Tikrit Journal of Pure Science (2024) 29 (1):163-173
Doi: https://doi.org/10.25130/tjps.v29i1.1365

TJPS

IASJ
المجلات الاكاديمية العراقية

Tikrit Journal of Pure Science
ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php

Linear Free Surface of Steady and Unsteady Flow Theoretical Study

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Keywords: free surface, steady, time-dependent, porous medium, freshwater, Tropical Island.

ARTICLEINFO.

Article history:

-Received:	16 Aug. 2023
-Received in revised form:	03 Oct. 2023
-Accepted:	04 Oct. 2023
-Final Proofreading:	24 Feb. 2024
-Available online:	25 Feb. 2024

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ABSTRACT

This study investigates the relationship between the crucial cone point in limiting steady and unsteady simulation flows inside a simple two-dimensional, vertical sand column subjected to water withdrawal. The analysis employs a versatile technique applicable to both linear steady and linear unsteady problems. By utilizing Fourier series techniques, the method finds the solution for a line sink in a vertical duct and extends the steady technique to achieve unsteady solutions. The evolution of the beneath-the-water table surface towards steady states is monitored when they exist. The key findings highlight that as the impermeable base moves further away, the distance between the interface and the sink increases. With increasing flow rates, the critical depth elevates; however, a minor decrease in the critical flow rate is observed when the sink approaches the base. The prior behaviour of the flow impacts the critical flow rate value for each interface height. The study provides steady and unsteady solutions, employing linearized formulations, which demonstrate good agreement with observed phenomena. The length scale of the spatial separation between the interface and the sink emerges as a critical factor in determining cone. These findings contribute to a better understanding of cone phenomena and emphasize the significance of considering specific dimensions in the analysis of flow dynamics within porous media.

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دراسة نظرية للتدفق المستقر وغير المستقر على السطح الحر الخطي

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الملخص

تهدف هذه الدراسة إلى استكشاف العلاقة بين نقطة الاختراق الحاسمة في التدفقات المحدودة المستقرة وغير المستقرة في المحاكاة على نموذج رملي ثنائي الأبعاد ورأسي بسيط تتم فيه سحب المياه ذات السطح الحر. تعتمد الدراسة على تقنية متعددة الاستخدامات يمكن تطبيقها على مشكلات التدفق المستقر الخطي وغير المستقر الخطي. باستخدام تقنيات سلسلة فورييه، يتم العثور على حل لمغطس في قناة رأسية ويتم توسيع التقنية المستقرة لتحقيق حلول غير مستقرة. يتم مراقبة تطور سطح الجدول المائي أسفل السطح نحو حالات مستقرة في حال توافرها. تشير النتائج الرئيسية إلى أن المسافة من الواجهة إلى المصدر تزداد عندما يكون المصدر بعيدًا عن القاعدة غير القابلة للتسريب. يرتفع العمق الحرج مع زيادة معدل التدفق، على الرغم من حدوث تقليل طفيف في معدل التدفق الحرج عندما يقترب المغطس من القاعدة. يؤثر سلوك التدفق السابق على قيمة معدل التدفق الحرج لكل ارتفاع واجهة. توفر الدراسة حلولًا مستقرة وغير مستقرة باستخدام صيغ خطية تظهر توافق جيد مع الظواهر المرصودة. تظهر المسافة من المصدر إلى الواجهة كمقياس طول هام في تحديد حدوث الاختراق. تساهم هذه النتائج في فهم أفضل لظواهر الاختراق وتؤكد أهمية اعتبار الأبعاد المحددة في تحليل ديناميكيات التدفق في الوسط المسامي

Nomenclature:

Term/Symbol	Definition
Steady-state flow	Fluid flow that is constant and unchanging with time
Time-dependent flow	Fluid flow that changes with time
Laplace's equation	A partial differential equation used to obtain solutions for the velocity potential
Linearized continuity and momentum equations	Equations used to obtain solutions for the stream function
Non-dimensional velocity	Velocity expressed in non-dimensional terms
Linear steady-state flow	Steady-state flow in which the nonlinear components can be ignored
Linear time-dependent flow	Time-dependent flow described by linearized continuity equations
Free surface wave	A wave with a tiny amplitude described by linearized continuity and momentum equations
Wave propagation	The movement of waves through a medium
Reflection	The bouncing back of waves from a surface
Diffraction	The bending of waves around an obstacle
Linear flow theory	A theory that assumes a linear relationship between the velocity of the fluid and pressure
Turbulence	Chaotic and irregular fluid motion
Chemical reactors	Devices used for carrying out chemical reactions
Circulatory system	The system of organs and vessels responsible for circulating blood through the body

1. Introduction

Pumping from a recovery well is a popular method of extracting fluid from an aquifer or an oil well [1]. However, this process can have certain unintended consequences, particularly when fluid is drained from strata of varying densities. The fluid withdrawn will originate from the layer surrounding the area of extracting until the rate of flow reaches a specific limit. When the next layer's fluid exceeds this crucial pace, the quality of the retrieved fluid can be damaged, such as when water enters an oil recovery well or when saline water infiltrates a freshwater well. A previous study showed that the flow rate must be lower than the critical value to remove fluid from the stratum where the extraction spot is identified [2]. The level of underground water at the interface of the saturated zone will stabilize at a specific depth within a limited area, contingent upon the surface boundary conditions, in the presence of a line sink. Extraction may occur from single or multiple layers in a stratified aquifer with changing salinity. Consequently, there is a potential for hazardous levels of salt to seep into the water used for drinking or irrigation [3]. The initiation of pumping in a phreatic aquifer can lead to a rapid deterioration of the strata interface, allowing undesired water or air to enter the pump. To avoid this, the flow rate can be reduced, or the pumps can be switched off and allowed to rebound before restarting [4]. Furthermore, in some cases, fluid extraction can cause subsidence, especially in locations with high porosity and permeability [5]. The removal of fluid from the earth can cause a decrease in pore pressure, leading the soil to compact and sink. To avoid such effects on the surrounding environment, rigorous monitoring and management of fluid extraction are required.

2. Formulation of Problem

In porous media, prominent contributors to the comprehension of fluid flow include Bear [6, 7], Dagan [8], Muskat [9], and Polubarinova-Kochina [10]. Their considerable research has established a solid foundation for the theoretical aspects of this discipline, thereby expanding the knowledge base. As a result, the flow dynamics within porous media are commonly characterised by Darcy's Law, which serves as a fundamental paradigm for understanding this fascinating flow phenomena. These well-known references have significantly contributed to the development of a complete framework for researching fluid flow in porous media.

$$q = -\kappa \nabla \Omega \qquad (2.1)$$

where Ω is the piezometric head that is defined as

$$\Omega = \frac{\kappa}{\mu}(p + \rho g y), \qquad (2.2)$$

The seepage velocity vector is represented by q, the dynamic viscosity is denoted by μ, and the permeability is denoted by κ, which describes the ease of water flows through the medium. Higher rates of flow are associated with higher permeability levels. Water flows from high-pressure areas to low-pressure areas, according to Darcy's Law. Assuming that pressure changes do not cause density changes, water can be treated as incompressible. This leads to the conservation equation, which is given by:

$$\nabla . q = 0 \qquad (2.3)$$

Equation (2.3) describes the conservation of mass in fluid flow by relating the mass flux, or flow rate (q) of the fluid to the divergence of the flow filed. The equation, which involves the operator denoted by ∇, states that the divergence of the mass flux is zero. The concept of mass conservation states that the amount of fluid entering a specific region is equal to the amount of fluid leaving it. The steady-state flow is obtained by balancing the rate of change of mass within a particular volume with the flow of mass across its boundaries. The notion is applied under the assumption that the fluid is incompressible, which means that its density remains constant independent of pressure variation. This simplifies the fluid flow analyses by ignoring any density changes caused by pressure variations. Darcy's Law, when applied, results in:

$$\nabla . (-\kappa \nabla \Omega) = 0 \qquad (2.4)$$

Darcy's Law is a fundamental idea that defines fluid movement through porous materials by connecting the flow rate to the medium hydraulic properties. The equation for a particular system incorporates the pressure head Ω and indicates that the negative multiplication of hydraulic conductivity and the divergence of the pressure head gradient is zero. This suggests that the flow of water through the porous medium is in a state of equilibrium, with the flow rate being counterbalanced:

$$\nabla^2 \Omega = 0. \qquad (2.5)$$

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Then, (2.4) or (2.5) must be solved to compute the flow in a porous material along with the required boundary conditions. The complementary equation will be derived for the undefined interface, which needs to satisfy the following equation:

$$\nabla^2\omega_1 = \frac{\partial^2\omega_1}{\partial x^2} + \frac{\partial^2\omega_1}{\partial y^2} = 0; \quad -L < x < L, \zeta(x) < y < 1. \qquad (2.6)$$

These equations are subject to the following boundary conditions:

$$f(x) = \begin{cases} \omega_1 = 1, y = 1, & -L < x < L \\ \omega_1 = y, x = \pm L, & 0 < y < 1. \end{cases} \qquad (2.7)$$

An example of boundary conditions is an impermeable barrier that prevents water from flowing through it. When a porous medium material encounters a solid boundary that does not allow the passages of fluids, it is necessary for motion of both the solid boundary and the fluid particle to align in order to prevent the flow of fluid. Mathematically, this can be represented as:

$$q.n = 0 \qquad (2.8)$$

This occurs when the fluid n is perpendicular to the boundary [11]. In other words, the fluid cannot penetrate the barrier while creating a vacuum at the same time. At any location, the velocity element corresponding to the boundary must be zero. This criterion can also be stated as the boundary must be a flow streamline. There is another type of boundary condition that occurs when the ground is saturated and interfaces with air, such as at the surface of the ground or the barriers of an earth dam. In these situations, the water flows through this surface, creating a seepage face that marks the point where the water exits the flow region and enters the atmosphere [12]. If the surface is not streamlined, the pressure on it will remain constant at atmospheric pressure, and an equipotential line will run along its boundary. It can be assumed that the air pressure is constant ($p = 0$), without loss of generality. As a result, the definition of ω given in equation (2.2) can be expressed as:

$$\omega = \frac{\kappa\rho g}{\mu} y. \qquad (2.9)$$

A solid air boundary is one from barrier in the context of saturation flow. A free surface barrier is another important type of barrier, which often takes the shape of an air-water interface or the interface between two denser layers. However, the specific location of the free surface is frequently unknown. The most common type of free surface comprises two requirements. First, as shown by (2.8), there must be a uniform pressure along the interface. Second, the free surface should be streamlined. The two-dimensional free surface can thus be defined by:

$$\omega_y - \zeta'(x)\Omega_x \qquad (2.10)$$

on the two-dimensional free surface given by $y = \zeta'(x)$. This condition is the most challenging to address because the boundary is unknown at the beginning, but it will play a significant role in this project. Different scenarios can present various conditions, including the case of a porous surface. In this section, the volume flow into the wall is often proportional to the pressure difference at the porous surface. Additional requirements may be necessary. For example, consider two layers in a porous media with no withdrawal of freshwater and saltwater of different densities. The length of the tropical island is $L, -L < x < L$. In the two layers, the flow satisfies the following equations:

$$\omega_1 = p_1 + \rho_1 g y, \quad y > \xi(x) \qquad (2.11)$$
$$\omega_2 = p_2 + \rho_2 g y, \quad y < \xi(x) \qquad (2.12)$$

When the interface is $y = \xi(x)$ between two fluids where the upper fluid has a density of ρ_1 and is bounded by air, the density of the lower fluid is ρ_2 and the pressure is p_i for layers $i = 1,2$. Pressure p_i and atmospheric pressure must be equal on the unknown surface, which indicates $\rho_1 = 0$. Given that the tropical island is high, there is:

$$\omega_1 = \rho_1 \, g \, z \qquad (2.13)$$

When $y = z$ along the bottom and between the two fluid layers, the pressures along the interface must be equal; therefore, $\rho_1 = \rho_2$ on $y = \xi(x)$. Furthermore, if it is assumed that there is no flow at the interface of the fluids and the lower layer, there exists $\omega_2 = 0$, implying that

$$\omega_2 = -\rho_2 \, g \, z. \qquad (2.14)$$

As a result, the two fluids at the interface have the following condition:

$$\omega_1 = (\rho_1 - \rho_2) g z, \text{ on } y = \xi(x). \qquad (2.15)$$

The following equation is derived by dividing (2.13) by $\rho_1 \, g$ and the non-dimensionless equations:

$$\omega_1 = \left(1 - \frac{\rho_2}{\rho_1}\right) g z, \qquad \text{on } y = \xi(x). \qquad (2.16)$$

To solve Laplace's equation (2.5) within the range $y > \xi(x)$, a form must be found that satisfies all of the parameters, except for the fact that the bottom interface is undefined:

$$\omega(x, y) = y + \sum_{k=0}^{N} S_k sinh\left[\left(\frac{k}{L}\right)\pi(y-1)\right] \, sin\left[\left(\frac{k}{L}\right)\pi k\right], \qquad (2.17)$$

defines additional conditions that must be met, including

$$\begin{cases} \omega = (1-\psi)y = 1\,, \ \ on \ \ y = \zeta(x) \\ \omega_y - \zeta'(x)\omega_x = 0, \end{cases} \qquad (2.18)$$

where a small value of ζ' is taken and a large value of ψ is assumed.

$$\omega_y = 1 + \sum_{k=0}^{N} S_k \left(\frac{k}{L}\right)\pi \cosh\left[\left(\frac{k}{L}\right)\pi(y-1)\right] \sin\left[\left(\frac{k}{L}\right)\pi x\right]. \qquad (2.19)$$

At y=0, equation (2.18) becomes:

$$-1 = \sum_{k=0}^{N} S_k \left(\frac{k}{L}\right)\pi \cosh\left(-\frac{k}{L}\right) \pi \sin\left[\left(\frac{k}{L}\right)\pi x\right]. \qquad (2.20)$$

To apply Orthogonal, both sides of (2.20) are multiplied by $sin\left(\frac{j}{L}\right)\pi x$, integrating over the range $-L$ to L, and having:

$$\int_{-L}^{L} sin\left(\frac{j}{L}\right)\pi x \, dx = \int_{-L}^{L} \sum_{k=0}^{N} -S_k \left(\frac{k}{L}\right)\pi \cosh\left(-\frac{k}{L}\right) \pi \sin^2\left(\frac{j}{L}\right)\pi x \ dx. \qquad (2.21)$$

Therefore, the Fourier series approximation is:

$$\Omega_1(x,y) = y + \sum_{k=0}^{N} S_k shin\left[\left(\frac{k}{L}\right)\pi(y-1)\right] sin\left[\left(\frac{k}{L}\right)\pi k\right] \qquad (2.22)$$

As a result, the interface shape from (2.15) can be determined as:

$$\zeta = \frac{\omega}{1-\psi}\left[\sum_{k=0}^{N} S_k shin\left(\frac{k}{L}\right)\pi \; sin\left[\left(\frac{k}{L}\right)\pi x\right]\right] \qquad (2.23)$$

3. Free Surface of Steady and Unsteady Flow

Steady-state and unsteady flows are two common types of free surface flows that are studied using linear flow theory. Steady-state flow refers to a situation where the fluid flow is constant and unchanging with time, while time-dependent flow refers to a situation where the fluid flow changes with time. In both cases, Laplace's equation or linearized continuity and momentum equations can be used to obtain solutions for the velocity potential and stream function. To analyse time-dependent flows, Laplace's equation or linearized continuity equations are often used, while steady-state flow requires more advanced techniques to solve the governing equations [13]. Numerical methods, such as finite difference, finite element, and boundary element methods have been used to solve the governing equations for various free surface problems [14]. Computational methods have become increasingly important in the study of free surface flows. In addition, experimental techniques, such as laser Doppler velocimetry and particle image velocimetry, have been used to measure the velocity and pressure distribution in the free surface flows [15]. The study of free surface flows using linear flow theory and computational methods has practical applications in various fields, including fluid mechanics, civil engineering, naval architecture and oceanography. For linear steady-state flow, it is assumed that the impact of $\zeta(x)$ on $y = z$ is minor. This is the case when the flow rate is insignificant, or the sink is located at a considerable distance from the interface. In non-dimensional terms, this means that the parameter m has a very small value. Based on this assumption, the nonlinear components in equation (2.10) can be ignored, and the conditions can be applied to $y = z$ instead of $y = \zeta(x)$. Consequently, these conditions undergo a transformation:

$$\omega = \zeta, on \quad y = z, 0 < x < 1 \qquad (3.1)$$

$$\omega_y = \frac{c}{2}, on \quad y = z, 0 < x < 1 \qquad (3.2)$$

It should be noted that the non-dimensional velocity of the recharge water is $c/2$. As for the linear time-dependent, the same fundamental method used for the steady problem can be adopted, but this time, the series coefficients are permitted to vary with time. To solve the complete equations, all surface conditions must be fulfilled at $y = f(x;t)$, and the coefficients are time-dependent functions. In Figure (3.1), the free surface for both cases is depicted, with the steady flow represented by the lower curve and the time-dependent flow represented by the top curve. Measurements for both steady and time-dependent cases are taken at the same height values of $y = z$ and the value of c. The free surface is modelled differently in steady and time-dependent conditions. In steady linear flow, the free surface is assumed to be a plane or a curve of constant slope, which is often referred to as a 'streamline'. The modelling of the free surface varies depending on the type of flow being studied. In the realm of steady linear flow, the free surface is presumed to conform to either a planar or curvilinear profile exhibiting a consistent gradient, commonly known as a streamline. This simplification facilitates the manipulation of the governing equations of motion, thereby facilitating the straightforward determination of the velocity potential

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Doi: https://doi.org/10.25130/tjps.v29i1.1365

and stream function. Within the purview of unsteady flow, a free surface wave pertains to a wave of infinitesimal amplitude that may be adequately characterized through the utilization of linearized continuity and momentum equations, thereby adhering to the principles of linear flow dynamic [16]. Consequently, the wave-like behaviour manifested by free surface engenders the potential to investigate diverse phenomena encompassing wave propagation, reflection, and diffraction [17]. Moreover, a profound temporal interrelationship exists between steady and time-dependent linear flow patterns over time, whereas steady linear flow proves particularly valuable when grappling with scenarios of pronounced, such as the steady of waves and tides in the field of coastal engineering [18]. Steady linear flow models are better suited for situations in which the flow is relatively constant and time-invariant, such as the analysis of steady-state fluid dynamics in pipelines and canals [19].

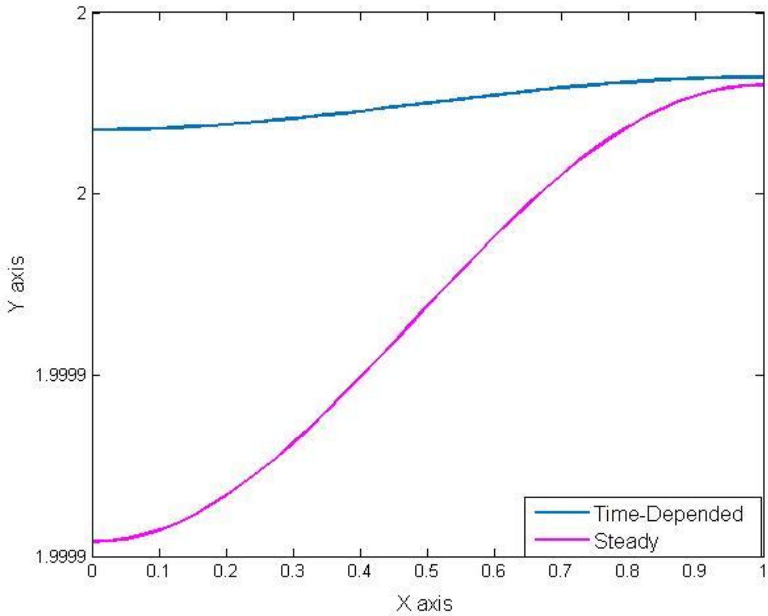

Figure (3.1) Free surface shape of the linear solution of (steady and unsteady) flow

The free surface modelling differs with respect to whether the flow is steady or erratic. Time-dependent linear flow models consider the free surface as a small-amplitude wave, whereas steady linear flow assumes a constant slope of the free surface. Time-dependent linear flow is better suited to modelling dynamic and changing fluid flows, whereas steady linear flow is more suited to modelling steady-state fluid dynamics. The employment of a steady or linear time-dependent flow theory is ultimately determined by the nature of the issue that is being studied. For example, steady linear flow theory is very beneficial in cases when the flow is uniform and constant, such as in the design of hydraulic elements, such as weirs and spillways. The computation of flow rate and water levels at several positions within the structure without the use of sophisticated simulation by using this theory. While the unsteady linear flow theory is better suited for modelling unsteady fluid flows and wave dynamics. For example, this theory can be used to predict the behaviour of waves in the ocean over time, such as wave propagation, reflection, and diffraction. It can also be used to inspect the impacts of ocean currents and tides on coastal structures, as well as the development of natural-force-resistant coastal defenses. In addition, both linear (steady and time-dependent) flow theories assume a linear relationship between the velocity of the fluid and pressure, but

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they approach turbulence differently. Both theories of linear steady and time-dependent flow assume a linear relationship between the velocity of the fluid and pressure, they take distinct approaches to turbulence. The theory of linear time-dependent flow can investigate laminar and turbulent flow regimes, whereas the theory of steady linear flow only considers laminar, or non-turbulent, flow regimes. The theory of unsteady linear flow proves that a complex system is highly valuable when analysing the behaviour of fluids, for example, chemical reactors and the circulatory system of humans. In conclusion, the two types of theories of linear steady and time-dependent flow have benefits and drawbacks. For understanding free surface behaviour and fluid dynamics in general, both of them are helpful tools. The theory to be used is ultimately determined by the application and problem under consideration. Comprehending the dynamics of free surface flows has a significant importance in a wide range of engineering and scientific domains. To model such flows, two primary methodologies are employed, steady linear flow and time-dependent linear flow. These approaches diverge in their assumptions and techniques for representing the free surface phenomenon. Steady linear flow assumes a flow state that remains unchanged with time, whereas time-dependent linear flow accounts for temporal variation by conceptualizing the free surface as a small-amplitude wave. Each approach possesses distinct advantages and limitations, and the selection characteristic of the problem under investigation. Prudent selection of the appropriate approach is pivotal for ensuring accurate modelling and analyses of fluid flows in a given context.

3.1 Outcomes and Discussion

The behaviour of a groundwater aquifer system was explored under both steady-state and time-dependent flow conditions in this work, with the goal of understanding the impact of flow rate, time-dependency, and other parameters on the aquifers free surface profiles. A code that simulated the steady-state and time-dependent free surface profiles supported this investigation. A specific solution was examined with aquifer thickness H of $2m$, aquifer length L of $1m$, hydraulic conductivity k of $0.2m$ per day, recharge water velocity c of $0.5m$ per day, and a series expansion with 10 terms N. By incorporating these parameters into the code, the behaviour of the aquifer was simulated. It was assumed that the effect of the function $\zeta(x)$ on the water surface $y = z$ was minimal for the steady-state flow analysis. This assumption was correct when the flow rate was modest or the sink was positioned a long distance from the water interface. The system's equilibrium state was examined under constant flow conditions by applying the steady-state conditions directly to y=z. The steady-state profile gave useful insights into the aquifer's behaviour and allowed for estimating the surface profile under steady-state flow conditions. The alignment between this work and other works [20] on similar systems validates the current findings. For the steady-state flow analysis, it was assumed that the effect of the function ζ(x) on the water surface y=z was negligible. This assumption was valid when the flow rate was small or when the sink was located at a significant distance from the water interface. By applying the steady-state conditions directly to y=z, the equilibrium state of the system was analysed under constant flow conditions. The steady-state profile provided valuable insights into the aquifer's behaviour and allowed for predicting the surface profile under steady flow conditions. To explore the effects of time dependence, the analysis was extended to time-dependent flow. By allowing the series coefficients to vary with time and solving the complete equations, time-dependent free surface profiles were obtained. These profiles captured the evolution of the water surface over time, considering changes in flow rate, recharge rate, and other system dynamics. The time-dependent profiles allowed for understanding the implications of time dependence on the aquifer system's behaviour. Comparing the steady-state and time-dependent profiles for the specific example,

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significant differences were observed. The non-dimensional velocity of the recharge water, represented by c/2, played a crucial role in determining the system's behaviour. It influenced the shape and magnitude of the surface profile, along with other factors, such as aquifer quality, recharge rate, and system geometry. The unsteady flow provided insights into the dynamic nature of the aquifer system, with changes in the surface profile occurring over time. In conclusion, the current findings highlighted the importance of considering time dependence and flow rate in understanding and predicting the behaviour of groundwater aquifer systems. The steady-state and time-dependent free surface profiles demonstrated the effects of these factors on the aquifer's surface behaviour, depletion rates, and the extent of drawdown. This knowledge is critical for efficient and sustainable utilization of groundwater resources, as it enables better management and decision-making in groundwater-related projects. Future research can be built upon these insights to explore more complex scenarios and optimize groundwater resource utilization in various settings. Furthermore, the non-dimensional velocity of the recharge water, with a value of, is an important metric. In the case of linear unsteady flow, the same basic method can be applied to the steady flow, but with allowing the series coefficient to vary with time. To achieve a complete solution for the equations, all surface requirements must be satisfied, and the coefficients must be time-dependent functions. The free surface for both the steady and time-dependent instances is shown in Figure (3.1). Readings at the same height values of z and c were taken to acquire measurements for both steady and time-dependent situations. This allowed comparing the system's behaviour under various settings and obtaining insight into the implications of time dependence on the surface profile. The non-dimensional velocity of the recharge water is an important factor that determines the system's behaviour. Other factors influencing flow characteristics include aquifer quality, recharge rate, and system geometry. Understanding these characteristics and their impact on system behaviour is critical for predicting the surface profile and ensuring efficient and sustainable groundwater resource utilization. The highest and lowest points of a line sink establish its surface, which is impacted by flow rate values at various sites along the sink. When the flow rate is high and at a critical value, the coning or drawdown happens extremely instantly, resulting in a narrow drawdown area. A lower flow rate, however, will cause the lowest points to take longer to decline, as shown in Figure (3.1). The pace and magnitude of the depletion are also affected by the pumping rate. A faster pumping rate results in a more rapid depletion, resulting in a more profound and widespread cone of depression. As a result, the flow rate and pumping rate are important elements in determining the surface profile and extent of drawdown. There are practical applications to this subject, such as groundwater resource management; therefore, knowing groundwater behaviour is critical for long term use. Flow conditions can be studied to learn about depletion rates, drawdown extent, and surface behaviour. Understanding helps this project management and decision-making. Furthermore, before embarking on infrastructure or land development project, it is critical to assess their impact on groundwater supplies. Analysing aquifer behaviour under different flow conditions predicts how changes in flow rate, recharge rate, and system dynamics affect the surface profile of the aquifer. This data informs environmental impact evaluations and mitigation measures for groundwater resource preservation. Understanding free surface fluxes in naval architecture is critical for designing and optimising ship hulls and offshore constructions. Linear flow theory can help forecast wave behaviour, propagation, and the effects of currents and tides on coastal constructions. The steady linear flow theory is useful for designing a uniform flow in hydraulic elements, such us weirs and spillways. It computes flow rate and water levels without the need for sophisticated simulations. For example, it aids in determining flow rate and water levels in various portion of spillway for dam design. Unsteady linear flow theory simulates dynamic fluid flows and wave dynamics. Over time, it predicts wave behaviour, propagation, reflection, and diffraction. It can help researcher's investigation of ocean wave behaviour during storms and estimate the influence on coastal buildings like breakwaters and seawalls. Both linear flow theories assume that fluid velocity and pressure have a linear relationship. However, they treat turbulence differently. Linear time-dependent flow theory investigates both laminar and turbulent flows, whereas steady linear flow theory accounts for turbulent flows and their effects. In conclusion, both steady and time-dependent linear flow theories offer advantages and disadvantages when it comes to understanding fluid dynamics and free surface behaviour. The theory chosen is dynamics and free surface behaviour. The theory chosen is determined by the precise application and problem at hand. Choosing the right theory ensures precise modelling and analysis of fluid flows, which helps to improve engineering and science.

4. Conclusion

The problem of an interface during withdrawal in porous medium flows has received a lot of attention. This study highlights the significance of utilizing linearized formulations to provide steady and time-dependent solutions, as well as the importance of considering nonlinear surface circumstances using Fourier techniques. Furthermore, it emphasises the need of taking into account the aquifer's finite dimensions as well as the impact of human activities on the system's behaviour. Anthropogenic activities, such as urbanisation, agriculture practises, and sea level rise caused by climate change can all have a considerable part to maintain the supply of freshwater resource of both human use and the preservation of biological system. In conclusion, this study sheds light on the behaviour of porous medium flows during withdrawal as well as the sensitive hydrological processes that occur in coastal aquifer system. The current findings can be used to establish effective and efficient management techniques for coastal aquifer systems, ensuring the availability of freshwater resources for both human use and ecological system preservation. Future research can be built upon these insights to explore more complex scenarios and further improve management techniques for coastal aquifer systems.

References

[1] Panday, S., Langevin, C. D., Niswonger, R. G., Ibaraki, M., and Hughes, J. D. (2013). An unstructured grid version of MODFLOW for simulating groundwater flow and tightly copled processes using a control volume finite-difference formulation. *US Geological Survey Techniques and Methods, book*, No.6-A45.

[2] Chen, K. (2018). Heat and Chemical Transport in Aquifers at Different Geological Setting. Texas A&M University (Doctoral dissertation).

[3] Sophocleous, M. (2005). Groundwater recharge and sustainability in the High Plains aquifer in Kansas, USA. *Hydrogeology Journal*, *13*, 351-365.

[4] Fetter, C. W., and Fetter, C. (2001). Applied Hydrogeology, 4th edition, Prentice Hall Upper Saddle River, USA.

[5] Krásný, J., and Sharp, J. M. (Eds.). (2007). *Groundwater in Fractured Rocks: IAH Selected Paper Series, volume 9* (Vol. 9). CRC Press.

[6] Bear, J. (1972). Dynamics of fluids in porous media. Elsevier, New York. *Dynamics of fluids in porous media. Elsevier, New York.*

[7] Bear, J. *(2013). Dynamics of fluids in porous media.* Courier Corporation, american elsevier publishing company INC. NEW YORK, pp: 757.

[8] Dagan, G. (2012). *Flow and transport in porous formations*. Springer-Verlag Berlin Heidelberg, GmbH.

[9] Muskat, M., & Wycokoff, R. D. (1935). An approximate theory of water-coning in oil production. *Transactions of the AIME*, *114*(01), 144-163.

[10] Ya, P. (1962). *Theory of Ground Water Movement*. Pelageya Yakovlevna Polubarinova-Kochina, USA.

[11] White, F. M., and Majdalani, J. (2006). *Viscous fluid flow* (Vol. 3, pp. 433-434). New York: McGraw-Hill.

[12] Harr, M. E. (2012). *Groundwater and seepage*. Courier Corporation, New York, San Francisco, Toronto, London.

[13] Stoker, J. J. (2017). Water waves: The mathematical theory with applications (Vol. 36). John Wiley and Sons.

[14] Boyun, G., & Lee, R. L. H. (1993). A simple approach to optimization of completion interval in oil/water coning systems. *SPE reservoir engineering*, *8*(04), 249-255.

[15] Kundu, P. K., Cohen, I. M., and Dowling, D. R. (2015). *Fluid mechanics*. 3th edition, Academic press, Elsevier.

[16] Debnath, L., and Debnath, L. (2005). Nonlinear partial differential equations for scientists and engineers, 4th edition, Birkhäuser, Boston, Berlin: 528-529.

[17] Whitham, G. B. (2011). Linear and nonlinear waves. John Wiley and Sons, INC, PP:629.

[18] Wang, Z., Countryman, A. M., Corbett, J. J., and Saebi, M. (2022). Economic and environmental impacts of ballast water management on Small Island Developing States and Least Developed Countries. *Journal of Environmental Management*, 301, 113779.

[19] Lucas, S. K., BLAKE, J., and KUCERA, A. (1988). Water coning in oil reservoirs. Univ. of Wollongong, Dep. Math., Honours Thesis, 23.

[20] Al-Ali, S., Hocking, G. C., Farrow, D. E., & Zhang, H. (2022). A spectral modelling approach for fluid flow into a line sink in a confined aquifer. *European Journal of Applied Mathematics*, *33*(5), 960-981.