Tikrit Journal of Pure Science Vol. 28 (3) 2023

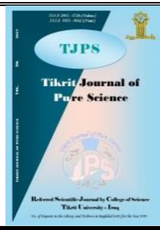
TJPS

https://doi.org/10.25130/tjps.v28i3.1435

Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php/j



BA-Semigroup

Mohammed Khalid Shahoodh

Al-Idrisi High School for Girls, General Directorate of Education in Ramadi, Ministry of Education, Iraq

ARTICLE INFO.

Article history:

-Received: 3 / 8 / 2022
-Received in revised form: 18 / 9 / 2022
-Accepted: 25 /9 / 2022
-Final Proofreading: 23 / 4 / 2023
-Available online: 25 / 6 / 2023

Keywords: BA-algebra, B-algebras, Semigroup, B-homomorphism, 0-commutative BA-algebra.

Corresponding Author:

\Name: **Mohammed Khalid Shahoodh**

E-mail: moha861122@yahoo.com

Tel:

©2022 COLLEGE OF SCIENCE, TIKRIT UNIVERSITY. THIS IS AN OPEN ACCESS ARTICLE UNDER THE CC BY LICENSE http://creativecommons.org/licenses/by/4.0/

ABSTRACT

Several algebraic structures have been studied by many authors to discuss the relationships among them. This article aims to study two algebraic structures namely semigroup and BA-algebra by combining them in one form namely BA-semigroup and investigate some of its properties. This paper studied the BA-sub-semigroup, an ideal and BA-homomorphism of a BA-semigroup with some of their properties. Some examples are given to illustrate the results.

شبه الزمرة-BA

محمد خالد شاهوذ

ثانوية الادريسي للبنات ، المديرية العامة للتربية في الرمادي ، وزارة التربية ، بغداد ، العراق

الملخص

العديد من التركيبات الجبرية قد تمت دراستها من قبل العديد من الباحثين من اجل مناقشة بعض العلاقات فيما بينها. هذه الدراسة تهدف الى دراسة تركيبتين جبريتين احداهما شبه الزمرة والاخرى جبر-BA من خلال دمج هاتين التركيبتين الجبريتين في تركيبة جبرية واحدة تسمى شبه الزمرة-BA. في هذا البحث تم دراسة بعض الخواص الجديدة مثل شبه الزمرة الجزئية-BA , المثالي و التشاكل-BA مع بعض خواصه على شبه الزمرة-BA. بعض الامثلة قد تم عرضها لتوضيح النتائج.

Introduction

Abstract algebra is one of the influential branches in the field of pure Mathematics. The name of algebra came from the book of the mathematician Muhammad ibn Musa al-Khwarizmi [1]. This field deals with study of the algebraic structures such as groups and rings and studied the relationships among them.

Many algebraic structures have been studied by many authors in order to present some new notations such as BA-algebra which has been introduced by Nouri [2]. Further, the same author presented the notation of a BS-algebra [3]. Then, some properties of this notation have been studied. Furthermore, the algebraic structure namely B-algebras has been provided by Neggers and Kim [4] and some of its properties are studied. A generalization of a B-algebras namely BG-algebras has been introduced by C. B. Kim and H. S. Kim [5]. Then, some of its properties are given such as BG-homomorphism, BG-isomorphism and quotient BG-algebras. Besides,

https://doi.org/10.25130/tjps.v28i3.1435

some other properties that concern on the BG-monomorphism and BG-epimorphism are investigated. Moreover, C. B. Kim and H. S. Kim [6] introduced the concept of BO-algebras with some of its properties. A paper by C. B. Kim and H. S. Kim [7] presented the idea of BM-algebras and studied some of its properties. Some other studies had investigated the direct product of some algebraic structures. For example, the concept of the direct product of a B-algebras has been introduced by Angeline et. al [8]. While, the direct product of a BF-algebras and BG-algebras are given in [9] and [10] respectively. In addition, the Lagrange's Theorem for the B-algebras has been studied by Bantug and Endam [11].Then, some important mappings which concentrated on the direct product of a B-algebras have been investigated by Angeline et. al [12]. On the other hand, some authors have been presented some new concepts by combining two algebraic structures and studied some of their properties. For instance, an algebraic structure namely BE-semigroups has been introduced by Ahn and Kim [13] with some of its properties. Another algebraic structure namely B-semigroups was defined by Çeven [14] and some of its properties have been discussed. Motivated by the work of previous researchers, we presented the notation of BA-semigroup and investigated some of its properties in this paper.

The present paper is structured as follows: In section two, some basic results which are important in this article are stated. The main results of this paper are given in section three. While, section four, followed by the conclusions and future research scope of the present paper.

Basic Concepts

Some basic results which are important for this study are given in this section. We started with the following definition.

Definition 2.1 [1] A system $(\mathcal{S},\boxtimes)$ is called semigroup, if $\boxtimes$ is an associative binary operation.

Definition 2.2 [2] The mathematical system $(\mathcal{S}, \boxplus, 0)$ is said to be BA-algebra, if it satisfying the following axioms.

i. $u \boxplus 0 = u$ for all $u \in \mathcal{S}$,

ii. $u \boxplus u = 0$ for all $u \in \mathcal{S}$,

iii. $u \boxplus (v \boxplus t) = t \boxplus (u \boxplus v)$ for all $u, v, t \in \mathcal{S}$.

Definition 2.3 [2] The system $(\mathcal{S}, \boxplus, 0)$ is said to be 0-commutative BA-algebra, if for every $u, v \in \mathcal{S}$ we have $u \boxplus (0 \boxplus v) = v \boxplus (0 \boxplus u)$.

BA-semigroup

This section presents the notation of a BA-semigroup with some of its properties. We commence by presenting the definition of a BA-semigroup which is presented as follows.

Definition 3.1 A mathematical system $(\mathcal{S}, \boxplus, \boxtimes, 0)$ is said to be BA-semigroup, if it satisfies the conditions below:

i. $(\mathcal{S}, \boxplus, 0)$ is a BA-algebra,

ii. $(\mathcal{S}, \boxtimes)$ is a semigroup,

iii. $u \boxtimes (v \boxplus t) = (u \boxtimes v) \boxplus (u \boxtimes t)$ and $(u \boxplus v) \boxtimes t = (u \boxtimes t) \boxplus (v \boxtimes t)$, $\forall u, v, t \in \mathcal{S}$.

Example 3.1 Consider the two binary operations $\boxplus$ and $\boxtimes$ which are defined on the non-empty set $\mathcal{S} = \{0,1,2\}$ by the following two Tables.

Table 3.1. BA-algebra

$\boxplus$	0	1	2
0	0	1	2
1	1	0	1
2	2	1	0

Table 3.2. Semigroup

$\boxtimes$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Then, it is easy to verify that $(\mathcal{S}, \boxplus, \boxtimes, 0)$ is a BA-semigroup. For the distributive property of $(\mathcal{S}, \boxplus, \boxtimes, 0)$, we have $0 \boxtimes (1 \boxplus 2) = 0 \boxtimes 1 = 0 = (0 \boxtimes 1) \boxplus (0 \boxtimes 2) = 0 \boxplus 0 = 0$. Similarly for the right side.

Example 3.2 Consider the two binary operations $\boxplus$ and $\boxtimes$ which are defined on the non-empty set $\mathcal{S} = \{0,1,2\}$ by the following two Tables.

Table 3.3. BA-algebra

$\boxplus$	0	1	2
0	0	1	2
1	1	0	2
2	2	2	0

Table 3.4. Semigroup

$\boxtimes$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

It's clear that $(\mathcal{S}, \boxplus, \boxtimes, 0)$ is a BA-semigroup.

Example 3.3 Consider the two binary operations $\boxplus$ and $\boxtimes$ which are defined on the non-empty set $\mathcal{S} = \{0,1,2,3\}$ by the following two Tables.

Table 3.5. BA-algebra

$\boxplus$	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	1	0	3
3	3	1	3	0

Table 3.6. Semigroup

$\boxtimes$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	0
3	0	3	0	0

Then, $(\mathcal{S}, \boxplus, \boxtimes, 0)$ is a BA-semigroup.

Lemma 3.1 For a semigroup $(\mathcal{S}, \boxtimes)$, $u \boxtimes 0 = 0 \boxtimes u = 0, \forall u \in \mathcal{S}$.

Proposition 3.1 Let $(\mathcal{S}, \boxplus, \boxtimes, 0)$ be a BA-semigroup. Then, $u \boxtimes 0 = 0 \boxtimes u = 0$ for every $u \in \mathcal{S}$.

https://doi.org/10.25130/tjps.v28i3.1435

Proof: Let $u \in \mathcal{S}$, then by Definitions 2.2, 3.1 we have $u \boxtimes 0 = u \boxtimes (0 \boxplus 0) = (u \boxtimes 0) \boxplus (u \boxtimes 0)$. By Lemma 3.1,we get $0 \boxplus 0 = 0$. Similarly, if we take the other side which is $0 \boxtimes u = (0 \boxplus 0) \boxtimes u = (0 \boxtimes u) \boxplus (0 \boxtimes u) = 0 \boxplus 0 = 0$. Therefore, $u \boxtimes 0 = 0 \boxtimes u = 0$ for every $u \in \mathcal{S}$. ■

Definition 3.2 Let $(\mathcal{S}, \boxplus, \boxtimes, 0)$ be a BA-semigroup. Then,

i. If $u \boxtimes v = v \boxtimes u$ for every $u, v \in \mathcal{S}$, then $\mathcal{S}$ is said to be commutative BA-semigroup.

ii. If $u \boxplus (0 \boxplus v) = v \boxplus (0 \boxplus u)$ for every $u, v \in \mathcal{S}$, then $\mathcal{S}$ is said to be 0-commutative BA-semigroup.

Remark 3.1 Example 3.1 shows that $\mathcal{S}$ is commutative and 0-commutative BA-semigroup. Furthermore, a BA-algebra given in Table 3.1 is a 0-commutative.

Definition 3.3 Let $(\mathcal{S}, \boxplus, 0)$ be a BA-Algebra, then its called an associative BA-algebra if for every $u, v, t \in \mathcal{S}$ we have $u \boxplus (v \boxplus t) = (u \boxplus v) \boxplus t$.

Example 3.4 For Example 3.3, a BA-algebra given in Table 3.5 is an associative BA-algebra.

Proposition 3.2 Let $(\mathcal{S}, \boxplus, \boxtimes, 0)$ be a BA-semigroup. Furthermore, let $\boxplus$ be an associative binary operation. If $u \boxtimes v = v \boxtimes u$, then $(u \boxplus v)^2 = u^2 \boxplus v^2$.

Proof: Let $u \boxtimes v = v \boxtimes u$ with $u, v \in \mathcal{S}$. Then, by Definition 3.1, we have $(u \boxplus v)^2 = (u \boxplus v) \boxtimes (u \boxplus v) = \big((u \boxplus v) \boxtimes u\big) \boxplus \big((u \boxplus v) \boxtimes v\big) = \big(u^2 \boxplus (v \boxtimes u)\big) \boxplus \big((u \boxtimes v) \boxplus v^2\big) = u^2 \boxplus (v \boxtimes u) \boxplus (v \boxtimes u) \boxplus v^2 = (u^2 \boxplus 0) \boxplus v^2 = u^2 \boxplus v^2$.■

Proposition 3.3 Let $(\mathcal{S}, \boxplus, \boxtimes, 0)$ be a BA-semigroup. Furthermore, let $\boxplus$ be an associative binary operation. If $u^{2k} \boxtimes v = v^{2k} \boxtimes u$, then $(u \boxplus v)^{2m} = u^{2m} \boxplus v^{2m}$ where $m \in Z^+$.

Proof: By using Mathematical induction on m. Let $m = 1$ then, by Proposition 3.2, the statement is true. Assume that the statement is true for $m = k$ in order to show that it's true for $k + 1$. That is mean, $(u \boxplus v)^{2k} = u^{2k} \boxplus v^{2k}$. Now, $(u \boxplus v)^{2k+1} = (u \boxplus v)^{2k} \boxtimes (u \boxplus v) = (u^{2k} \boxplus v^{2k}) \boxtimes (u \boxplus v) = \big((u^{2k} \boxplus v^{2k}) \boxtimes u\big) \boxplus \big((u^{2k} \boxplus v^{2k}) \boxtimes v\big) = (u^{2k+1} \boxplus (v^{2k} \boxtimes u)) \boxplus (u^{2k} \boxtimes v) \boxplus v^{2k+1}) = u^{2k+1} \boxplus (v^{2k} \boxtimes u) \boxplus (u^{2k} \boxtimes v) \boxplus v^{2k+1}$. Thereafter, $u^{2k+1} \boxplus (u^{2k} \boxtimes v) \boxplus (u^{2k} \boxtimes v) \boxplus v^{2k+1} = (u^{2k+1} \boxplus 0) \boxplus v^{2k+1} = u^{2k+1} \boxplus v^{2k+1}$. Thus, the statement is true for $k + 1$. Therefore, as required. ■

Definition 3.4 Let $(\mathcal{S}, \boxplus, \boxtimes, 0)$ be a BA-semigroup and let $\mathfrak{D} \subseteq \mathcal{S}$ then, $\mathfrak{D}$ is said to be BA-sub-semigroup of $\mathcal{S}$, if for every $u, v \in \mathfrak{D}$ we have $u \boxplus v \in \mathfrak{D}$ and $u \boxtimes v \in \mathfrak{D}$.

Example 3.5 For Example 3.1, let $\mathfrak{D} = \{0,1\}$, then $\mathfrak{D}$ is a BA-sub-semigroup of $\mathcal{S}$.

Definition 3.5 Let $(\mathcal{S}, \boxplus, \boxtimes, 0)$ and $(\mathcal{S}', \boxplus', \boxtimes', 0')$ be two BA-semigroups. Then, the mapping $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$ is said to be BA-homomorphism, if for every $u, v \in \mathcal{S}$ we have

i. $\mathcal{T}(u \boxplus v) = \mathcal{T}(u) \boxplus' \mathcal{T}(v)$,

ii. $\mathcal{T}(u \boxtimes v) = \mathcal{T}(u) \boxtimes' \mathcal{T}(v)$.

Remark 3.2 Let $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$ be a BA-homomorphism. If $\mathcal{T}$ is (1-1), then $\mathcal{T}$ is called BA-monomorphism. Moreover, If $\mathcal{T}$ is an onto, then $\mathcal{T}$ is called BA-epimophism and if $\mathcal{T}$ is a bijection, then it's called BA-isomorphism.

Proposition 3.4 Let $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$ be a BA-homomorphism. Then, $\mathcal{T}(0) = 0'$.

Proof: By Definition 2.2, we have $\mathcal{T}(0) = \mathcal{T}(0 \boxplus 0) = \mathcal{T}(0) \boxplus' \mathcal{T}(0) = 0' \boxplus' 0' = 0'$. ■

Proposition 3.5 Let $\mathcal{S}$ and $\mathcal{S}'$ be two BA-semigroups. Furthermore, let $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$ be a BA-homomorphism. Then,

i. If $\mathfrak{D}$ is a BA-sub-semigroup of $\mathcal{S}$, then $\mathcal{T}(\mathfrak{D})$ is a BA-sub-semigroup of $\mathcal{S}'$.

ii. If $\mathfrak{D}$ is a BA-sub-semigroup of $\mathcal{S}'$, then $\mathcal{T}^{-1}(\mathfrak{D})$ is a BA-sub-semigroup of $\mathcal{S}$.

Proof: (i) Since $\mathfrak{D}$ is a BA-sub-semigroup of $\mathcal{S}$, then by Definition 3.4, we have $u \boxplus v \in \mathfrak{D}$ and $u \boxtimes v \in \mathfrak{D}$ for every $u, v \in \mathfrak{D}$ and let $\mathcal{T}(\mathfrak{D}) = \{\mathcal{T}(u): u \in \mathcal{S}\}$. Since $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$ is a BA-homomorphism, then $\mathcal{T}(u \boxplus v) = \mathcal{T}(u) \boxplus' \mathcal{T}(v) \in \mathcal{T}(\mathfrak{D})$ and $\mathcal{T}(u \boxtimes v) = \mathcal{T}(u) \boxtimes' \mathcal{T}(v) \in \mathcal{T}(\mathfrak{D})$. (ii) $\mathcal{T}^{-1}(\mathfrak{D}) = \{u \in \mathcal{S}: \mathcal{T}(u) \in \mathfrak{D}\} \subseteq \mathcal{S}$. That is mean $u \in \mathcal{T}^{-1}(\mathfrak{D}) \Leftrightarrow \mathcal{T}(u) \in \mathfrak{D}$. Next, let $u, v \in \mathcal{T}^{-1}(\mathfrak{D})$, then $\mathcal{T}(u), \mathcal{T}(v) \in \mathfrak{D}$. Since $\mathfrak{D}$ is a BA-sub-semigroup of $\mathcal{S}'$, then we have $\mathcal{T}(u) \boxplus \mathcal{T}(v) \in \mathfrak{D}$ and $\mathcal{T}(u) \boxtimes \mathcal{T}(v) \in \mathfrak{D}$. This gives us $u \boxplus v \in \mathcal{T}^{-1}(\mathfrak{D})$ and $u \boxtimes v \in \mathcal{T}^{-1}(\mathfrak{D})$. Therefore, as required. ■

Proposition 3.6 The composition of two BA-homomorphisms is a BA-homomorphism.

Proof: Let $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$ and $\mathcal{T}^*: \mathcal{S} \longrightarrow \mathcal{S}^\star$ be two BA-homomorphisms. By definition 3.5, for every $u, v \in \mathcal{S}$ we have $\mathcal{T}(u \boxplus v) = \mathcal{T}(u) \boxplus' \mathcal{T}(v)$ and $\mathcal{T}(u \boxtimes v) = \mathcal{T}(u) \boxtimes' \mathcal{T}(v)$ and similarly for the second BA-homomorphism $\mathcal{T}^*$. Thus, $(\mathcal{T} \circ \mathcal{T}^*)(u \boxplus v) = \mathcal{T}(\mathcal{T}^*((u \boxplus v)) = \mathcal{T}\big(\mathcal{T}^*(u) \boxplus' \mathcal{T}^*(v)\big) = \mathcal{T}\big(\mathcal{T}^*(u)\big) \boxplus' \mathcal{T}\big(\mathcal{T}^*(v)\big) = (\mathcal{T} \circ \mathcal{T}^*)(u) \boxplus' (\mathcal{T} \circ \mathcal{T}^*)(v)$. Also, $(\mathcal{T} \circ \mathcal{T}^*)(u \boxtimes v) = \mathcal{T}(\mathcal{T}^*\big((u \boxtimes v)\big) = \mathcal{T}\big(\mathcal{T}^*(u) \boxtimes' \mathcal{T}^*(v)\big) = \mathcal{T}\big(\mathcal{T}^*(u)\big) \boxtimes' \mathcal{T}\big(\mathcal{T}^*(v)\big) = (\mathcal{T} \circ \mathcal{T}^*)(u) \boxtimes' (\mathcal{T} \circ \mathcal{T}^*)(v)$. Therefore, this completed the proof. ■

Definition 3.6 Let $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$ be a BA-homomorphism. Then, the kernel of $\mathcal{T}$ is defined to be the set $\{u \in \mathcal{S}: \mathcal{T}(u) = 0\_k\}$.

Proposition 3.7 Let $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$ be a BA-homomorphism. Then, kernel $\mathcal{T}$ is a BA-sub-semigroup of $\mathcal{S}$.

Proof: Let $u, v \in Ker\,\mathcal{T}$, then $\mathcal{T}(u) = 0\_k$ and $\mathcal{T}(v) = 0\_k$. Since $\mathcal{T}$ is a BA-homomorphism, then by Definition 3.5, for every $u, v \in \mathcal{S}$ we have $\mathcal{T}(u \boxplus v) = \mathcal{T}(u) \boxplus' \mathcal{T}(v) = 0 \boxplus' 0 = 0$. That is mean $u \boxplus v \in Ker\,\mathcal{T}$. Again, from Definition 3.5, we have $\mathcal{T}(u \boxtimes v) = \mathcal{T}(u) \boxtimes' \mathcal{T}(v) = 0 \boxtimes' 0 = 0$

https://doi.org/10.25130/tjps.v28i3.1435

which implies that $\mathcal{u} \boxtimes \mathcal{v} \in Ker\, \mathcal{T}$. Therefore, $Ker\, \mathcal{T}$ is a BA-sub-semigroup of $\mathcal{S}$. ■

Definition 3.7 Let $\mathcal{S}$ be a BA-semigroup. Furthermore, let $\Phi \neq \mathfrak{D} \subseteq \mathcal{S}$, then $\mathfrak{D}$ is said to be right (left) ideal of $\mathcal{S}$, if it satisfies the following conditions.

i. For each $\mathcal{u} \in \mathfrak{D}, \mathcal{v} \in \mathcal{S}$, $\mathcal{u} \boxplus \mathcal{v} \in \mathfrak{D}$ implies $\mathcal{v} \in \mathfrak{D}$,

ii. For each $\mathcal{u} \in \mathfrak{D}, \mathcal{v} \in \mathcal{S}$, $\mathcal{v} \boxtimes \mathcal{u} \in \mathfrak{D}$ ($\mathcal{u} \boxtimes \mathcal{v} \in \mathfrak{D}$).

Moreover, if $\mathfrak{D}$ is both right and left ideal from $\mathcal{S}$, then we say it an ideal from $\mathcal{S}$.

Example 3.6 Consider the BA-semigroup which is given in Example 3.3. Furthermore, let $\mathfrak{D} = \{0,2\}$, then $\mathfrak{D}$ is an ideal of $\mathcal{S}$.

Proposition 3.8 Let $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$be a BA-homomorphism. Then, kernel $\mathcal{T}$ is an ideal of $\mathcal{S}$.

Proof: Let $\mathcal{u} \in Ker\, \mathcal{T}$ with $\mathcal{v} \in \mathcal{S}$, then $\mathcal{T}(\mathcal{u}) = 0\_k$. Without loss of generality, let $\mathcal{u} \boxplus \mathcal{v} \in Ker\, \mathcal{T}$, then by Definition 3.6, we have $\mathcal{T}(\mathcal{u} \boxplus \mathcal{v}) = \mathcal{T}(\mathcal{u}) \boxplus' \mathcal{T}(\mathcal{v}) = 0$. From the other side we have $\mathcal{T}(\mathcal{v}) = 0$. Thus, we get $\mathcal{v} \in Ker\, \mathcal{T}$. Now, since $\mathcal{u} \in Ker\, \mathcal{T}$, $\mathcal{v} \in \mathcal{S}$ and $\mathcal{T}$ is a BA-homomorphism, then we get $\mathcal{T}(\mathcal{u} \boxtimes \mathcal{v}) = \mathcal{T}(\mathcal{u}) \boxtimes' \mathcal{T}(\mathcal{v}) = 0 \boxtimes' \mathcal{T}(\mathcal{v}) = 0$ which gives us $\mathcal{u} \boxtimes \mathcal{v} \in Ker\, \mathcal{T}$. In same way, $\mathcal{T}(\mathcal{v} \boxtimes \mathcal{u}) = \mathcal{T}(\mathcal{v}) \boxtimes' \mathcal{T}(\mathcal{u}) = \mathcal{T}(\mathcal{v}) \boxtimes' 0 = 0$ which gives us $\mathcal{v} \boxtimes \mathcal{u} \in Ker\, \mathcal{T}$. Therefore, $Ker\, \mathcal{T}$ is an ideal of $\mathcal{S}$. ■

Proposition 3.9 Let $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$ be a BA-homomorphism. Furthermore, let $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$ be an epimorphism. If $\mathfrak{D}$ is an ideal of $\mathcal{S}$, then $\mathcal{T}(\mathfrak{D})$ is an ideal of $\mathcal{S}'$.

Proof: Let $\mathfrak{D}$ be an ideal of $\mathcal{S}$, then by Definition 3.7, for each $\mathcal{u} \in \mathfrak{D}, \mathcal{v} \in \mathcal{S}$ then $\mathcal{u} \boxplus \mathcal{v} \in \mathfrak{D}$ implies $\mathcal{v} \in \mathfrak{D}$. Also, for each $\mathcal{u} \in \mathfrak{D}, \mathcal{v} \in \mathcal{S}$ then $\mathcal{u} \boxtimes \mathcal{v} \in \mathfrak{D}$ and $\mathcal{v} \boxtimes \mathcal{u} \in \mathfrak{D}$. Since $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$ be an epimorphism, then by Remark 3.2, $\mathcal{T}: \mathcal{S} \longrightarrow \mathcal{S}'$ is an onto. That is mean, if $\mathcal{u}, \mathcal{v} \in \mathcal{S}$ there exist $\mathcal{s}, \mathcal{t} \in \mathcal{S}'$ such that $\mathcal{T}(\mathcal{u}) = \mathcal{s}$ and $\mathcal{T}(\mathcal{v}) = \mathcal{t}$. Next, let $\mathcal{s} \in \mathcal{T}(\mathfrak{D})$ and $\mathcal{t} \in \mathcal{S}'$, then $\mathcal{s} \boxplus \mathcal{t} = \mathcal{T}(\mathcal{u}) \boxplus \mathcal{T}(\mathcal{v}) = \mathcal{T}(\mathcal{u} \boxplus \mathcal{v}) \in \mathcal{T}(\mathfrak{D})$. Thus, we have $\mathcal{s} \boxplus \mathcal{t} \in \mathcal{T}(\mathfrak{D})$. Further, $\mathcal{u} \boxtimes \mathcal{v} \in \mathfrak{D} \Longrightarrow \mathcal{T}(\mathcal{u} \boxtimes \mathcal{v}) \in \mathcal{T}(\mathfrak{D}) \Longrightarrow \mathcal{T}(\mathcal{u}) \boxtimes \mathcal{T}(\mathcal{v}) \in \mathcal{T}(\mathfrak{D}) \Longrightarrow \mathcal{s} \boxtimes \mathcal{t} \in \mathcal{T}(\mathfrak{D})$. Similarly we can prove the other side. Therefore, $\mathcal{T}(\mathfrak{D})$ is an ideal of $\mathcal{S}'$. ■

Conclusion

As a conclusion, this paper presented the notation of a BA-semigroup and studied some of its properties. This study proved that the kernel of a BA-homomorphism is an ideal. Moreover, we showed that the kernel of a BA-homomorphism is a BA-sub-semigroup of a BA-semi group. Also we proved that the image (pre-image) of a BA-sub-semi group of a BA-semigroup is a BA-sub-semi group. For future work, this notation can be applied to other algebraic structures such as D-algebra and BG-algebra.

References

[1] N. Jackson, A Course in Abstract Algebra, 2017, pp.3.

[2] A. H. Nouri. " BA-algebra". International Journal of Algebra, vol. 13, no.7, pp. 323-330, 2019.

[3] A. H. Nouri and L. S. Mahdi. "BS-algebra and Pseudo Z-algebra". 2nd International Conference on Physics and Applied Sciences (ICPAS), pp. 1- 6, 2021.

[4] J. Neggers and H. S. Kim. "On B-algebras". Matematiqki Vesnik, vol.54, pp. 21-29, 2002.

[5] C. B. Kim and H. S. Kim. "On BG-algebras". Demonsteatio Mathematica, vol. 41, no. 3, pp. 497-505, 2008.

[6] C. B. Kim and H. S. Kim. "On BO-algebras". Mthematica Slovaca, vol 62, no. 5, pp.855-864, 2012.

[7] C. B. Kim and H. S. Kim. "On BM-algebras". Scientiae Mathematicae Japonicae, vol.63, pp. 215-221, 2006.

[8] J. Angeline, V. Lingcong and J. C. Endam. "Direct Product of B-algebras". International Journal of Algebra, vol. 10, no. 1, pp. 33-40, 2016.

[9] R. C. Teves and J. C. Endam. "Direct Product of BF-Algebras". International Journal of Algebra, vol. 10, no. 3, pp. 125-132, 2016.

[10] S. Widianto, S. Gemawati, and Kartini. "Direct Product in BG-Algebras". International Journal of Algebra, vol. 13, no. 5, pp. 239-247, 2019.

[11] J. S. Bantug and J. C. Endam. "Lagrange's Theorem for B-algebras". International Journal of Algebra, vol. 11, no. 1, pp. 15-23, 2017.

[12] J. Angeline, V. Lingcong and J. C. Endam. "Mappings of the Direct Product of B-algebras". International Journal of Algebra, vol. 10, no. 3, pp. 133-140, 2016.

[13] S. S. Ahn and Y. H. Kim. "On BE-Semigroups". International Journal of Mathematics and Mathematical Sciences, vol. 2011, pp.1-8, 2011.

[14] Y. Çeven. "On B-Semigroups". Journal of Science, vol.30, no. 4, pp.404-411, 2017.