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A Sufficient Descent 3-Term Conjugate Gradient Method for Unconstrained Optimization Algorithm

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ABSTRACT

In recent years, 3-term conjugate gradient algorithms (TT-CG) have sparked interest for large scale unconstrained optimization algorithms due to appealing practical factors, such as simple computation, low memory requirement, better sufficient descent property, and strong global convergence property. In this study, minor changes were made to the BRB-CG method used for addressing the optimization algorithms discussed. Then, a new 3-term BRB-CG (MTTBRB) was presented. This new method solved large-scale unconstrained optimization problems. Despite the fact that the BRB algorithm achieved global convergence by employing a modified strong Wolfe line search, in this new MTTBRB-CG method the researchers employed the classical strong Wolfe-Powell condition (SWPC). This study also attempted to quantify how much better 3-term efficiency is than 2-term efficiency. As a result, in the numerical analysis, the new modification was compared to an effective 2-term CG-method. The numerical analysis demonstrated the effectiveness of the proposed method in solving optimization problems.

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الانحدار الكافي لخوارزمية التدرج المترافق ثلاثي الحدود في الامثلية غير المقيدة

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الملخص

في الســـنوات الأخيرة، اثارت خوارزميات التدرج المترافق ثلاثي الحدود في خوارزميات الامثلية غير المقيدة الاهتمام على نطاق واســع بســبب عوامل علمية جذابة مثل حسـابات بسـيطة ومتطلبات الذاكرة المنخفضــة، وخاصـية الانحدار الكافية، وخاصـية التقارب القوية. قمنا بالبداية بإجراء تغيير بسـيط لطريقة BRB-CG لاسـتخدامها بهذا العمل. ثم بعد ذلك، قمنا باسـتخدام هذه الصـيغة المقترحة في خوارزمية التدرج المترافق ثلاثي الحدود (MTTBRB) BRB-CG، الطريقة الجديدة اســتخدمت في مســائل التدرج المترافق ذات القياس العالي. ان طريقة BRB-CG تحقق تقارباً شـمولي من خلال اسـتخدام بحث خط Wolfe القوي المعدل. في طريقتنا اسـتخدمنا خط البحث القوي الكلاسـيكي. في هذا البحث أثبتنا مدى كفاءة طريقة التدرج المترافق الثلاثي مقارنة بطريقة التدرج ثنائية الحدود. النتائج العددية وضـــحت مدى فعالية الطريقة المقترحة في حل مســـائل الامثلية.

I-Introduction

Consider the following unconstrained nonlinear optimization problem:

$$Min\, w(x), x \in \Re^n \tag{1.1}$$

with $w: \Re^n \to \Re$ being soft and $g(x) = \nabla w(x)$. One method for obtaining the smallest amount (1.1) [1] is the nonlinear CG-method, which does not demand any matrices. This is how iterative CG-methods look.

$$x_k = x_{k-1} + \rho_{k-1} d_{k-1}, \qquad k = 0,1,2,3, \ldots, \tag{1.2}$$

Where ρ_{k-1} denotes a positive step size and d_k represents the search direction. Typically, the direction is defined as follows:

$$d_k = \begin{cases} -g_k\,. & k = 0 \\ -g_k + \beta_k d_{k-1}. & k \geq 1' \end{cases} \tag{1.3}$$

$\beta_k \in \Re$ denotes the CG-method as a scalar parameter. Typically, the parameter β_k is chosen in such a way that (1.2) - (1.3) can be reverted to the linear CG-method [2]. Similarly, if w(x) is a strongly convex quadratic function and an exact line search (ELS) is used, all parameters β_k in these methods will be identical. However, for non-quadratic functions, each parameter β_k results in significantly different performances of the corresponding methods. Although FR, DY, and CD methods have strong convergent properties, they may not perform well in practice due to jamming. Despite their poor convergent properties, PRP, HS, and LS methods frequently perform well. Furthermore, for many years, the PRP method, because it essentially restarts if a bad direction occurs, has been regarded as one of the most efficient CG methods in practical computation [3]. The research results of the CG algorithm are very rich, including HS, FR, PRP, CD, LS, and DY [4-10], respectively.

$$\beta_k^{HS} = \frac{g_k^T y_{k-1}}{y_{k-1}^T d_{k-1}}; \quad \beta_k^{FR} = \frac{g_k^T g_k}{g_{k-1}^T g_{k-1}}; \quad \beta_k^{PRP} = \frac{g_k^T y_{k-1}}{g_{k-1}^T g_{k-1}};$$
$$\beta_k^{CD} = \frac{g_k^T g_k}{y_{k-1}^T d_{k-1}}; \quad \beta_k^{LS} = \frac{g_k^T y_{k-1}}{-g_{k-1}^T d_{k-1}}; \quad \beta_k^{DY} = \frac{g_k^T g_k}{y_{k-1}^T d_{k-1}}. \tag{1.4}$$

However, because an ELS is usually not possible for large-scale problems, the SWPC conditions are widely used in the CG-method for establishing convergence results.

$$w(x_k + \rho_k d_k) \leq w(x_k) + \gamma_1 \rho_k d_k \tag{1.5}$$

$$|g(x_k + \rho_k d_k)^T d_k| \leq \gamma_2 g_k^T d_k \tag{1.6}$$

Where $0 < \gamma_1 < \gamma_2 < 1$,and d_k that is a path to the minimum must be descent [11].

The remainder of this work is structured as follows. The second section is devoted to the evolution of TT-CG. Following the introduction of the TT-CG proposed by many researchers, an MTTBRB-CG is presented in the third section. SWPC provides a corresponding algorithm as well as descent properties. The fourth section presents obtained preliminary numerical results with SWPC. Finally, the closing remarks are included in the final Section.

II-REVIEW OF CONNECTED WORKS

Many researchers have investigated β_k choices because it is well known that the choice of β_k affects the numerical performance of the method. (1.2) and (1.3) define the classical algorithms, which calculate the CG parameter as shown in (1.4). As modifications to the classical CG algorithms, many researchers have recently proposed a plethora of TT-CG methods for unconstrained optimization problems. [12] generalizes the CD method to produce (NTTCD), defined by:

$$d_k = \begin{cases} -g_k . & k = 0 \\ -g_k + \frac{(-\|g_k\|^2)}{g_{k-1}^T d_{k-1}} d_{k-1} - \frac{g_k^T d_{k-1}}{g_{k-1}^T d_{k-1}} y_{k-1} . & k \geq 1 \end{cases},$$

[13] proposes an MTT-PRP procedure and demonstrates its global convergence using the Armijo line search.

$$d_k = \begin{cases} -g_k . & k = 0 \\ -g_k + \frac{g_k^T y_{k-1}}{g_{k-1}^T g_{k-1}} d_{k-1} - \frac{g_k^T d_{k-1}}{g_{k-1}^T g_{k-1}} y_{k-1} . & k \geq 1 \end{cases} \quad (1.7)$$

Where $y_{k-1} = g_k - g_{k-1}$,the TT-HS method is created by [14] in a similar content. This is written as:

$$d_k = \begin{cases} -g_k . & k = 0 \\ -g_k + \frac{g_k^T y_{k-1}}{y_{k-1}^T d_{k-1}} d_{k-1} - \frac{g_k^T d_{k-1}}{y_{k-1}^T d_{k-1}} y_{k-1} . & k \geq 1 \end{cases} \quad (1.8)$$

The TT-HS method has the steepest descent capability; when an ELS is used, it is reduced to the classic HS method. Furthermore, a modified TT-HS algorithm on the search direction is employed to ensure the global convergence properties of the direction clarified in (1.7):

$$d_k = \begin{cases} -g_k . & k = 0 \\ -g_k + \frac{g_k^T z_{k-1}}{z_{k-1}^T d_{k-1}} d_{k-1} - \frac{g_k^T d_{k-1}}{z_{k-1}^T d_{k-1}} y_{k-1} . & k \geq 1 \end{cases} \quad (1.9)$$

Given that the modified TT-HS algorithm is used to indicate the search direction's global convergence properties in (1.7), it is easy to understand why (1.7) is not used to prove the search direction's global convergence properties. Instead of ignoring (1.7), it should be made efficient and globally convergent. As a result of this, (1.7) can be changed to satisfy the global convergence criteria. This modification is expected to outperform the modified TT-CG algorithm in terms of numerical effectiveness. [15] proposes a new Dai-Liao-based TT-CG method motivated by this appealing descent property:

$$d_k = -g_k + \frac{{g_k}^T (y_{k-1} - \tau s_{k-1})}{s_{k-1}^T \, y_{k-1}} s_{k-1} - \frac{g_k^T \, s_{k-1}}{s_{k-1}^T \, y_{k-1}} (y_{k-1} - \tau s_{k-1}) \quad (1.10)$$

Where $d_0 = -g_0$ and $\tau \geq 0$.

Although the PRP-CG method is widely viewed as one of the most productive CG-methods in practical computation, its convergence properties are not all that great. [16] proposes the VPRP method, a variant of the PRP method, and its parameter β_k is determined by:

$$d_k = -g_k + \beta_k^{VPRP} s_{k-1} - \vartheta_k (y_{k-1} - \tau s_{k-1}) \quad (1.11)$$

$$\beta_k^{VPRP} = \frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|} g_k^T g_{k-1}}{\|g_{k-1}\|^2} - t \frac{\|y_{k-1}\|^2 . {g_k}^T d_{k-1}}{(\|g_{k-1}\|^2 + \epsilon |{g_k}^T d_{k-1}|)^2} \text{ and } \vartheta_k = \frac{g_k^T \, d_{k-1}}{\|g_{k-1}\|^2 + \epsilon |{g_k}^T d_{k-1}|} .$$

The general convergence results of the proposed formula with certain line searches, including the ELS, the Wolfe Powell line search, and the Grippo-Lucidi line search, are discussed. [17] proposes two TT-CG methods, TT-SMAR and TT-SMARZ, which are defined as follows:

$$d_k = -g_k + \beta_k^{\text{SMARZ}} d_{k-1} - \vartheta_k \varphi_{k-1} .$$

$$d_k = -g_k + \beta_k^{\text{SMAR}} d_{k-1} - \vartheta_k \varphi_{k-1}$$

$$\beta_k^{\text{SMARZ}} = \frac{g_k^T \left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|} d_{k-1} \right)}{d_{k-1}^T (d_{k-1} - g_k)},$$

$$\beta_k^{\text{SMAR}} = \frac{g_k^T \left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|} d_{k-1} \right)}{\|d_{k-1}\|^2},$$

$$\vartheta_k = \frac{g_k^T \, d_{k-1}}{\|d_{k-1}\|^2} \text{ and } \varphi_{k-1} = g_k - \frac{\|g_k\|}{\|g_{k-1}\|} d_{k-1} .$$

Not only do the modified methods have a good computational effect, but also they have all of the identical interesting properties as the FR method.

In addition, [18] expands on the approach by proposing a TT-CG method. Using formulas, the researchers create new search directions.

$$d_k = \begin{cases} -g_k . & k = 0 \\ -g_k + \beta_k^{BZAU} s_{k-1} - \vartheta_k^{BZAU} y_{k-1} . & k \geq 1 \end{cases} \quad (1.12)$$

$$\beta_k^{BZAU} = \frac{{g_k}^T y_{k-1}}{-\alpha g_{k-1}^T \, d_{k-1} + \tau |{g_k}^T d_{k-1}|} \text{ and } \vartheta_k^{BZAU} = \frac{g_k^T \, d_{k-1}}{-\alpha g_{k-1}^T \, d_{k-1} + \tau |{g_k}^T d_{k-1}|} .$$

Where $\alpha \in [1, +\infty), \tau \in [\alpha, +\infty)$, TTBZAU is the method's name. TTBZAU uses Wolfe Powell line search to satisfy global convergence properties with convex and nonconvex functions. The method meets the sufficient descent condition regardless of the line search utilized.

[19] recently suggests a TTCG method of RMIL-CG-method. The suggested technique is called TTRMIL method. The strategy's search direction is characterized by:

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$d_k = -g_k + \frac{g_k{}^T y_{k-1}}{\|d_{k-1}\|^2} d_{k-1} - \frac{g_k^T d_{k-1}}{\|d_{k-1}\|^2} y_{k-1}$

The proposed method by [20] has the following direction:

$$d_k = \begin{cases} -g_k \,. & k = 0 \\ -g_k + \beta_k^{\#\#+} d_{k-1} + \vartheta_k^{\#\#} g_{k-1}. & k \geq 1 \end{cases} \quad (1.13)$$

$\beta_k^{\#\#+} = max\{0, \beta_k^{\#\#}\}$,

$\beta_k^{\#\#} = \left[\beta_k^{BZAU} - \frac{\|g_{k-1}\|^2 . g_k{}^T s_{k-1}}{\left(-\alpha g_{k-1}^T d_{k-1} + \tau |g_k{}^T d_{k-1}|\right)^2}\right]$, $\vartheta_k^{\#\#} = \vartheta_k^{BZAU}$, $s_{k-1} = x_k - x_{k-1}$, $\alpha \in [1, +\infty)$, and finally, $\tau \in [\alpha, +\infty)$.

III-THE NEW MTTBRB-CG METHOD AND THEORETICAL RESULTS

First, the researchers refer to the BRB-CG [21] method; the direction is defined by (3), and the formula β_k^{BRB} is defined by:

$$\beta_{k-1}^{BRB} = \frac{\|g_k\|^2}{\|d_{k-1}\|^2}, \quad (2.1)$$

The current TT-CG algorithm focuses on making minor changes to β_k^{BRB} by:

$$d_k = \begin{cases} -g_k \,. & k = 0 \\ -g_k + \beta_{k-1}^{**} d_{k-1} + \vartheta_{k-1}^{**} y_{k-1}. & k \geq 1 \end{cases} \quad (2.2)$$

$$\beta_{k-1}^{**} = \frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|} g_k^T g_{k-1}}{\|d_{k-1}\|^2 + |g_k{}^T d_{k-1}|} \text{ and } \vartheta_{k-1}^{**} = \frac{g_k^T d_{k-1}}{\|d_{k-1}\|^2 + |g_k{}^T d_{k-1}|}. \quad (2.3)$$

The MTTBRB-CG method is the proposed TT-CG method, and the algorithm is explained further below.

The MTTBRB-CG Algorithm

Step 1: Choose $x_0 \in \Re^n, , \varepsilon > 0,\ d_k = -g_k$, put $k = 0$.

Step 2: If $\|g_k\| \leq \varepsilon$, then end; or else, go on to the next step.

Step 3: Estimate ρ_{k-1} using SWPC described in the equations (1.5) and (1.6).

Step 4: Estimate x_k by the equation (1.2), and Estimate g_k & f_k.

Step 5: Estimate the direction d_k by the equations (2.2) and (2.3).

Step 6: If $\|g_k\| \leq \varepsilon$, stop; instead, move on to the next step.

Step 7: If k=n or $|g_k^T g_{k-1}| \geq 0.2\ (\|g_k\|^2)$ is satisfied, go to

Step 1; or else, move on to the next step.

Step 8: Put $k = k + 1$ and proceed to Step (3).

Note that:

$$0 \leq \frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|} g_k^T g_{k-1}}{\|d_{k-1}\|^2 + |g_k{}^T d_{k-1}|} \leq \frac{\|g_k\|^2 + \frac{\|g_k\|}{\|g_{k-1}\|} |g_k^T g_{k-1}|}{\|d_{k-1}\|^2 + |g_k{}^T d_{k-1}|} \leq \frac{\|g_k\|^2}{\|d_{k-1}\|^2}, \quad (2.4)$$

The researchers maintain the descent condition and global convergence properties of the MTTBRB-CG method. First, standard Presumptions are made about the objective function. These Presumptions are going to be applied throughout paper.

Presumptions (A)

A1-The level set $\Upsilon = \{x \in \Re^n . w(x) \leq w(x_0)\}$, is bounded.

A2-The function r is smooth and its gradient is Lipschitz continuous in a specific neighborhood $\mathbb{N}$ of Υ; notably, there is a constant $\mathcal{L}$ greater than zero so that:

$$\|\nabla w(x_1) - \nabla w(x_2)\| \leq \mathcal{L}\|x_1 - x_2\| . \forall x_1 . x_2 \in \mathbb{N} . \quad (2.5)$$

Since $\{w(x_k)\}$ is decreasing, it is obvious that the sequence $\{x_k\}$ produced by the MTTBRB-CG method is stored in Υ. Furthermore, Presumption (A) results in that ν is a positive constant using MTTBRB-CG algorithm, resulting in $0 < \|g_k\| \leq \upsilon .\ \forall x \in \Upsilon$ [22].

This section discusses some of the most important properties of sufficient descent, as well as the global convergence of the MTTBRB-CG algorithm. The next theorem will be used to demonstrate that the presented MTTBRB-CG method meets the descent condition. From Presumption (A), it can be deducted that there is a small positive constant ξ greater than zero, resulting in

$$0 < \|g_k\| \leq \xi .\ \forall x \in \Upsilon . \quad (2.6)$$

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Theorem: (Descent condition)

Suppose the calculation of g_k and d_k is done by using MTTBRB-CG's algorithm under ILS in calculating the step length of ρ_{k-1} and the sufficient descent condition stands true for all $k \geq 0$

$$g_k^T d_k \leq -\omega \|g_k\|^2, \qquad (2.7)$$

Proof: For k = 0, it is obvious that the formulas of (2.6) are true. Take a look at the condition k ≥ 1. Similarly, (2.2) and (2.3) are obtained.

$$g_k^T d_k = g_k^T \left[-g_k + \frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|} g_k^T g_{k-1}}{\|d_{k-1}\|^2 + |g_k{}^T d_{k-1}|} d_{k-1} + \frac{g_k^T d_{k-1}}{\|d_{k-1}\|^2 + |g_k{}^T d_{k-1}|} y_{k-1} \right]$$

$$= -\|g_k\|^2 + \frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|} g_k^T g_{k-1}}{\|d_{k-1}\|^2 + |g_k{}^T d_{k-1}|} g_k^T d_{k-1} + \frac{g_k^T d_{k-1}}{\|d_{k-1}\|^2 + |g_k{}^T d_{k-1}|} g_k^T y_{k-1}$$

From (1.6), ξ is small positive number,

$$\gamma_2 g_{k-1}^T d_{k-1} \leq g_k^T d_{k-1} \leq -\gamma_2 g_{k-1}^T d_{k-1} \leq \gamma_2 \omega_1 \|g_{k-1}\|^2, \qquad (2.8)$$

By using the restart criteria, the following formula is obtained:

$$g_k^T y_{k-1} = \|g_k\|^2 - g_k^T g_{k-1} \leq 1.2\|g_k\|^2 \qquad (2.9)$$

and when combining (2.4), (2.8) and (2.9) with arithmetic operations, the following formula is obtained:

$$g_k^T d_k \leq -\|g_k\|^2 + \frac{\|g_{k-1}\|^2}{\|d_{k-1}\|^2} \|g_k\|^2 + \frac{\gamma_2 \omega_1 \|g_{k-1}\|^2}{\|d_{k-1}\|^2 + \gamma_2 \omega_1 \|g_{k-1}\|^2} (1.2\|g_k\|^2)$$

$$\leq -\left[1 - \frac{1}{(\eta^2/\xi^2)} - \frac{1{,}2\gamma_2\omega_1}{(\eta^2/\xi^2) + \gamma_2\omega_1}\right] \|g_k\|^2$$

Since the third term is postive , so

$$g_k^T d_k \leq -\left[1 - \frac{1}{(\eta^2/\xi^2)}\right] \|g_k\|^2$$

$$\leq -\omega \|g_k\|^2.$$

Following that, the study demonstrates that the proposed MTTBRB-CG -algorithm converges globally.

Theorem:

Consider whether Presumption (A) is true, whether the MTTBRB-CG is satisfying (2.2), and whether the step-size ρ_k satisfies (1.5) and (1.6). If

$$\sum_{k \geq 1} \frac{1}{\|d_k\|^2} = \infty \qquad (2.10)$$

Then,

$$\lim_{k \to \infty} inf \|g_k\| = 0 . \qquad (2.11)$$

Theorem: (Global convergent condition)

Suppose that Presumption (A) is correct. Let $\{x_k\}$ be a point sequence generated by TTMMMWA. Then, there is $\lim_{k\to\infty} \inf \|g_k\| = 0$.

Proof: Assume that the conclusion is not true, then $\|g_k\| \neq 0$, as previously mentioned, there exists constants, $\xi, \bar{\xi} > 0$ such that $0 < \bar{\xi} \leq g_k \leq \xi, \forall k \geq 0$. Now, by taking the norm of both sides of the proposed new direction, the following formula is obtained:

$$\|d_k\| = \left\| -g_k + \frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|} g_k^T g_{k-1}}{\|d_{k-1}\|^2 + |g_k{}^T d_{k-1}|} d_{k-1} + \frac{g_k^T d_{k-1}}{\|d_{k-1}\|^2 + |g_k{}^T d_{k-1}|} y_{k-1} \right\|$$

By using (2.4), (2.8) and $\|y_{k-1}\| = \|g_k - g_{k-1}\| \leq \|g_k + g_{k-1}\| \leq \|g_k\| + \|g_{k-1}\| \leq \|g_k\|$, the following formula is obtained:

$$\|d_k\| \leq \|g_k\| + \frac{\|g_k\|^2}{\|d_{k-1}\|^2} \|d_{k-1}\| + \frac{\gamma_2 \omega_1 \|g_{k-1}\|^2}{\|d_{k-1}\|^2 + \gamma_2 \omega_1 \|g_{k-1}\|^2} \|g_k\|$$

$$\leq \left[1 + \frac{\xi}{\eta} + \frac{\gamma_2 \omega_1}{(\eta^2/\xi^2) - \gamma_2 \omega_1}\right] \|g_k\|$$

$$\leq \varphi\, \xi = W.$$

where $\varphi = \left[1 + \frac{\xi}{\eta} + \frac{\gamma_2 \omega_1}{(\eta^2/\xi^2) - \gamma_2 \omega_1}\right]$

Square both sides of $\|d_k\| \leq W$, the following formula is obtained:

$$\|d_k\|^2 \leq W^2 \,. \tag{2.12}$$

By adding the sums on both sides of (2.12), the following formula is obtained:

$$\sum_{k\geq1} \frac{1}{\|d_k\|^2} \geq \frac{1}{W^2} \sum_{k\geq1} 1 = +\infty.$$

The above formula is obtained as a result of the inconsistency with the Zountendijk theorem [23].

IV-NUMERICAL RESULTS

This section's primary responsibility is to notify on the effectiveness of the MTTBRB-CG algorithm on a collection of test problems. The codes, which used double precision arithmetic, were written in Fortran77.

The researchers tried small dimensions (n=100) and large dimensions (n=1000) of the variables with SWPC with $\delta_1 = 10^{-3}$and $\delta_2 = 0.9$, respectively, in their experiments with 26 nonlinear unconstrained problems. A same test functions were used to compare the dependability of the current study algorithms to the well-known routines of Dx [24], LS [9], MMAU [20], RMIL [26], TTRMIL [19] and BRB [21]. All of these methods were terminated when the following stopping criteria were met: $\|g_{k+1}\| \leq 1 \times 10^{-6}$. If the iteration count is greater than 600, these routines are also forced to stop.

The tests were based on number of iterations (NOI) and the number of function evaluations (NOF). For better comparison, the Dolan and More's method [27] was used to plot these results. The results are shown in the figures below:

Fig. (1) depicts the progress of the new MTTBRB-CG algorithm relative to the calculated (NOI) of the test functions during the implementation of Dolan-More method with (n=100 & n=1000).

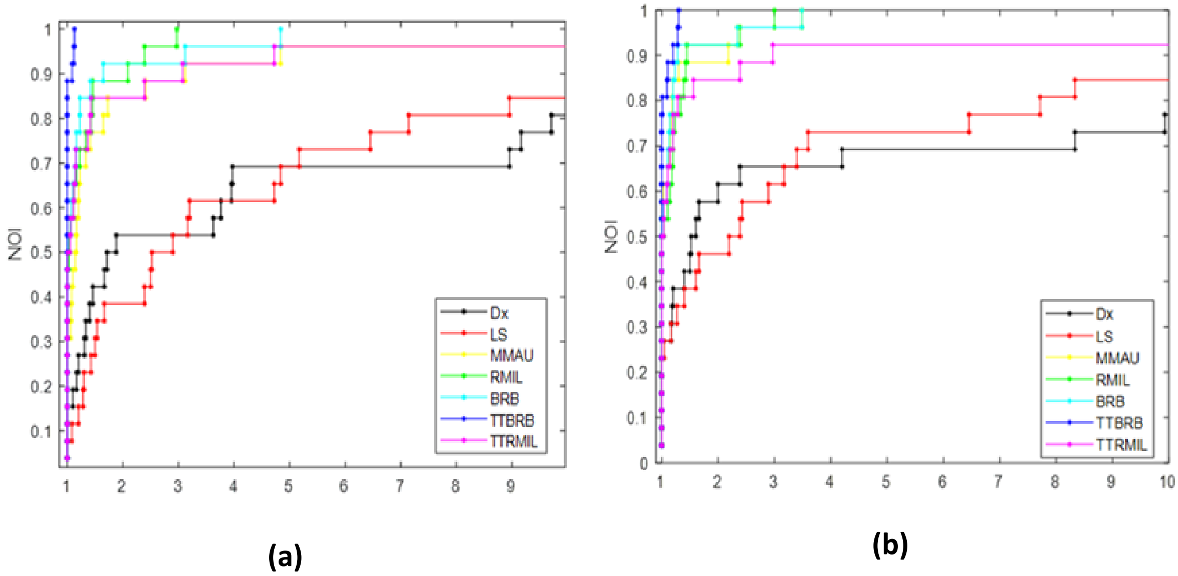


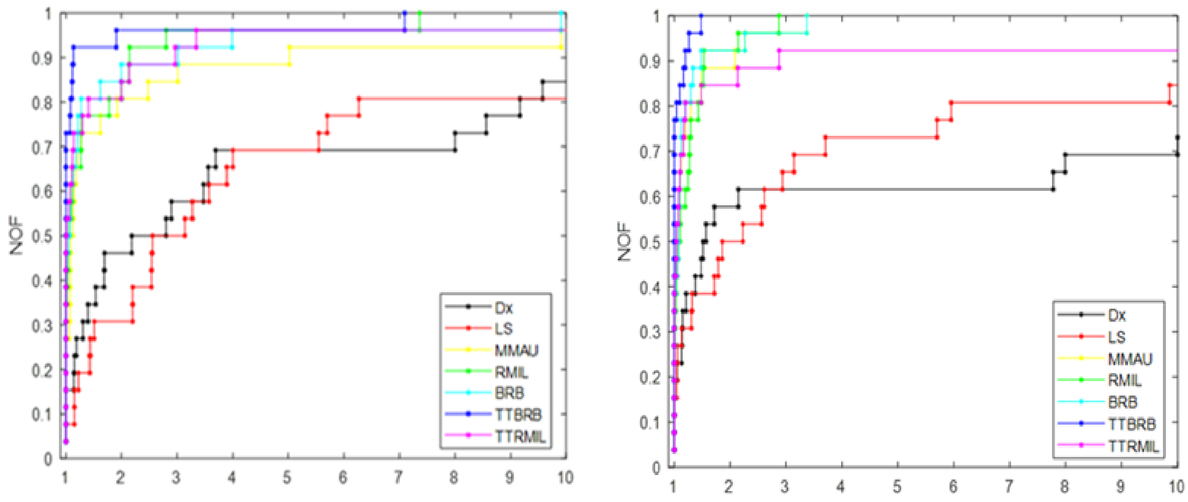
Fig. 1: The performance results for (NOI) between the algorithms (a) n=100 and (b) n=1000

Table (1) compares the proposed method TTBRB to DX, LS, MMAU, RMIL, TTRMIL, and BRB in terms of performance percentage. In the case of NOI, the DX method saves (10.23%), the MMUA method saves (55.52%), the RMIL method saves (69.68%), the TTRMIL method saves (53.25%), the BRB method saves (69.49%), and the TTBRB method saves (78.64%) when compared to the LS method. In the case of NOF, the DX method saves (15.84%), the MMUA method saves (52.91%), the RMIL method saves (69.54%), the TTRMIL method saves (56.91%), the BRB method saves (69.54%), and the TTBRB method saves (76.35%) when compared to the LS method.

Table 1: The proposed method's percentage performance between the algorithms

LS	DX	MMUA	BRB	RMIL	TTRMIL	TTBRB	LS
NOI	100%	89.77%	44.48%	34.51%	30.32%	46.75%	21.36%
NOF	100%	84.16%	47.09%	35.77%	30.46%	43.09%	23.65%

Fig. (2) depicts the progress of the new MTTBRB-CG algorithm relative to the calculated (NOI) of the test functions during the implementation of Dolan-More method with (n=100 & n=1000.



(c) (d)

Fig. 2: The performance results for (NOF) between the algorithms (c) n=100 and (d) n=1000

Table (2) compares the proposed method TTBRB to DX, LS, MMAU, RMIL, TTRMIL, and BRB in terms of percentage performance. In the case of NOI, the LS method saves (1.5%), the MMUA method saves (40.55%), the RMIL method saves (42.46%), the TTRMIL method saves (26.25%), the BRB method saves (40.95%), and the TTBRB method saves (48.3%) when compared to the DX method. In the case of NOF, the LS method saves (3.26%), the MMUA method saves (39.78%), the RMIL method saves (41.45%), the TTRMIL method saves (26.96%), the BRB method saves (41.08%), and the TTBRB method saves (46.55%) when compared to the DX method.

Table 2: The proposed method's percentage performance between the algorithms

	DX	LS	MMUA	BRB	RMIL	TTRMIL	TTBRB
NOI	100%	98.50%	59.45%	59.05%	57.54%	73.75%	51.70%
NOF	100%	96.74%	60.22%	59.92%	58.55%	73.04%	53.45%

CONCLUSION

The recent CG method research has resulted in a number of modifications to this method. TTCG methods are a fascinating computational innovation that yields efficient conjugate gradient algorithms. A new MTTBRB-CG, a modification of the BRB formula, provides enough descent directions for the objective function in this work when combined with a SWPC. Using the same line search, the modified method's global convergence is discovered. Furthermore, numerical experiments show that the suggested methods are effective and outperform some traditional conjugate gradient methods, which use some test functions and inexact line search.

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