s_k} - \frac{(y_k^T g_{k+1})(s_k^T g_{k+1})}{(y_k^T s_k)^2}\right) s_k, & k > 1 \end{cases} \quad (10)$$

Our motivation to select the parameter θ_k in such a manner that the defection d_{k+1} given (10) is equal to the Newton direction $d_{k+1}^T = -\nabla^2 f(x_{k+1})^{-1} g_{k+1}$ .therefore

$$-\nabla^2 f(x_{k+1})^{-1} g_{k+1} = -g_{k+1} + \left((1-\theta_k)\beta_k^{DY} + \theta_k \beta_k^{CGSD}\right) s_k$$

$$-\nabla^2 f(x_{k+1})^{-1} g_{k+1} = -g_{k+1} + (1-\theta_k)\frac{g_{k+1}^T g_{k+1}}{y_k^T s_k} s_k + \theta_k \left(\frac{g_{k+1}^T g_{k+1}}{y_k^T s_k} - \frac{(y_k^T g_{k+1})(s_k^T g_{k+1})}{(y_k^T s_k)^2}\right) s_k$$

$$-\nabla^2 f(x_{k+1})^{-1} g_{k+1} = -g_{k+1} + \frac{g_{k+1}^T g_{k+1}}{y_k^T s_k} s_k - \theta_k \left(\frac{(y_k^T g_{k+1})(s_k^T g_{k+1})}{(y_k^T s_k)^2}\right) s_k \quad (11)$$

Therefore, in order to have an algorithm for solving large scale problems we assume that pair (s_k , y_k) satisfies the secant equation

$$y_k = \nabla^2 f(x_{k+1}) s_k \quad (12)$$

From (12) we get

$$s_k^T \nabla^2 f(x_{k+1}) = y_k^T$$

Multiplying (11) by $s_k^T \nabla^2 f(x_{k+1})$ then we get

$$-s_k^T g_{k+1} = -s_k^T \nabla^2 f(x_{k+1}) g_{k+1} + \frac{g_{k+1}^T g_{k+1}}{y_k^T s_k} s_k^T \nabla^2 f(x_{k+1}) s_k - \theta_k \left(\frac{(y_k^T g_{k+1})(s_k^T g_{k+1})}{(y_k^T s_k)^2}\right) s_k^T \nabla^2 f(x_{k+1}) s_k$$

$$-s_k^T g_{k+1} = -y_k^T g_{k+1} + \frac{g_{k+1}^T g_{k+1}}{y_k^T s_k} y_k^T s_k - \theta_k \left(\frac{(y_k^T g_{k+1})(s_k^T g_{k+1})}{(y_k^T s_k)^2}\right) y_k^T s_k$$

After some algebra, we get Denoting $\theta_k^{HZI} = \theta_k$,

$$\theta_k^{HZI} = \frac{y_k^T s_k (y_k^T g_{k+1} - s_k^T g_{k+1} - g_{k+1}^T g_{k+1})}{(y_k^T g_{k+1})(s_k^T g_{k+1})} \quad (13)$$

Algorithm HZI

Step 1 : choose $x_0 \in R^n$, $\epsilon > 0$, compute $f(x_0)$ and $g_0 = -\nabla f(x_0)$, set $d_0 = -g_0$, When $k = 0$

Step 2 : Stopping criteria, if $\| g_n \| \le \epsilon$, then its stop.

Step 3 : Compute α_k by using Strong Wolf Powell condition in eq(3) and eq(4).

Step 4 : Compute $x_{k+1} = s_k + \alpha_k d_k$, and $g_{k+1} = g(x_{k+1})$ .Compute $s_k = x_{k+1} - x_k$, and
$y_k = g_{k+1} - g_k$
Step 5 : If $\theta_k \geq 1$ then put $\theta_k = 1$. If $\theta_k \leq 0$, then put $\theta_k = 0$, otherwise Compute θ_k as (13)
Step 6 : Compute β_k^{HZI} By (8), and Generate $d_{k+1} = -g_{k+1} + \beta_k^{HZI} s_k$
Step 7 : If the criteria of Powell $|g_{k+1}^T g_k| \geq 0.2 \parallel g_{k+1} \parallel^2$ Satisfied put the set $d_{k+1} = -g_{k+1}$, otherwise,
we define $d_{k+1} = d_k$.
Step 8 : Evaluate α_k , set $k = k + 1$, and go to step2.

3. The Sufficient Descent Condition

In this section, we use to the following theorem to clear up that the search direction $d_k$obtained by HZI satisfies the sufficient descent condition. For additional deliberations, we need the assumptions below:

3.1. Assumption

The level sets $Q = \{x \in R^n : f(x) \leq f(x_0)\}$ at x_0 is bounded where x_0 is starting point, that there exists $w > 0$, such that $\parallel x \parallel \leq w, \forall x \in Q$

3.2. Assumption

In a neighborhood $N \ of \ Q$, the function f is continuously differentiable and its gradient is Lipschitz continuous, i.e. there exists a constant $q > 0$, such that
$\parallel \nabla f(x) - \nabla f(y) \parallel \leq q \parallel x - y \parallel, \forall x, y \in N$
Under assumption (3.1) and (3.2), there exists positive constant $(n, \bar{n}, z, \bar{z})$,such that
$\bar{n} \leq \|g_{k+1}\| \leq n, and\ \bar{z} \leq \|g_k\| \leq z \ \ \forall x \in Q$

3.3. Theorem

Let the sequences $\{g_k\}$ and $\{d_k\}$ be generated by HZI method. Then d_k is the search direction satisfies the sufficient descent condition

$$g_{k+1}^T d_{k+1} \leq -c \parallel g_{k+1} \parallel^2, \forall\ c \geq 0 \quad (14)$$

with $c = c_3 c_2 + (1 - c_3) c_1$

Proof

We demonstration that search direction d_k shall satisfies the sufficient descent condition embraces for $k = 0$, the proof is a trivial one, i.e. $d_0 = -g_0$ and so $g_0^T d_0 = -\parallel g_0 \parallel^2$. and that can be concluded that (14) holds for $k = 0$.
Next is to show that it holds for > 0 .

$$d_{k+1} = -g_{k+1} + \beta_k^{HZI} s_k$$

Obviously
$d_{k+1} = -g_{k+1} + ((1 - \theta_k)\beta_k^{DY} + \theta_k \beta_k^{CGSD}) s_k$
We can rewrite the direction by the following below:

$$d_{k+1} = -(\theta_k g_{k+1} + (1 - \theta_k) g_{k+1}) + ((1 - \theta_k)\beta_k^{DY} + \theta_k \beta_k^{CGSD}) s_k$$

It follows that

$$d_{k+1} = \theta_k(-g_{k+1} + \beta_k^{CGSD} s_k) + (1 - \theta_k)(-g_{k+1} + \beta_k^{DY} s_k)$$

Wherefrom

$$d_{k+1} = \theta_k d_{k+1}^{CGSD} + (1 - \theta_k) d_{k+1}^{DY} \quad (14)$$

Multiplying the (14) from the left by g_{k+1}^T , we get

$$g_{k+1}^T d_{k+1} = \theta_k g_{k+1}^T d_{k+1}^{CGSD} + (1 - \theta_k) g_{k+1}^T d_{k+1}^{DY} \quad (15)$$

Firstly, let $\theta_k = 0$, then $d_{k+1} = d_{k+1}^{DY}$. Remember that

$$d_{k+1}^{DY} = -g_{k+1} + \beta_k^{DY} s_k$$

$$g_{k+1}^T d_{k+1} = g_{k+1}^T(-g_{k+1} + \beta_k^{DY} s_k)$$

$$g_{k+1}^T d_{k+1} = g_{k+1}^T\left(-g_{k+1} + \left(\frac{g_{k+1}^T g_{k+1}}{y_k^T s_k}\right) s_k\right)$$

$$g_{k+1}^T d_{k+1} = -\parallel g_{k+1} \parallel^2 \left(1 - \frac{g_{k+1}^T s_k}{y_k^T s_k}\right)$$

$$g_{k+1}^T d_{k+1} = -\parallel g_{k+1} \parallel^2 \left(\frac{y_k^T s_k - g_{k+1}^T s_k}{y_k^T s_k}\right) \quad (16)$$

For (16) satisfy sufficient descent condition we have

$$\left|\frac{y_k^T s_k - g_{k+1}^T s_k}{y_k^T s_k}\right| \leq M \quad , M > 0$$

So that
$g_{k+1}^T d_{k+1}^{DY} = -M \parallel g_{k+1} \parallel^2$
We denote $c_1 = M > 0$, then we can write

$$g_{k+1}^T d_{k+1}^{DY} = -c_1 \|g_{k+1}\|^2 \quad (17)$$

We are done with $\theta_k = 0$
Now, let $\theta_k = 1$, then $d_{k+1} = d_{k+1}^{CGSD}$. Remember that

$$d_{k+1}^{CGSD} = -g_{k+1} + \beta_k^{CGSD} s_k$$

$$g_{k+1}^T d_{k+1} = g_{k+1}^T(-g_{k+1} + \beta_k^{CGSD} s_k)$$

$$g_{k+1}^T d_{k+1} = g_{k+1}^T\left(-g_{k+1} + \left(\frac{g_{k+1}^T g_{k+1}}{y_k^T s_k} - \frac{(y_k^T g_{k+1})(s_k^T g_{k+1})}{(y_k^T s_k)^2}\right) s_k\right)$$

$$g_{k+1}^T d_{k+1} = g_{k+1}^T\left(-g_{k+1} + \left(\frac{(g_{k+1}^T g_{k+1})(y_k^T s_k) - (y_k^T g_{k+1})(s_k^T g_{k+1})}{(y_k^T s_k)^2}\right) s_k\right)$$

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \frac{(g_{k+1}^T g_{k+1})(y_k^T s_k)(g_{k+1}^T s_k) - (y_k^T g_{k+1})(s_k^T g_{k+1})(g_{k+1}^T s_k)}{(y_k^T s_k)^2} \quad (18)$$

For (18) satisfy sufficient descent condition we have

$$\left|\frac{(g_{k+1}^T g_{k+1})(y_k^T s_k)(g_{k+1}^T s_k) - (y_k^T g_{k+1})(s_k^T g_{k+1})(g_{k+1}^T s_k)}{(y_k^T s_k)^2}\right| \leq L\|g_{k+1}\|^2 \ , where\ 0 < L < 1$$

So that

$$g_{k+1}^T d_{k+1}^{CGSD} \leq -\|g_{k+1}\|^2 + L \|g_{k+1}\|^2$$

And

$$g_{k+1}^T d_{k+1}^{CGSD} \leq -(1 - L) \|g_{k+1}\|^2$$

We denote $c_2 = (1 - L) > 0$ then we can write

$$g_{k+1}^T d_{k+1}^{CGSD} \leq -c_2 \parallel g_{k+1} \parallel^2 \quad (19)$$

Next, we are going to prove the direction satisfy the sufficient descent condition when $0 < \theta_k < 1$, we have $g_{k+1}^T s_k \leq y_k^T s_k \leq p\|s_k\|^2$ and $y_k = g_{k+1} - g_k$, then from (13) we get

$$\theta_k^{HZI} \leq \frac{(p\|s_k\|^2)(\|g_{k+1}\|^2 - g_{k+1}^T g_k - p\|s_k\|^2 - \|g_{k+1}\|^2)}{(\|g_{k+1}\|^2 - g_{k+1}^T g_k)(p\|s_k\|^2)}$$

We have $|g_{k+1}^T g_k| \geq 0.2 \|g_{k+1}\|^2$ then

$$\theta_k^{HZI} \leq \frac{-0.2\|g_{k+1}\|^2 - p\|s_k\|^2}{\|g_{k+1}\|^2 - 0.2\|g_{k+1}\|^2} \quad (20)$$

From assumption (3.1), (3.2) and we know that

$$s_k = x_{k+1} - x_k \Rightarrow \|s_k\| \leq \|x_{k+1} - x_k\| \leq \|x_{k+1}\| - \|x_k\| \leq w$$

Set the overhead in (20) become

$$\theta_k^{HZI} \leq \frac{-0.2\,\bar{n}^2 - p\,w^2}{0.8\,\bar{n}^2} = c_3 \qquad (21)$$

From (15), (17), (19), and (21) we get

$$\therefore\ g_{k+1}^T d_{k+1} \leq -[c_3 c_2 + (1-c_3)c_1]\|g_{k+1}\|^2$$

$$\therefore\ g_{k+1}^T d_{k+1} \leq -c\,\|g_{k+1}\|^2\,,\quad with\ c = c_3 c_2 + (1-c_3)c_1$$

Therefore, it is showed that d_{k+1} satisfied the sufficient descent condition.

4. Global Convergence analysis

For any conjugate gradient method with strong wolf line search, the convergence holds. But for general function, only weak form of the zoutendijk condition is needed (Dai and Liao, 2001).

4.1. Lemma

Let Assumption (3.1) and (3.2) holds. Consider the method (2) and (3) where d_k is a descent direction α_k is established from the SWP if

$$\sum_{k\geq 1} \frac{1}{\|d_k\|^2} = \infty$$

Then

$$\lim_{k\to\infty} inf\ \|\ g_k\ \| = 0$$

4.2. Theorem

Suppose that Assumption (3.1) and (3.2) holds. Consider the algorithm HZI were $0 \leq \theta_k \leq 1$, and α_k is achieved by the strong wolfe line search and d_{k+1} is the descent direction. Then

$$\lim_{k\to\infty} inf\ \|\ g_k\ \| = 0$$

Proof

For the descent condition holds, we have $d_{k+1} \neq 0$. So using lemma (4.1) it is sufficient to prove that $\|\ d_{k+1}\ \|$ is confined above. From (9)

$$d_{k+1} = -g_{k+1} + \beta_k^{NEW} s_k$$

$$\|\ d_{k+1}\ \| = \|\ g_{k+1}\ \| + \|\ \beta_k^{NEW}\ \|\|\ s_k\ \|$$

$$\|\ d_{k+1}\ \| \leq \|\ g_{k+1}\ \| + \left[|1-\theta_k|.\,|\beta_k^{DY}| + |\theta_k|\left|\beta_k^{CGSD}\right|\right].\|\ s_k\ \|$$

By using Lipschitz condition and assumption (3.2) we get

$$|\beta_k^{DY}| \leq \frac{\|g_{k+1}\|^2}{\|y_k\|\|s_k\|} \leq \frac{n^2}{Aw} = k_1$$

And

$$\left|\beta_k^{CGSD}\right| \leq \frac{\|g_{k+1}\|^2}{\|y_k\|\|s_k\|} - \frac{\|y_k\|\,.\|g_{k+1}\|\,.\|s_k\|\,.\|g_{k+1}\|}{(\|y_k\|\|s_k\|)^2}$$

$$\leq \frac{n^2}{B(A\,w)^2} - \frac{A\,.n\,.w.n}{(A\,w)^2}$$

$$\leq \frac{n^2(1-A\,w\,B)}{B(A\,w)^2} = k_2$$

Now, we have

$$|\theta_k| \leq \left|\frac{y_k^T s_k(y_k^T g_{k+1} - s_k^T g_{k+1} - g_{k+1}^T g_{k+1})}{(y_k^T g_{k+1})(s_k^T g_{k+1})}\right|$$

Using SWC, we get $y_k^T s_k \leq \mathrm{p}\ \alpha_k\ \|s_k\|^2$

$$\leq \frac{\mathrm{p}\ \alpha_k \|s_k\|^2\ (\|y_k\|\|g_{k+1}\| - \|s_k\|\|g_{k+1}\| - \|g_{k+1}\|^2)}{\|y_k\|\|g_{k+1}\| - \|s_k\|\|g_{k+1}\|}$$

$$\leq \frac{\mathrm{p}\ \alpha_k\ \mathrm{w}^2(\mathrm{A\ n - w\ n - n^2})}{(\mathrm{A\ n - w\ n})}$$

$$\leq \frac{\mathrm{p}\ \alpha_k\ \mathrm{w}^2(\mathrm{A - w - n})}{(\mathrm{A - w})} = \mathrm{k}_3$$

$$\therefore \|\ d_{k+1}\ \| \leq \|\ g_{k+1}\ \| + [(1-k_3)k_1 + k_3 k_2].\|\ s_k\ \| \leq \mathrm{n + k\ w} = \eta$$

$$\Rightarrow \sum_{k\geq 1} \frac{1}{\|s_k\|^2} \geq \frac{1}{\eta_1^2} \sum_{k\geq 1} 1 = \infty$$

$$\Rightarrow \lim_{n\to\infty} inf \|g_k\| = 0$$

5. Numerical Results

In this section, we present the computational performance of a FORTRAN implementation of the (HZI) algorithm on a set of 75 unconstrained optimization test problems. And compiled with f77 (default compiler settings) on a Intel core i7. The test problems are the unconstrained problems in CUTE [Bongratz et al, 1995] [16] library. Along with other large-scale optimization problems presented in [17]. with the number of variables (n=1000, and 10000). Along with other large-scale optimization test problems in [Andrei, 2008].All algorithm implement the wolf line search condition (4) and (5) with $\varepsilon_1 = 0.0001\ and\ \varepsilon_2 = 0.001$ and the stopping criterion $\|g_k\|_\infty \leq (10)^{-6}$ where $\|\,.\,\|_\infty$ is the maximum absolute component of a vector. The criterion used here is CPU time.

From the figures below, we can conclude that (HZI) algorithm behaves equally to or healthier than the other algorithms.

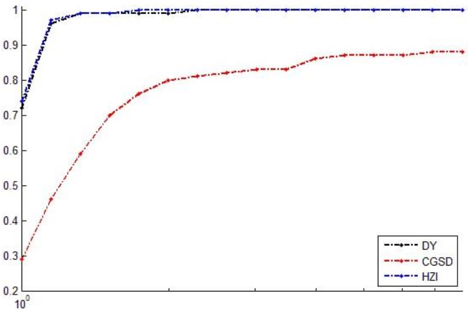

Fig. 1: Performance profiles based on iterations (PI)

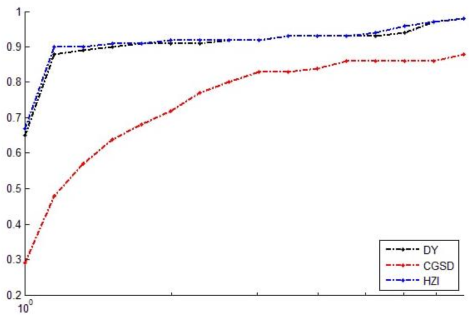

Fig. 2: Performance profiles based on function evaluations (PFE)

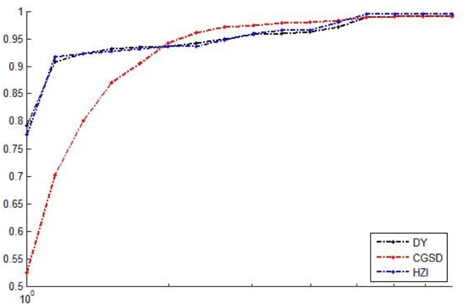

Fig. 3: Performance profiles based on CPU time (PT)

Conclusions

In this research, we proposed a new hybrid algorithm in which we used the Newton direction method
to improve the constraints of the unconstrained problem in the nonlinear optimization. The algorithm was
also treated from the theoretical and practical sides, with encouraging results in this field. The condition of sufficient regression and universal convergence was achieved under some assumptions. The new algorithm has the property of generalization, while the standard algorithm does not have such a property. Therefore, we can say that the new algorithm is effective for all dimensions of unconstrained problems.

The new algorithms included improvements in Number of iterations (NOI)), and the total number of computed functions (Number of Function Evaluations NOF), in addition, Time is a comparison of algorithm performance based on CPU time.

References

[1] Dai, Y. H., & Yuan, Y. (1999). A nonlinear conjugate gradient method with a strong global convergence property. *SIAM Journal on optimization*, *10*(1), 177-182.

[2] Wolfe, P. (1969). Convergence conditions for ascent methods. *SIAM review*, *11*(2), 226- 235.

[3] Wolfe, P. (1971). Convergence conditions for ascent methods. II: Some corrections. SIAM review, 13(2), 185-188.

[4] Hestenes, M. R., & Stiefel, E. (1952). Methods of conjugate gradients for solving linear systems. *Journal of research of the National Bureau of Standards*, *49*(6), 409-436.

[5] Fletcher, R., & Reeves, C. M. (1964). Function minimization by conjugate gradients. *The computer journal*, *7*(2), 149-154.

[6] Polak, E., & Ribiere, G. (1969). Note sur la convergence de méthodes de directions conjuguées. *ESAIM: Mathematical Modelling and Numerical Analysis-Modélisation Mathématique et Analyse Numérique*, *3*(R1), 35-43.

[7] Polyak, B. T. (1969). The conjugate gradient method in extremal problems. *USSR Computational Mathematics and Mathematical Physics*, *9*(4), 94-112.

[8] Fletcher, R. (1987). Practical methods of optimization john wiley & sons. *New York*, *80*, 4.

[9] Liu, Y., & Storey, C. (1991). Efficient generalized conjugate gradient algorithms, part 1: theory. *Journal of optimization theory and applications*, *69*(1), 129-137.

[10] Jardow, F. N., & Al-Naemi, G. M. (2020). A new hybrid conjugate gradient algorithm for unconstrained optimization with inexact line search. *Indonesian Journal of Electrical Engineering and Computer Science*, *20*(2), 939-947.

[11] Dai, Y. H., & Yuan, Y. (2001). An efficient hybrid conjugate gradient method for unconstrained optimization. *Annals of Operations Research*, *103*(1), 33-47.

[12] Dai, Y., & Yuan, Y. (2003). A class of globally convergent conjugate gradient methods. *Science in China Series A: Mathematics*, *46*(2), 251-261.

[13] Andrei, N. (2008). A Dai–Yuan conjugate gradient algorithm with sufficient descent and conjugacy conditions for unconstrained optimization. *Applied Mathematics Letters*, *21*(2), 165-171.

[14] Jardow, F. N., & Al-Naemi, G. M. (2020). A new hybrid conjugate gradient algorithm for unconstrained optimization with inexact line search. *Indonesian Journal of Electrical Engineering and Computer Science*, *20*(2), 939-947.

[15] Dai, Y. H., & Liao, L. Z. (2001). New conjugacy conditions and related nonlinear conjugate gradient methods. *Applied Mathematics and Optimization*, *43*(1), 87-101.

[16] I. Bongartz, A.R. Conn, N.I.M. Gould, P.L. Toint, CUTE: Constrained and unconstrained testing environments, ACM Trans. Math. Software 21 (1995) 123–160.

[17] N. Andrei, Test functions for unconstrained optimization. http://www.ici.ro/camo/neculai/SCALCG/evalfg.for.

تهجين طريقة جديدة للتدرج المترافق لكل من DY و CGSD باستخدام طريقة اتجاه نيوتن

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الملخص

قدمنا في هذا البحث طريقة جديدة للتدرج المترافق الهجينة لحل مسائل الابعاد الكبيرة في الامثلية اللاخطية غير المقيدة. وهذه الطريقة عبارة عن مزيج محدب من التدرج المترافق بين Dai-Yuan و Andrei-sufficient descent condition، وهذه الخوارزمية الجديدة تفي بتحقيق شرط اقتران D-L الشهير، وباستخدام صيغة اتجاه نيوتن ينتج ان الطريقة الجديدة تولد دائمًا اتجاه بحث منحدر كاف عند كل تكرار له. وبتقارب شامل باستخدام شرط وولف القوي (SWP) ، وأخيرًا تم تطبيق هذه الخوارزمية الجديدة على مجموعة من الدوال الاختبار للأمثلية اللاخطية غير المقيدة المعروفة في هذا المجال لغرض تقييم كفاءة هذه الخوارزمية بمقارنة نتائجها بنتائج الخوارزميات الاساسية ، والتي اظهرت النتائج التي حققناها جيدة وان طريقتنا هذه قوية وفعالة.