Anew Types of Contra Continuity in Bi-Supra Topological Space

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Abstract:

In this paper we introduce a new class of functions in bi-supra topological space called (contra-*i*[contra-*ii*]-continuous, contra-*g*-*i*[contra-*g*-*ii*]-continuous,contra-*gα*-*i*[contra-*gα*-*ii*]-continuous, contra-*gr*-*i*[contra-*gr*-*ii*]-continuous, contra-*gb*-*i*[contra-*gb*-*ii*]-continuous, contra-*πg*-*i*[contra-*πg*-*ii*]-continuous, contra-*πgα*-*i*[contra-*πgα*-*ii*]-continuous, contra-*πgr*-*i*[contra-*πgr*-*ii*]-continuouos, contra-*πgb*-*i*[contra-*πgb*-*ii*]-continuous) and we study the relation among these functions and the composition of these functions. At last many important theorems are proved.

1.Introduction

In 1996, J. Dontchev[12] introduced the notion of contra continuity. In 2007, Caldas et al.[16] introduced and investigated the notion of contra *g*-continuity.In 2012,S.I. Mahmood [23] introduced the notion of contra *gr*-continuity. In 1991, H. Maki, P.Sundaram and K. Balachandran[8] are introduced the notion of contra *gα*-continuity. In 2012, Metin Akdag and Alkan Ozkan[17] introduced and investigated the notion of contra generalized *b*−continuity (contra *gb* − continuity).In 2008, Ekici.E[7] introduced the notion of contra *πg*-continuity. In 2013,C.Janaki, V. Jeyanthi[4] are introduced the nothion of contra *πgr*- continuity.In 2008, I. Arokiarani, K. Balachandran and C. Janaki, [10] introduced the notion of contra *πgα*-continuity.In 2011, Sreeja .D and Janaki.C[24] are introduced the notion of contra *πgb*-continuity.In 1963,Kelly[14] introduced the concept of bi-topological space where a set X equipped with two topologies and denoted by $(X,\mathcal{T}_1,\mathcal{T}_2)$ where $\mathcal{T}_1,\mathcal{T}_2$ are two topologies defined on X.Al mashhour[15] in (1983) introduced the concept of supra topological space as a subfamily $\mathcal{T}$ of a family of all subset of X is said to be a supra topology on X if :

1. $\varphi, X \in \mathcal{T}$
2. If $A_i \in \mathcal{T}$ for all $i \in I$ then $\cup A_i \in \mathcal{T}$, where I is index set.

$(X, \mathcal{T})$ is called a supra topological space . The elements of $\mathcal{T}$ are called supra open sets in $(X,\mathcal{T})$ and the complement of supra open set is called a supra closed set .In this paper we introduce a new Types of Contra Continuity in Bi-Supra Topological Space.

2.preliminaries

Let us recall the definitions and results which are used in the sequel.

Definition 2.1:

A subset *A* of a topological space (X,τ) is called:

1. regular-open[18]if *A*=*int*(*cl*(*A*)) and regular-closed if *A*=*cl*(*int*(*A*)).
2. semi-open[20] if $A \subseteq cl(int(A))$ and a semi-closed if $int(cl(A)) \subseteq A$.
3. pre-open[2] if $A \subseteq int(cl(A))$ and a pre-closed if $cl(int(A)) \subseteq A$.
4. *α*-open[21] if $A \subseteq int(cl(int(A)))$ and an *α*-closed if $cl(int(cl(A))) \subseteq A$.
5. *π*-open[25] if it is the finite union of regular open set.

Definition 2.2:[9]

Let *A* be a subset of a topological space (X,τ), then:

1. $Scl(A) = \cap\{ F: A \subseteq F$, F is *s*-closed set }.
2. $Pcl(A) = \cap\{ F: A \subseteq F$, F is *p*-closed set }.
3. $\alpha cl(A) = \cap\{ F: A \subseteq F$, F is *α*-closed set }.
4. $rcl(A) = \cap\{ F: A \subseteq F$, F is *r*-closed set }.

Lemma 2.3:

Let *A* be a subset of topological space (X,τ),then:

1. $\alpha cl(A) = A \cup cl(int(cl(A)))$.[6]
2. $bcl(A) = Scl(A) \cap Pcl(A) = A \cup [int(cl(A)) \cap cl(int(A))]$.[5]

Definition 2.4:

A subset *A* of a topological space (X,τ) is called:

1. *g*-closed [19] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X.
2. *gr*-closed [22] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X.
3. *gα*-closed [9] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *α*-open set in X.
4. *gb*-closed [1] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X.
5. *πg*-closed [11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *π*-open set in X.
6. *πgr*-closed [13] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is *π*-open set in X.
7. *πgα*-closed [3] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *π*-open set in X.
8. *πgb*-closed [24] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is *π*-open set in X.

3.Bi-supra topological space

Definition 3.1:

Let X be a non-empty set. Let $\mathcal{ST}$ be the set of all semi open subsets of X(for short $\mathcal{S}o(X)$ [20] and Let $\mathcal{PT}$ be the set of all pre-open subsets of X (for short $\mathcal{P}o(X)$)[2], then we say that $(X,\mathcal{ST}, \mathcal{PT})$ is a bi-supra topological space. where each of $(X,\mathcal{ST})$ and $(X, \mathcal{PT})$ are supra topological spaces.

Remark 3.2:

It is clear that $\mathcal{ST}$, $\mathcal{PT}$ was independent .

Example 3.3:

Let X={a,b,c,d} with $\mathcal{T}$={φ,{c},{a,b},{a,b,c},X} therefore

$\mathcal{S}o(X) = \mathcal{ST}$ ={φ,{c},{a,b}, {c,d},{a,b,c},{a,b,d},X}.

$\mathcal{Po}$(X) = $\mathcal{PT}$={φ,{c},{a,b},{a,b,c},{a},{b},{a,c},{b,c},{a,c,d},{b,c,d},X}.

Hence (X,$\mathcal{ST}$, $\mathcal{PT}$) is bi-supra topological space.

Now we introduce the definition of the type of open sets in bi-supra topological space .

Definition 3.4:

Let (X,$\mathcal{ST}$, $\mathcal{PT}$) be a bi-supra topological space and let G be a subset of X. Then G is said to be:

1. ($\mathcal{ST}$, $\mathcal{PT}$)-supra open set (briefly *i*-open set) if G=(A∪B) ∪ φ where A∈$\mathcal{ST}$ and B∈ $\mathcal{PT}$.The complement ($\mathcal{ST}$, $\mathcal{PT}$)-supra open set is called ($\mathcal{ST}$, $\mathcal{PT}$)-supra closed set (briefly *i*-closed set).

2. ($\mathcal{ST}$, $\mathcal{PT}$)*-supra open set(briefly *ii*-open set) if G=A∪B where $A \in \mathcal{ST}$, $B \in \mathcal{PT}$ such that $A \notin \mathcal{PT}$ and A∩B≠ φ. The complement of ($\mathcal{ST}$, $\mathcal{PT}$)*-supra open set is called ($\mathcal{ST}$, $\mathcal{PT}$)*-supra closed set (briefly *ii*-closed set).

Proposition 3.5:

1.Every *ii*-open [*ii*-closed] set is *i*-open [*i*-closed] set but the converse is not true.

2.Notice that if $A \in \mathcal{ST}$, $B \in \mathcal{PT}$ such that $B \notin \mathcal{ST}$ and A∩B≠ φ is equivalent to (2) in Definition 3.4 .

Proof: Directly from definition.

Remark 3.6:

Observe that

The set of all *i*[*ii*]-open set and *i*[*ii*]-closed set is need not necessarily form a topology it is a supra topology.

Now we give an example to explain the types of open sets in bi-supra topological space.

Example 3.7:

Let X={a,b,c,d}

$\mathcal{T}$={φ,{a},{b},{a,b},{a,c},{a,b,c},{a,b,d},X}.$\mathcal{ST}$ ={φ,{a},{b},{a,b},{a,c},{a,d},{b,d},{a,b,c},{a,b,d} {a,c,d},X}.

$\mathcal{PT}$ ={φ,{a},{b},{a,b},{a,c},{a,b,c},{a,b,d},X}.

i-open set s={φ,{a},{b},{a,b},{a,c},{a,d},{b,d},{a,b,c},{a,b,d}, {a,c,d},X}.

i-closed sets={φ,{b},{c},{d},{c,d},{b,d} {b,c},{a,c},{a,c,d},{b,c,d},X}.

ii-open sets={φ,{a,d },{a,b,d},{a,c,d},{b,d},X}.

ii-closed sets ={φ,{a,c},{b,c},{c},{b},X}.

4.Some types of sets in bi-supra topological space

Definition 4.1:

Asubset A of bi-supra topological space (X,$\mathcal{ST}$, $\mathcal{PT}$) is called:

1.regular *i*[regular *ii*]-open if A=*i-int*[*ii-int*](*i-cl*(A)[*ii-cl*(A)]) and regular *i*[regular *ii*]-closed if A=*i-cl*[*ii-cl*](*i-int*(A)[*ii-int*(A)]).

2.semi-*i*[semi-*ii*]-open if A⊆*i-cl*[*ii-cl*](*i-int*(A)[*ii-int*(A)]) and semi-*i*[semi-*ii*]-closed if *i-int*[*ii-int*](*i-cl*(A)[*ii-cl*(A)]) ⊆A.

3.pre-*i*[pre-*ii*]-open if A⊆ *i-int*[*ii-int*](*i-cl*(A)[*ii-cl*(A)]) and pre-*i*[pre-*ii*]-closed if *i-cl*[*ii-cl*](*i-int*(A)[*ii-int*(A)]) ⊆ A.

4.α-*i*[α-*ii*]-open if A⊆ *i-int*[*ii-int*](*i-cl*[*ii-cl*](*i-int*(A)[*ii-int*(A)])) and an α-*i*[α-*ii*]-closed if *i-cl*[*ii-cl*](*i-int*[*ii-int*](*i-cl*(A)[*ii-cl*(A)])) ⊆ A.

5. π-*i*[π-*ii*]-open if it is the finite union of regular *i*[regular *ii*] open sets.

Definition 4.2:

A subset A of bi-supra topological space (X,$\mathcal{ST}$, $\mathcal{PT}$) is called:

1. *S-i-cl*(A)[*S-ii-cl*(A)] = ∩{ F: A ⊆ F , F is semi-*i*[semi-*ii*]-closed set }.

2. *P-i-cl*(A)[*P-ii-cl*(A)] = ∩{ F: A ⊆ F , F is pre-*i*[pre-*ii*]-closed set }.

3. α-*i-cl*(A)[α-*ii-cl*(A)] = ∩{ F: A ⊆ F , F is α-*i*[α-*ii*]-closed set }.

4. *r-i-cl*(A)[*r-ii-cl*(A)] = ∩{ F: A ⊆ F , F is regular *i*[regular *ii*]-closed set }.

Lemma 4.3:

Let A be a subset of bi-supra topological space (X,$\mathcal{ST}$, $\mathcal{PT}$) then:

1. α-*i-cl*(A)[α-*ii-cl*(A)] = A ∪ *i-cl*[*ii-cl*](*i-int*[*ii-int*](*i-cl*(A)[*ii-cl*(A)])).

2. *b-i-cl*(A)[*b-ii-cl*(A)] = *S-i-cl*(A)[*S-ii-cl*(A)] ∩ *P-i-cl*(A)[*P-ii-cl*(A)] =A ∪ [*i-int*[*ii-int*] (*i-cl*(A)[*ii-cl*(A)]) ∩ *i-cl*[*ii-cl*](*i-int*(A)[*ii-int*(A)])].

Definition 4.4:

A subset A of bi-supra topological space (X,$\mathcal{ST}$, $\mathcal{PT}$) is called:

1. *g-i*[*g-ii*]-closed if *i-cl*(A)[*ii-cl*(A)] ⊆ U whenever A ⊆ U and U is *i*[*ii*]-open set in X.

2.*gr-i*[*gr-ii*]-closed if *r-i-cl*(A)[*r-ii-cl*(A)] ⊆ U whenever A ⊆ U and U is *i*[*ii*]-open set in X.

3.*gα-i*[*gα-ii*]-closed if α-*i-cl*(A)[α-*ii-cl*(A)] ⊆U whenever A⊆U and U is α-*i*[α-*ii*]-open set in X.

4. *gb-i*[*gb-ii*]-closed if *b-i-cl*(A)[*b-ii-cl*(A)] ⊆ U whenever A ⊆ U and U is *i*[*ii*]-open set in X.

5. *πg-i*[*πg-ii*]-closed if *i-cl*(A)[*ii-cl*(A)]⊆ U whenever A ⊆ U and U is π-*i*[π-*ii*]-open set in X.

6. *πgr-i*[*πgr-ii*]-closed if *r-i-cl*(A)[*r-ii-cl*(A)] ⊆ U whenever A ⊆ U and U is π-*i*[π-*ii*]-open set in X.

7. *πgα-i*[*πgα-ii*]-closed if α-*i-cl*(A)[α-*ii-cl*(A)] ⊆ U whenever A ⊆ U and U is π-*i*[π-*ii*]-open set in X.

8. *πgb-i*[*πgb-ii*]-closed if *b-i-cl*(A)[*b-ii-cl*(A)] ⊆ U whenever A ⊆ U and U is π-*i*[π-*ii*]-open set in X.

5.Contra Continuity in bi-supra topological space

Definition 5.1:

A function f: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Y, $\mathcal{ST}_Y$, $\mathcal{PT}_Y$) is called:

1. Contra-*i*[contra-*ii*] continuous if f^{-1}(V) is *i*[*ii*]-closed in X for each *i*[*ii*]-open set V of Y.

2. Contra *g-i*[contra-*g-ii*]-continuous if f^{-1}(V) is *g-i*[*g-ii*]-closed in X for each *i*[*ii*]-open set V of Y.

3. Contra *gr-i*[contra-*gr-ii*]-continuous if f^{-1}(V) is *gr-i*[*gr-ii*]-closed in X for each *i*[*ii*]-open set V of Y.

4. Contra *gα-i*[contra-*gα-ii*]-continuous if f^{-1}(V) is *gα-i*[*gα-ii*]-closed in X for each *i*[*ii*]-open set V of Y.

5. Contra *gb-i*[contra-*gb-ii*]-continuous if f^{-1}(V) is *gb-i*[*gb-ii*]-closed in X for each *i*[*ii*]-open set V of Y.

6. Contra *πg-i*[contra-*πg-ii*]-continuous if f^{-1}(V) is *πg-i*[*πg-ii*]-closed in X for each *i*[*ii*]-open set V of Y.

7. Contra πgr-i[contra-πgr-ii]-continuous if f^{-1}(V) is πgr-i[πgr-ii]-closed in X for each i[ii]-open set V of Y.
8. Contra $\pi g\alpha$-i[contra-$\pi g\alpha$-ii]-continuous if f^{-1}(V) is $\pi g\alpha$-i[$\pi g\alpha$-ii]-closed in X for each i[ii]-open set V of Y.
9. Contra πgb-i[contra-πgb-ii]-continuous if f^{-1}(V) is πgb-i[πgb-ii]-closed in X for each i[ii]-open set V of Y.

Proposition 5.2:

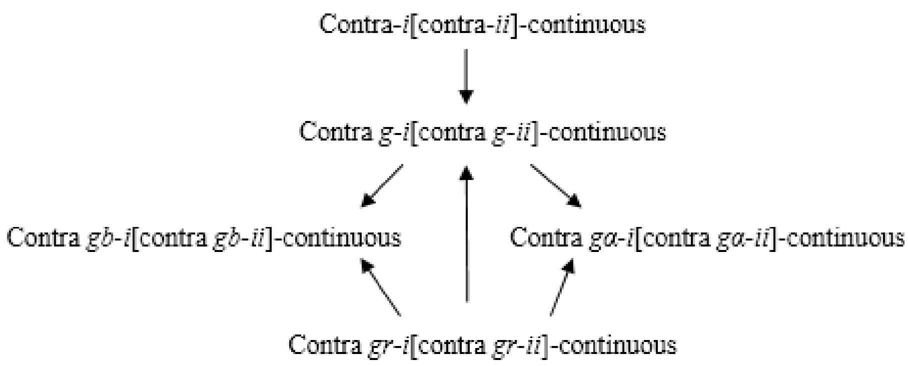
For a function f: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Y, $\mathcal{ST}_Y$, $\mathcal{PT}_Y$) the following conditions are hold

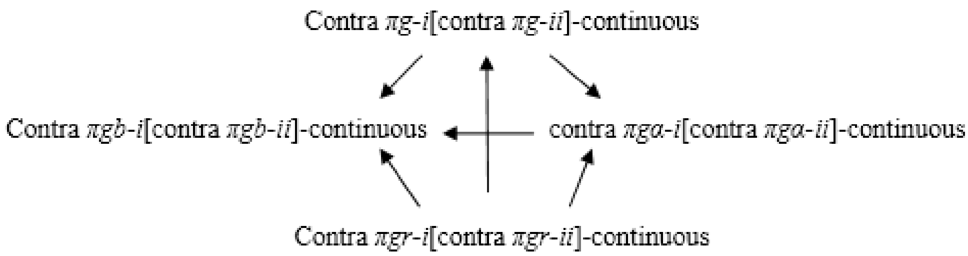
1. Every contra-i[contra-ii]- continuous is contra g-i[contra g-ii]-continuous.
2. Every contra g-i[contra g-ii]- continuous is contra $g\alpha$-i[contra $g\alpha$-ii]-continuous.
3. Every contra gr-i[contra gr-ii]- continuous is contra $g\alpha$-i[contra $g\alpha$-ii]-continuous.
4. Every contra g-i[contra g-ii]- continuous is contra gb-i[contra gb-ii]-continuous.
5. Every contra gr-i[contra gr-ii]- continuous is contra gb-i[contra gb-ii]-continuous.
6. Every contra gr-i[contra gr-ii]- continuous is contra g-i[contra g-ii]-continuous.
7. Every contra πg-i[contra πg-ii]- continuous is contra πgb-i[contra πgb-ii] continuous.
8. Every contra $\pi g\alpha$-i[contra $\pi g\alpha$-ii]- continuous is contra πgb-i[contra πgb-ii]-continuous.
9. Every contra πgr-i[contra πgr-ii]- continuous is contra πg-i[contra πg-ii]-continuous.
10. Every contra πgr-i[contra πgr-ii]- continuous is contra $\pi g\alpha$-i[contra $\pi g\alpha$-ii]- continuous.
11. Every contra πgr-i[contra πgr-ii]- continuous is contra πgb-i[contra πgb-ii]-continuous.
12. Every contra πg-i[contra πg-ii]- continuous is contra $\pi g\alpha$-i[contra $\pi g\alpha$-ii]- continuous.

Proof:

1. Let V be i[ii]-open set in Y.Since f is contra-i[contra-ii]-continuous , f^{-1}(V) is i[ii]-closed in X. Thus f^{-1}(V) is g-i[g-ii]-closed in X.(since every i[ii]-closed is g-i[g-ii]-closed).
Hence f is contra g-i[contra g-ii]-continuous.
2. Let V be i[ii]-open set in Y.Since f is contra g-i[contra g-ii]-continuous , f^{-1}(V) is g-i [g-ii]-closed in X. Thus f^{-1}(V) is $g\alpha$-i[$g\alpha$-ii]-closed in X.(since every g-i[g-ii]-closed is $g\alpha$-i[$g\alpha$-ii]-closed).
Hence f is contra $g\alpha$-i[contra $g\alpha$-ii]-continuous.
3. Let V be i[ii]-open set in Y.Since f is contra gr-i[contra gr-ii]-continuous , f^{-1}(V) is gr-i[gr-ii]-closed in X. Thus f^{-1}(V) is $g\alpha$-i[$g\alpha$-ii]-closed in X.(since every gr-i[gr-ii]-closed is $g\alpha$-i[$g\alpha$-ii]-closed).
Hence f is contra $g\alpha$-i[contra $g\alpha$-ii]-continuous.
4. Let V be i[ii]-open set in Y.Since f is contra g-i[contra g-ii]-continuous , f^{-1}(V) is g-i [g-ii]-closed in X. Thus f^{-1}(V) is gb-i[gb-ii]-closed in X.(since every g-i[g-ii]-closed is gb-i[gb-ii]-closed).
Hence f is contra gb-i[contra gb-ii]-continuous.
5. Let V be i[ii]-open set in Y.Since f is contra gr-i[contra gr-ii]-continuous , f^{-1}(V) is gr-i[gr-ii]-closed in X. Thus f^{-1}(V) is gb-i[gb-ii]-closed in X.(since every gr-i[gr-ii]-closed is gb-i[gb-ii]-closed).
Hence f is contra gb-i[contra gb-ii]-continuous.
6. Let V be i[ii]-open set in Y.Since f is contra gr-i[contra gr-ii]-continuous , f^{-1}(V) is gr-i[gr-ii]-closed in X. Thus f^{-1}(V) is g-i[g-ii]-closed in X.(since every gr-i[gr-ii]-closed is g-i[g-ii]-closed).
Hence f is contra g-i[contra g-ii]-continuous.
7. Let V be i[ii]-open set in Y.Since f is contra πg-i[contra πg-ii]-continuous , f^{-1}(V) is πg-i[πg-ii]-closed in X. Thus f^{-1}(V) is πgb-i[πgb-ii]-closed in X.(since every πg-i[πg-ii]-closed is πgb-i[πgb-ii]-closed).
Hence f is contra πgb-i[contra πgb-ii]-continuous.
8. Let V be i[ii]-open set in Y.Since f is contra $\pi g\alpha$-i[contra $\pi g\alpha$-ii]-continuous , f^{-1}(V) is $\pi g\alpha$-i[$\pi g\alpha$-ii]-closed in X. Thus f^{-1}(V) is πgb-i[πgb-ii]-closed in X.(since every $\pi g\alpha$-i [$\pi g\alpha$-ii]-closed is πgb-i[πgb-ii]-closed).
Hence f is contra πgb-i[contra πgb-ii]-continuous.
9. Let V be i[ii]-open set in Y.Since f is contra πgr-i[contra πgr-ii]-continuous , f^{-1}(V) is πgr-i[πgr-ii]-closed in X. Thus f^{-1}(V) is πg-i[πg-ii]-closed in X.(since every πgr-i[πgr-ii]-closed is πg-i[πg-ii]-closed).
Hence f is contra πg-i[contra πg-ii]-continuous.
10. Let V be i[ii]-open set in Y.Since f is contra πgr-i[contra πgr-ii]-continuous , f^{-1}(V) is πgr-i[πgr-ii]-closed in X. Thus f^{-1}(V) is $\pi g\alpha$-i[$\pi g\alpha$-ii]-closed in X.(since every πgr-i[πgr-ii]-closed is $\pi g\alpha$-i[$\pi g\alpha$-ii]-closed).
Hence f is contra $\pi g\alpha$-i[contra $\pi g\alpha$-ii]-continuous.
11. Let V be i[ii]-open set in Y.Since f is contra πgr-i[contra πgr-ii]-continuous , f^{-1}(V) is πgr-i[πgr-ii]-closed in X. Thus f^{-1}(V) is πgb-i[πgb-ii]-closed in X.(since every πgr-i[πgr-ii]-closed is πgb-i[πgb-ii]-closed).
Hence f is contra πgb-i[contra πgb-ii]-continuous.
12. Let V be i[ii]-open set in Y.Since f is contra πg-i[contra πg-ii]-continuous , f^{-1}(V) is πg-i[πg-ii]-closed in X. Thus f^{-1}(V) is $\pi g\alpha$-i[$\pi g\alpha$-ii]-closed in X.(since every πg-i[πg-ii]-closed is $\pi g\alpha$-i[$\pi g\alpha$-ii]-closed).
Hence f is contra $\pi g\alpha$-i[contra $\pi g\alpha$-ii]-continuous.

Remark 5.3: The implication between some types in proposition 5.2 are given in the following diagrams.
Contra-i[contra-ii]-continuous.





Remark 5.4: The converse of some of the above statements is not ture as shown in the following examples.

Example 5.5:

Let X={a,b,c,d}=Y

$\mathcal{T}_X$={φ,{a},{c},{a,c},{b,c},{a,b,c},X}.$\mathcal{T}_Y$={φ,{a,b},{a,b,d},{a,b,c},Y}.

Let f: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Y, $\mathcal{ST}_Y$, $\mathcal{PT}_Y$) be identity function. Hence f is contra *g-ii*-continuous but not contra-*ii*-continuous.

Example 5.6:

Let X={a,b,c,d}=Y

$\mathcal{T}_X$={φ,{a},{c},{a,c},{b,c},{a,b,c},X}.

$\mathcal{T}_Y$={φ,{a},{d},{a,d},{a,c},{a,c,d},Y}.

Let f: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Y, $\mathcal{ST}_Y$, $\mathcal{PT}_Y$) be identity function. Hence f is contra *gb-ii*-continuous but not contra *g-ii*-continuous and not contra *gr-ii*-continuous.

Example 5.7:

Let X={a,b,c,d}=Y

$\mathcal{T}_X$={φ,{a,b.c},X}.

$\mathcal{T}_Y$={φ,{c},{d},{c,d},{a,c,d},{b,c,d},Y}.

Let f: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Y, $\mathcal{ST}_Y$, $\mathcal{PT}_Y$) be identity function. Hence f is contra *g-i*-continuous ,contra *gb-i*-continuous and contra *gα-i*-continuous but not contra *gr-i*-continuous.

Example 5.8:

Let X={a,b,c,d}=Y

$\mathcal{T}_X$={φ,{a},{b},{a,b},{a,c},{a,b.c},{a,b,d},X}.

$\mathcal{T}_Y$={φ,{d},{a,d},Y}.

Let f: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Y, $\mathcal{ST}_Y$, $\mathcal{PT}_Y$) be identity function. Hence f is contra *πg-i*-continuous ,contra *πgb-i*-continuous and contra *πgα-i*-continuous but not contra *πgr-i*-continuous.

6.The Composition of some types of functions in bi-supra topological space

Proposition 6.1:

Let f: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Y, $\mathcal{ST}_Y$, $\mathcal{PT}_Y$) and g : (Y,$\mathcal{ST}_Y$, $\mathcal{PT}_Y$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) such that *gof*: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is the composition function, then:

1. If f is *g-i*[*g-ii*]-continuous and g is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *g-i*[contra *g-ii*]-continuous.
2. If f is *gα-i*[*gα-ii*]-continuous and g is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *gα-i*[contra *gα-ii*]-continuous.
3. If f is *gr-i*[*gr-ii*]-continuous and g is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *gr-i*[contra *gr-ii*]-continuous.
4. If f is *gb-i*[*gb-ii*]-continuous and g is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *gb-i*[contra *gb-ii*]-continuous.
5. If f is *πg-i*[*πg-ii*]-continuous and g is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *πg-i*[contra *πg-ii*]-continuous.
6. If f is *πgr-i*[*πgr-ii*]-continuous and g is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *πgr-i*[contra *πgr-ii*]-continuous.
7. If f is *πgα-i*[*πgα-ii*]-continuous and g is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *πgα-i*[contra *πgα-ii*]-continuous.
8. If f is *πgb-i*[*πgb-ii*]-continuous and g is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *πgb-i*[contra *πgb-ii*]-continuous.

Proof:

1. Let V be $i[ii]$-open set in Z. Since g is contra-i[contra-ii]-continuous, g^{-1}(V) is $i[ii]$-closed in Y. Since f is g-i[g-ii] - continuous, $f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is g-i[g-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra g-i[contra g-ii]-continuous.
2. Let V be $i[ii]$-open set in Z. Since g is contra-i[contra-ii]-continuous, g^{-1}(V) is $i[ii]$-closed in Y. Since f is $g\alpha$-i[$g\alpha$-ii]-continuous,$f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is $g\alpha$-i[$g\alpha$-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra $g\alpha$-i[contra $g\alpha$-ii]-continuous.
3. Let V be $i[ii]$-open set in Z. Since g is contra-i[contra-ii]-continuous, g^{-1}(V) is $i[ii]$-closed in Y. Since f is gr-i[gr-ii]-continuous,$f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is gr-i[gr-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra gr-i[contra gr-ii]-continuous.
4. Let V be $i[ii]$-open set in Z. Since g is contra-i[contra-ii]-continuous, g^{-1}(V) is $i[ii]$-closed in Y. Since f is gb-i[gb-ii] - continuous ,$f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is gb-i[gb-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra gb-i[contra gb-ii]-continuous.
5. Let V be $i[ii]$-open set in Z. Since g is contra-i[contra-ii]-continuous, g^{-1}(V) is $i[ii]$-closed in Y. Since f is πg-i[πg-ii] - continuous, $f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is πg-i[πg-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra πg-i[contra πg-ii]-continuous.
6. Let V be $i[ii]$-open set in Z. Since g is contra-i[contra-ii]-continuous, g^{-1}(V) is $i[ii]$-closed in Y. Since f is πgr-i[πgr-ii] - continuous, $f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is πgr-i[πgr-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra πgr-i[contra πgr-ii]-continuous.
7. Let V be $i[ii]$-open set in Z. Since g is contra-i[contra-ii]-continuous, g^{-1}(V) is $i[ii]$-closed in Y. Since f is $\pi g\alpha$-i[$\pi g\alpha$-ii] - continuous, $f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is $\pi g\alpha$-i[$\pi g\alpha$-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra $\pi g\alpha$-i[contra $\pi g\alpha$-ii]-continuous.
8. Let V be $i[ii]$-open set in Z. Since g is contra-i[contra-ii]-continuous, g^{-1}(V) is $i[ii]$-closed in Y. Since f is πgb-i[πgb-ii] - continuous, $f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is πgb-i[πgb-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra πgb-i[contra πgb-ii]-continuous.

Proposition 6.2:

Let f: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Y, $\mathcal{ST}_Y$, $\mathcal{PT}_Y$) and g : (Y,$\mathcal{ST}_Y$, $\mathcal{PT}_Y$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) such that gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is the composition function,then:

1. If f is contra g-i[contra g-ii]-continuous and g is $i[ii]$-continuous, then gof is contra g-i[contra g-ii]-continuous.
2. If f is contra $g\alpha$-i[contra $g\alpha$-ii]-continuous and g is $i[ii]$-continuous, then gof is contra $g\alpha$-i[contra $g\alpha$-ii]-continuous.
3. If f is contra gr-i[contra gr-ii]-continuous and g is $i[ii]$-continuous, then gof is contra gr-i[contra gr-ii]-continuous.
4. If f is contra gb-i[contra gb-ii]-continuous and g is $i[ii]$-continuous, then gof is contra gb-i[contra gb-ii]-continuous.
5. If f is contra πg-i[contra πg-ii]-continuous and g is $i[ii]$-continuous, then gof is contra πg-i[contra πg-ii]-continuous.
6. If f is contra πgr-i[contra πgr-ii]-continuous and g is $i[ii]$-continuous, then gof is contra πgr-i[contra πgr-ii]-continuous.
7. If f is contra $\pi g\alpha$-i[contra $\pi g\alpha$-ii]-continuous and g is $i[ii]$-continuous, then gof is contra $\pi g\alpha$-i[contra $\pi g\alpha$-ii]-continuous.
8. If f is contra πgb-i[contra πgb-ii]-continuous and g is $i[ii]$-continuous, then gof is contra πgb-i[contra πgb-ii]-continuous.

Proof:

1. Let V be $i[ii]$-open set in Z. Since g is $i[ii]$-continuous, g^{-1}(V) is $i[ii]$-open in Y. Since f is contra g-i[contra g-ii] - continuous, $f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is g-i[g-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra g-i[contra g-ii]-continuous.
2. Let V be $i[ii]$-open set in Z. Since g is $i[ii]$-continuous, g^{-1}(V) is $i[ii]$-open in Y. Since f is contra $g\alpha$-i[contra $g\alpha$-ii]-continuous, $f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is $g\alpha$-i[$g\alpha$-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra $g\alpha$-i[contra $g\alpha$-ii]-continuous.
3. Let V be $i[ii]$-open set in Z. Since g is $i[ii]$-continuous, g^{-1}(V) is $i[ii]$-open in Y. Since f is contra gr-i[contra gr-ii] - continuous, $f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is gr-i[gr-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra gr-i[contra gr-ii]-continuous.
4. Let V be $i[ii]$-open set in Z. Since g is $i[ii]$-continuous, g^{-1}(V) is $i[ii]$-open in Y. Since f is contra gb-i[contra gb-ii] - continuous, $f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is gb-i[gb-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra gb-i[contra gb-ii]-continuous.
5. Let V be $i[ii]$-open set in Z. Since g is $i[ii]$-continuous, g^{-1}(V) is $i[ii]$-open in Y. Since f is contra πg-i[contra πg-ii] - continuous, $f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is πg-i[πg-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra πg-i[contra πg-ii]-continuous.
6. Let V be $i[ii]$-open set in Z. Since g is $i[ii]$-continuous, g^{-1}(V) is $i[ii]$-open in Y. Since f is contra πgr-i[contraπgr-ii] - continuous, $f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is πgr-i[πgr-ii]-closed in X. Hence gof: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra πgr-i[contra πgr-ii]-continuous.
7. Let V be $i[ii]$-open set in Z. Since g is $i[ii]$-continuous, g^{-1}(V) is $i[ii]$-open in Y. Since f is contra $\pi g\alpha$-i[contra $\pi g\alpha$-ii] - continuous, $f^{-1}(g^{-1}$(V))=$(gof\ )^{-1}$(V) is $\pi g\alpha$-i[$\pi g\alpha$-ii]-closed in

X. Hence *gof*: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra *πgα-i*[contra *πgα-ii*]-continuous.

8. Let V be *i*[*ii*]-open set in Z. Since *g* is *i*[*ii*]-continuous, g^{-1}(V) is *i*[*ii*]-open in Y. Since *f* is contra *πgb-i*[contra*πgb-ii*] - continuous, f^{-1}(g^{-1}(V))=(gof)$^{-1}$(V) is *πgb-i*[*πgb-ii*]-closed in X. Hence *gof*: (X,$\mathcal{ST}_X$, $\mathcal{PT}_X$) → (Z, $\mathcal{ST}_Z$, $\mathcal{PT}_Z$) is contra *πgb-i*[contra *πgb-ii*]-continuous.

Remark 6.3:

1. Notice that If *f* is contra *g-i*[contra *g-ii*]-continuous and *g* is *i*[*ii*]-continuous, then *gof* is contra *g-i*[contra *g-ii*]-continuous is equivalent to If *f* is *g-i*[*g-ii*]-continuous and *g* is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *g-i*[contra *g-ii*]-continuous.
2. Notice that If *f* is contra *gα-i*[contra *gα-ii*]-continuous and *g* is *i*[*ii*]-continuous, then *gof* is contra *gα-i*[contra *gα-ii*]-continuous is equivalent to If *f* is *gα-i*[*gα-ii*]-continuous and *g* is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *gα-i*[contra *gα-ii*]-continuous
3. Notice that If *f* is contra *gr-i*[contra *gr-ii*]-continuous and *g* is *i*[*ii*]-continuous, then *gof* is contra *gr-i*[contra *gr-ii*]-continuous is equivalent to If *f* is *gr-i*[*gr-ii*]-continuous and *g* is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *gr-i*[contra *gr-ii*]-continuous.
4. Notice that If *f* is contra *gb-i*[contra *gb-ii*]-continuous and *g* is *i*[*ii*]-continuous, then *gof* is contra *gb-i*[contra *gb-ii*]-continuous is equivalent to If *f* is *gb-i*[*gb-ii*]-continuous and *g* is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *gb-i*[contra *gb-ii*]-continuous.
5. Notice that If *f* is contra *πg-i*[contra *πg-ii*]-continuous and *g* is *i*[*ii*]-continuous, then *gof* is contra *πg-i*[contra *πg-ii*]-continuous is equivalent to If *f* is *πg-i*[*πg-ii*]-continuous and *g* is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *πg-i*[contra *πg-ii*]-continuous.
6. Notice that If *f* is contra *πgr-i*[contra *πgr-ii*]-continuous and *g* is *i*[*ii*]-continuous, then *gof* is contra *πgr-i*[contra *πgr-ii*]-continuous is equivalent to If *f* is *πgr-i*[*πgr-ii*]-continuous and *g* is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *πgr-i*[contra *πgr-ii*]-continuous.
7. Notice that If *f* is contra *πgα-i*[contra *πgα-ii*]-continuous and *g* is *i*[*ii*]-continuous, then *gof* is contra *πgα-i*[contra *πgα-ii*]-continuous is equivalent to If *f* is *πgα-i*[*πgα-ii*]-continuous and *g* is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *πgα-i*[contra *πgα-ii*]-continuous.
8. Notice that If *f* is contra *πgb-i*[contra *πgb-ii*]-continuous and *g* is *i*[*ii*]-continuous, then *gof* is contra *πgb-i*[contra *πgb-ii*]-continuous is equivalent to If *f* is *πgb-i*[*πgb-ii*]-continuous and *g* is contra-*i*[contra-*ii*]-continuous, then *gof* is contra *πgb-i*[contra *πgb-ii*]-continuous.

Proof:

1. directly by proposition 6.2.1 and proposition 6.1.1 .
2. directly by proposition 6.2.2 and proposition 6.1.2 .
3. directly by proposition 6.2.3 and proposition 6.1.3 .
4. directly by proposition 6.2.4 and proposition 6.1.4 .
5. directly by proposition 6.2.5 and proposition 6.1.5 .
6. directly by proposition 6.2.6 and proposition 6.1.6 .
7. directly by proposition 6.2.7 and proposition 6.1.7 .
8. directly by proposition 6.2.8 and proposition 6.1.8 .

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أنواع جديدة من الاستمرارية العكسية في الفضاء ثنائي التبولوجي الفوقي

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الملخص

في هذا البحث قدمنا صف جديد من الدوال في الفضاء ثنائي التبولوجي الفوقي (عكوس-i] عكوس-ii] المستمرة , عكوس-i] عكوس-ii] المستمرة من النمط g , عكوس- i] عكوس-ii] المستمرة من النمط $g\alpha$, عكوس-i] عكوس-ii] المستمرة من النمط gr , عكوس-i] عكوس-ii]المستمرة من النمط gb , عكوس-i] عكوس-ii] المستمرة من النمط πg , عكوس-i] عكوس-ii] المستمرة من النمط $\pi g\alpha$, عكوس-i] عكوس-ii] المستمرة من النمط πgr , عكوس- i] عكوس-ii] المستمرة من النمط πgb, ودرسنا العلاقات بين هذه الدوال وتركيباتها وأخيرا قدمنا مبرهنات مهمه حول الموضوع.