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Class AK-manifold of Concircular curvature tensor (V)<br>Yaseen K. Abass, A. A. Shihab<br>Mathematics Department, College of Education for Pure Science, University of Tikrit, Tikrit, Iraq<br>https://doi.org/10.25130/tips.v28i1.1272

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## ABSTRACT

The current deals with new Three classes of almost Kahler manifold W2 of Concircular curvature tensor are Calculating differential geometrical but also topological parameters appropriate for new classes $\overline{\mathrm{V}} 1, \overline{\mathrm{~V}} 2$, and $\overline{\mathrm{V}} 3$, are the focus of the paper. Through it, an equivalence relationship was obtained between these classes and one of or more the Tensor compound in the adjoint G -structure space and then construct a relation between this new classes.

## Introduction

Three different kinds of almost Hermitian manifolds, each of which is defined in terms of the Riemannian curvature tensor, were established by A.Gray [1].These classes are designated as $R_{1}, R_{2}$, and $R_{3}$. The class $R_{1}$ stipulates what a parakahler manifold [2]. The $\mathrm{R}_{3}$ class includes RK-manifolds [3].The identities of $R_{1}, R_{2}$, and $R_{3}$ were demonstrated by $A$. Gray [1],[4] and [5] to the fundamental concept for comprehending the differential-geometrical properties of Kahler manifolds. Following are the components that make up the Riemannian curvature tensor:
$\mathrm{R}_{1}:<\mathrm{R}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}, \kappa_{4}>=<\mathrm{R}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \kappa_{3}, \kappa_{4}>$;
$\mathrm{R}_{2}:<\mathrm{R}\left(\kappa_{1}, \kappa_{2}\right) \quad \kappa_{3}, \kappa_{4}>=<\mathrm{R}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \quad \kappa_{3}, \kappa_{4}>+<$ $\mathrm{R}\left(\mathrm{J}_{1}, \kappa_{2}\right) \mathrm{J}_{3}, \kappa_{4}>+<\mathrm{R}\left(\mathrm{J}_{1}, \kappa_{2}\right) \kappa_{3}, \mathrm{~J}_{4}>$
$\left.\mathrm{R}_{3}:<\mathrm{R}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}, \kappa_{4}>=<\mathrm{R}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}, \mathrm{~J} \kappa_{4}\right\rangle$
The AH-structures belonging to the class $R_{i}$ have a tensor R that fulfills the identity $\mathrm{R}_{\mathrm{i}}$. If AH -any subclass of H -structures is named $\cap \mathrm{R}_{\mathrm{i}}=0$, where i is 1,2 , or 3 , then it exists. [5]. It is common knowledge that $V, R_{1}, R_{2}, R_{3}$ [6]. As a result, it makes sense to expect that the manifold class, $\mathrm{R}_{1}$ manifold class, and lastly the manifold of class $\mathrm{R}_{3}$ are among the AH-
manifolds that are closest to the Kahler manifold class for differential - geometrical and topological properties. AH -structures, Concircular tensor ( $V$ ) which satisfies to identity $V_{i}$, are referred to as the structures. of class $V_{i}$. If $\theta \subset A H-$ any sub class of AHstructures designations $\cap R_{i}=s$ where $i=1,2,3$ well - known that $V \subset V_{1} \subset V_{2} \subset V_{3}$ [4]. In light of this, it makes sense to anticipate that the H -manifolds with the closest geometrical and topological features will be the ones to the Vaishman - Gray manifold class, manifold class $V_{1}$, manifold class $V_{2}$, but rather finally, manifold of class $V_{3}$. In this paper, we will generalize these relationships, definitions and theories related to them for almost Kahler manifold $\mathrm{W}_{2}$ of Concircular curvature tensor

## Preliminaries

Assuming $M$ is a smooth manifold of size $2 n, C^{\wedge \infty}(M)$ is represents an algebra of smooth functions on $M$, and $\mathrm{X}(\mathrm{M})$ is really a module of smooth vector fields on M. The following assumes that all objects are of class $\mathrm{C}^{\wedge \infty}(\mathrm{M})$ and include manifold, tensor fields, Therefore, J -almost complex structure ( $\mathrm{J}^{\wedge 2}=\mathrm{id}$ ) on M , $\mathrm{g}=<.$, . $>$ and Almost Hermition (is short, AH)
structure on a manifold M the couple（ $\mathrm{J}, \mathrm{g}$ ）．pseudo metric Riemannian on M ．In this instance＜J丹，J $\alpha>$ $=\langle\vartheta, \alpha\rangle ; \vartheta, \alpha \in \mathrm{X}(\mathrm{M})$ ．

## Definition 1 ［4］

If the basic form $\Omega(\tau, \mu)=<\tau$ ，J $\mu>$ closed i．e $d \Omega$ $=0$ ，the Hermition manifold is said to be approximately＜M，J，g＝．，．＞has an almost Kahler structure（AK structure）．A manifold that is smooth M with an AK－chassis is referred to as a roughly Kahler manifold（AK－manifold）．

## Definition 2 ［3］

A class manifold is denoted by the letters（ $\mathrm{M}, \mathrm{J}, \mathrm{g}$ ） such that：
1） $\bar{V}_{1}$ if $\left\langle\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}, \kappa_{4}\right\rangle=\left\langle\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \mathrm{J} \kappa_{3}, \mathrm{~J} \kappa_{4}\right\rangle$ ； 2）$\overline{\mathrm{V}}_{2}$ if $\left\langle\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}, \kappa_{4}\right\rangle=\left\langle\mathrm{V}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \kappa_{3}, \kappa_{4}\right\rangle+$ $\left.\left\langle\mathrm{V}\left(\mathrm{J}_{1}, \kappa_{2}\right) \mathrm{J} \kappa_{3}, \kappa_{4}\right\rangle+\left\langle\mathrm{V}\left(\mathbf{J} \kappa_{1}, \kappa_{2}\right) \kappa_{3}, \mathrm{~J}_{4}\right\rangle ; 3\right) \bar{V}_{3}$ if $\left\langle\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}, \kappa_{4}\right\rangle=\left\langle\mathrm{V}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}, \mathrm{~J} \kappa_{4}\right\rangle ;$

## Note3：

From history theorem which states（（The following are our inclusion relationships：i）$V_{0}=V_{3}$ ，ii）$V_{1}=V_{2}$ ，iii） $\mathrm{V}_{4}=\mathrm{V}_{7}$ ，iv） $\mathrm{V}_{5}=\mathrm{V}_{6}$ ））．
We follows that AK－manifold of class $\mathrm{V}_{0}=\mathrm{V}_{3}=\mathrm{V}_{5}$ $=\mathrm{V}_{6}$ are also class $\bar{V}_{3}$ manifolds．The meaning of the specified curvature identities of is most obvious when expressed in terms of an a spectrum Concircular curvature tensor．

## Theorem 4

Consider $\mathrm{W}=(\mathrm{J}, \mathrm{g}=\langle., .>)$ represents almost Kahler manifold ．Then the following statements are identical in this case ：
1）W denote a class＇s structure of $\bar{V}_{3}$ ；
2）$V_{(0)}=0$ and
（3）The identities $V_{b c d}^{a}=0$ are reasonable in space of adjont G－structure space．

## Proof：

Consider $W$ denote a class＇s structure of $\bar{V}_{3}$ ．W without doubt，the same as identity $\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{J}$ $\mathrm{V}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}=0 ; \kappa_{1}, \kappa_{2}, \kappa_{3} \in \mathrm{X}(\mathrm{M})$
Spectral tensors are defined as follows：
$\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(1)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(2)}\left(\kappa_{1}\right.$ ， $\left.\kappa_{2}\right) \quad \kappa_{3}+V_{(3)}\left(\kappa_{1}, \quad \kappa_{2}\right) \quad \kappa_{3}+V_{(4)}\left(\kappa_{1}, \quad \kappa_{2}\right) \quad \kappa_{3}$ $+\mathrm{V}_{(5)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(6)}\left(\kappa_{1}, \quad \kappa_{2}\right) \quad \kappa_{3}$ $+\mathrm{V}_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3} ; \kappa_{1}, \kappa_{2}, \kappa_{3} \in \mathrm{X}(\mathrm{M})$
$\mathrm{J} \circ \mathrm{V}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}=\mathrm{J} 。 \mathrm{~V}_{(0)}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{J}_{\circ} \mathrm{V}_{(1)}\left(\mathrm{J} \kappa_{1}\right.$, $\left.\mathbf{J} \kappa_{2}\right) \mathrm{J}_{\kappa_{3}}+\mathbf{J} 。 \mathrm{~V}_{(2)}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{J} 。 \mathrm{~V}_{(3)}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \mathbf{J} \kappa_{3}+$ $\mathrm{J}_{\circ} \mathrm{V}_{(4)}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{J}_{\circ} \mathrm{V}_{(5)}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{J} 。 \mathrm{~V}_{(6)}\left(\mathrm{J} \kappa_{1}\right.$, $\left.\mathrm{J}_{2}\right) \mathrm{J}_{3}+\mathrm{J}_{\circ} \mathrm{V}_{(7)}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}$
$=\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(1)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(2)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+$ $\mathrm{V}_{(3)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(4)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(5)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+$ $\mathrm{V}_{(6)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3} ; \kappa_{1}, \kappa_{2}, \kappa_{3} \in \mathrm{X}(\mathrm{M})$
These identities will be defined as follows：
$\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{JV}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}=\left\{\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}\right.$ $\left.+\mathrm{V}_{(3)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(5)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(6)}\left(\kappa_{1}, \kappa_{2}\right)\right\}$
The identity is shaped by resources $\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{J}$ $\mathrm{V}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}=0$ is equal $\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(3)}\left(\kappa_{1}\right.$ ， $\left.\kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(5)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(6)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}$ and this identity is the same as other identities $V_{(0)}=V_{(3)}=$ $V_{(5)}=V_{(6)}=0$ ．

The following relations can be derived from the obtained identities on the adjoint G－space：structure＇s， according to characteristics（3）：
$V_{b c d}^{a}=V_{b \hat{c} \hat{d}}^{a}=V_{\hat{b} c \hat{d}}^{a}=0$ ．
Because of materiality tensorCand its properties（3） received relations that are identical to $V_{b c d}^{a}=0$ ，this mean identity $\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=0$ According to ［5］，the exact opposite is true．

## Theorem 5

Consider $\mathrm{W}=(\mathrm{J}, \mathrm{g}=$＜．，．$>$ ）represents almost Kahler structure．Then the following statements are identical in this case：
（1）W denote a class＇s structure of $\bar{V}_{2}$ ；
（2）$V_{(0)}=V_{(7)}=0$ ；and
（3）On the associated $G$－structure identities space $V_{b c d}^{a}=V_{\hat{b} \hat{c} \hat{d}}^{a}=0$ are they reasonable $V_{b c d}^{a}=V_{\hat{b} \hat{c} \hat{d}}^{a}=$ 0

## Proof：

Consider $W$－structure of a class $\bar{V}_{2}$ be the case identity $\bar{V}_{2}$ will be copied in the following format．
The identity will compute based on the notion of a spectrum tensor once every one has been assembled：
1）$\quad \mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(1)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+$ $\mathrm{V}_{(2)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(3)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(4)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}$ $+\mathrm{V}_{(5)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(6)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3} ; \kappa_{1}$ ， $\kappa_{2}, \kappa_{3} \in X(M)$
2）$\quad \mathrm{V}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \kappa_{3}=\mathrm{V}_{(0)}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(1)}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right)$ $\kappa_{3}+V_{(2)}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(3)}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(4)}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right)$ $\kappa_{3} \quad+\mathrm{V}_{(5)}\left(\mathrm{J} \kappa_{1}, \quad \mathrm{~J} \kappa_{2}\right) \quad \kappa_{3}+\mathrm{V}_{(6)}\left(\mathrm{J}_{1}, \quad \mathrm{~J}_{2}\right) \quad \kappa_{3}$ $+\mathrm{V}_{(7)}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \kappa=-\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(1)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+$ $\mathrm{V}_{(2)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(3)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(4)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(5)}\left(\kappa_{1}\right.$ ， $\left.\kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(6)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3} ; \kappa_{1}, \kappa_{2}$ ， $\kappa_{3} \in \mathrm{X}(\mathrm{M})$
3）$\quad \mathrm{V}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \mathrm{J} \kappa_{3}=\mathrm{V}_{(0)}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \mathrm{J}_{3}+\mathrm{V}_{(1)}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{V}_{( }$ ${ }_{2)}\left(\mathrm{J}_{1}, \kappa_{2}\right) \mathrm{J}_{3}+\mathrm{V}_{(3)}\left(\mathrm{J}_{1}, \kappa_{2}\right) \mathrm{J}_{3}+\mathrm{V}_{(4)}\left(\mathrm{J}_{1}, \kappa_{2}\right) \mathrm{J}_{3}+\mathrm{V}_{(5)}\left(\mathrm{J}_{1}\right.$ ， $\left.\kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{V}_{(6)}\left(\mathrm{J}_{1}, \kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{V}_{(7)}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \mathrm{J} \kappa_{3}=-\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right)$ $\kappa_{3}-\mathrm{V}_{(1)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(2)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(3)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}$ $+\mathrm{V}_{(4)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(5)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(6)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-$ $\mathrm{V}_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3} ; \kappa_{1}, \kappa_{2}, \kappa_{3} \in \mathrm{X}(\mathrm{M})$
4） $\mathrm{JV}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \kappa_{3}=\mathrm{JV}_{(0)}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{JV} \mathrm{V}_{(1)}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \kappa_{3}$ $+\mathrm{JV}_{(2)}\left(\mathrm{J}_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{JV}_{(3)}\left(\mathrm{J}_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{JV}_{(4)}\left(\mathrm{J}_{1}, \kappa_{2}\right) \kappa_{3}$ $+\mathrm{V}_{(5)}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{JV}_{(6)}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{JV}_{(7)}\left(\mathrm{J}_{1}, \kappa_{2}\right)$ $\kappa_{3}=-\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(1)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(2)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}$ $+\mathrm{V}_{(3)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(4)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(5)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+$ $\mathrm{V}_{(6)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3} ; \kappa_{1}, \kappa_{2}, \kappa_{3} \in \mathrm{X}(\mathrm{M})$
If we replace these decompositions in the previous equality，we get： $\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \kappa_{3}$－ $\mathrm{V}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{JV}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \kappa_{3}=2\left\{\mathrm{~V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}\right.$ $+\mathrm{V}_{(3)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(5)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(6)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}$ $\left.+V_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}\right\}$
This identity is equivalent to that
$\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=\mathrm{V}_{(3)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=\mathrm{V}_{(5)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=$ $\mathrm{V}_{(6)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=\mathrm{V}_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=0$
$V_{b c d}^{a}=V_{b \hat{c} \hat{d}}^{a}=V_{\hat{b} c \hat{d}}^{a}=V_{\hat{b} \hat{c} d}^{a}=V_{\hat{b} \hat{c} \hat{d}}^{a}=0$ ．
Additionally，these identities in the adjacent G－space structures are identical to those in the adjacent G－ space structures．

The received relations are equal to relations due to the materiality tensor V and by history characteristics we have $V_{b c d}^{a}=V_{\hat{b} \hat{d} \hat{d}}^{a}=0$ i．e．to identities $\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right)$ $\kappa_{3}=\mathrm{V}_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}$ ．Allow for AK＇s several identities once more $\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=\mathrm{V}_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=0$ are executed．
Then by history from we have： $\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}\left(\kappa_{1}\right.$ ， $\left.\mathbf{J} \kappa_{2}\right) \mathbf{J} \kappa_{3}-\mathrm{V}\left(\mathbf{J} \kappa_{1}, \kappa_{2}\right) \mathbf{J} \kappa_{3}-\mathrm{V}\left(\mathbf{J} \kappa_{1}, \mathbf{J} \kappa_{2}\right) \kappa_{3}=0$ ；i．e．
$\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=\mathrm{V}\left(\kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{V}\left(\mathbf{J} \kappa_{1}, \kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{V}\left(\mathrm{J} \kappa_{1}\right.$, $\left.\mathrm{J}_{2}\right) \kappa_{3}$ ．
In the received identity instead of $\mathrm{V}\left(\kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}$ we shall put the value received history from replacement
$\kappa_{2} \rightarrow \mathbf{J} \kappa_{2}$ and $_{3} \rightarrow \mathbf{J} \kappa_{3}$ ，i．e
$\mathrm{V}\left(\kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}=-\mathrm{JV}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \kappa_{3}$
Then
$\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=\mathrm{V}\left(\mathbf{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \kappa_{3}+\mathrm{V}\left(\mathbf{J} \kappa_{1}, \kappa_{2}\right) \mathrm{J} \kappa_{3}-$ $\mathrm{JV}\left(\mathbf{J} \kappa_{1}, \mathbf{J} \kappa_{2}\right) \kappa_{3}$
i．e．
$\left\langle\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}, \kappa_{4}\right\rangle=\left\langle\mathrm{V}\left(\mathrm{J}_{1}, \mathrm{~J}_{2}\right) \kappa_{3}, \kappa_{4}\right\rangle+\left\langle\mathrm{V}\left(\mathrm{J}_{1}\right.\right.$, $\left.\left.\kappa_{2}\right) \mathrm{J} \kappa_{3}, \kappa_{4}\right\rangle+\left\langle\mathrm{V}\left(\mathrm{J} \kappa_{1}, \kappa_{2}\right) \kappa_{3}, \mathrm{~J} \kappa_{4}\right\rangle$ The outcome is that the identical requirement is met by the manifold $\bar{V}_{2}$ ．
In a similar manner，the next theorem is demonstrated．

## Theorem 6

Consider $\mathrm{W}=(\mathrm{J}, \mathrm{g}=<.,\rangle$.$) represents almost$ Kahler manifold ．Then the following statements are identical in this：
（1）W denote a class＇s structure of $\bar{V}_{1}$
（2）$V_{(0)}=V_{(4)}=V_{(7)}=0$ ；
（3）associated identities on the G－structure space $V_{b c d}^{a}=V_{\hat{b} c d}^{a}=V_{\hat{b} \hat{c} \hat{d}}^{a}$ are reasonable ．

## proof ：

Take，for example，a class＇s W－structure $\bar{V}_{1}$ ．It＇s obvious that it＇s the same as identity $<\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}$ ，

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$\left.\kappa_{4}\right\rangle=\left\langle\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \mathrm{J} \kappa_{3}, \mathrm{~J} \kappa_{4}\right\rangle \quad$ as well as we have
$\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{J} V\left(\kappa_{1}, \kappa_{2}\right) \mathrm{J} \kappa_{3}=0 ; \kappa_{1}, \kappa_{2}, \kappa_{3} \in \mathrm{X}(\mathrm{M})$
Spectral tensors are through definition：
1） $\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(1)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+$ $\mathrm{V}_{(2)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(3)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(4)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}$ $+\mathrm{V}_{(5)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(6)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3} ; \kappa_{1}$ ， $\kappa_{2}, \kappa_{3} \in X(M)$
2） $\mathrm{J} \circ \mathrm{V}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}=\mathrm{J} 。 \mathrm{~V}_{(0)}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{J} 。$ $\mathrm{V}_{(1)}\left(\mathrm{J}_{1}, \mathrm{~J}_{2}\right) \mathrm{J}_{3}+\mathrm{J} 。 \mathrm{~V}_{(2)}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J}_{3}+\mathrm{J} 。 \mathrm{~V}_{(3)}\left(\mathrm{J} \kappa_{1}\right.$ ， $\left.\mathrm{J}_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{J} \circ \mathrm{V}_{(4)}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{J} 。 \mathrm{~V}_{(5)}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}+$ $\mathrm{J} \circ \mathrm{V}_{(6)}\left(\mathrm{J} \kappa_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}+\mathrm{J}_{\circ} \mathrm{V}_{(7)}\left(\mathrm{J}_{1}, \mathrm{~J} \kappa_{2}\right) \mathrm{J} \kappa_{3}$
$=-\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(1)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(2)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-$ $\mathrm{V}_{(3)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(4)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-\mathrm{V}_{(5)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}-$ $\mathrm{V}_{(6)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3} ; \kappa_{1}, \kappa_{2}, \kappa_{3} \in \mathrm{X}(\mathrm{M})$ ． Putting（1）and（2）in
$\mathrm{V}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{JV}\left(\kappa_{1}, \kappa_{2}\right) \mathrm{J} \kappa_{3}$ means，this identity is equivalent to that $\mathrm{V}_{(0)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+\mathrm{V}_{(4)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}+$ $\mathrm{V}_{(7)}\left(\kappa_{1}, \kappa_{2}\right) \kappa_{3}=0$
This identity is also equivalent to other identities $\mathrm{V}_{(0)}$ $=V_{(4)}=V_{(7)}=0$ ．So according their history properties the adjoint G－structure＇s received identities in space are identical to relation $V_{b c d}^{a}=V_{\hat{b} c d}^{a}=V_{\hat{b} \hat{c} \hat{d}}^{a}=0$ ．

## Theorem 7

Consider $\mathrm{W}=(\mathrm{J}, \mathrm{g}=\langle., .>)$ is an AK－structure， then the next class insertion $\bar{V}_{1} \subset \bar{V}_{2} \subset$ $\bar{V}_{3}$ are resonable．

## Proof：

Let＇s say a class＇s W－structure is $\bar{V}_{1}$ ．By history theory，it is identical to $\bar{V}_{0}=\bar{V}_{4}=\bar{V}_{7}=0$ ．
As a result of history theorem class $\bar{V}_{0}=\bar{V}_{7}=0$ ， is identical to class $\bar{V}_{2}$ ．Then $\bar{V}_{1} \subset \bar{V}_{2}$ ．Furthermore， the class $\bar{V}_{3}$ is the same as class $\bar{V}_{0}$ ．As shown by history theorem so $\bar{V}_{1} \subset \bar{V}_{2} \subset \bar{V}_{3}$
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## صفوف تنزرالانحناء الهرمتي الدائري لمنطوي كوهلر التقريبي

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الملخص
تم تعريف ثلاث فئات جديدة من منطوي كوهلر التقريبي من تتزر الانحناء الهرمتي الدائري , ويهدف هذا البحث الى حساب الخصائص التفاضلية - الهندسية والتبلوجية الاقرب للفئات الجديدة مركبات تنزر الانحناء الهرمتي الدائري لمنطوي كوهلر التقريبي , واخيرا تم ايجاد علاقة بين V1, V2, V3 ومع بعضها البعض .

