On ideal supra topological space

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Abstract
In this paper we introduces via of ideal supra topological space classes of concepts (I-supra -dense, ∗-supra dense, ∗-closed supra, ∗-perfect supra) in ideal supra topological space. At last we study and investigate some characterization of these concept for there more we introduces some continuous mapping, and there properties.

1- Introduction
The concept of ideal in topological space was first introduced by Kuratowski[10] and Vaidyananthswamy [15]. They also have defined local function in ideal topological space. Further Hamlett and Jankovic in [19] and [5] studied the properties of ideal topological spaces and introduced the notion of I-open set in topological space and they have introduced set operator called $(\ )^{*\mu}$, μ-local function .and introduced μ-codense ideal .[6] further investigated I-open sets and I-continuous functions. Dontchev [9] introduced the notion of pre-I-open sets and obtained a decomposition of I-continuity. The notion of semi-I-open sets to obtain decomposition of continutity was introduced by Hatir and Noiri [7, 8].In addition to this, Caksu Guler and Aslim [1] have introduced the notion of b-I-open sets and b-I-continuous functions. Different types of generalized open sets like semi-open[12] ,preopen [3], α-open[13] already are there in literature and these generalized sets have a common Property which is closed under arbitrary union. Mashhour et al [2] put all of the sets in a pocket and defined a generalized space which is supra topological space. In the light of the above results, the purpose of this paper is study b-I-supra open sets and b-I-supra continuous functions and to obtain several characterizations and properties of these concepts.

2- Preliminaries
Definition 2.1 [17]
A nonempty collection I of subset of X is called an ideal on X if:
i. $A \in I$ and $B \subset A$ implies $B \in I$ (heredity);
ii. $A \in I$ and $B \in I$ implies $A\cup B \in I$ (finite additivety).

Definition 2.2 [4]
A sub family μ of the power set P(X) of nonempty set X is called a supra topology on X if μ satisfies the following condition:
1. **μ** contains ϕ and X;
2. **μ** is closed under the arbitrary union.

The pair (X, **μ**) is called a supra topological space. In this respect, the member of **μ** is called supra open set in (X, **μ**). The complement of supra open set is called supra closed set.

Definition 2.3 [4]
The (X, μ , I) is called ideal supra topological space .if (X, μ) is supra topological space and I is ideal on X.

Example 2.4
Let X = {a ,b, c} , and μ = { ϕ , X ,{a}{b, c},{a, c},{a, b, c}} , I ={ ϕ ,{a},{b},{a , b}} .
Then (X, μ, I) is ideal supra topological space.

Definition 2.5 [17]
Let (X, μ, I) be an ideal supra topological space. A set operator $(\ )^{*\mu}$: p(X)⟶ p(X), is called the μ-local function of I on X with respect to μ , is defined as :
$(A)^{*\mu}(I , \mu)= \{ x \in X: (U\cap A)\notin I, \text{ for every } U \in \mu(x) \}$,
Where $\mu(X) = \{U\in \mu: x\in U\}$.This is simply called μ-local function and simply denoted as $A^{*\mu}$.
We have discussed the properties of μ-local function in the following theorem .

Theorem 2.6 [17]
Let (X, μ, I) be an ideal supra topological space, and let A, B, A_1 , A_2 ,----, A_i ,----
Be subsets of X, then
1. $\phi^{*\mu} = \phi$;
2. $A\subset B$ implies $A^{*\mu} \subset B^{*\mu}$;
3. For another ideal $J \supset I$ on X , $A^{*\mu}(J) \subset A^{*\mu}(I)$;
4. $A^{*\mu} \subset cl^{\mu}(A)$;
5. $A^{*\mu}$ is a supra closed set;
6. $(A^{*\mu})^{*\mu} \subset A^{*\mu}$;
7. $A^{*\mu} \cup B^{*\mu} \subset (A\cup B)^{*\mu}$;
8. $\cup_i A^{*\mu} \subset (\cup_i A_i)^{*\mu}$
9. $(A\cap B)^{*\mu} \subset A^{*\mu} \cap B^{*\mu}$;
10. For $V\in \mu$, $V\cap (V \cap A)^{*\mu} \subset V \cap A^{*\mu}$,
11. For $I \in$ **I,** $(A\cup I)^{*\mu} = A^{*\mu} = (A\text{-}I)^{*\mu}$.

Following example shows that $A^{*\mu} \cup B^{*\mu} = (A\cup B)^{*\mu}$ does not hold in general

Example 2.7 [17]
Let X = {a ,b ,c ,d} , μ = { ϕ ,X ,{a},{b},{a, b},{a, c},{a, d},{b, c},{b, d},{a, b, d},{a ,b, c},{a, c, d},{b, c, d}}
I = {ϕ, {c}}, then
Conceder A = {a, c}, B = {b, c}, then $A^{*\mu}$ = {a} , $B^{*\mu}$ ={ a ,b, c}
Now $(A\cup B)^{*\mu} = \{ a ,b, c\}^{*\mu} = \{a ,b ,c ,d\}$
Hence $A^{*\mu} \cup B^{*\mu} \neq (A\cup B)^{*\mu}$.

Definition 2.8 [17]
An ideal I in a space (X, μ, I) is called **μ-codense ideal if** $\mu \cap I =\{ \phi \}$.

Example 2.9
Let X = {a ,b ,c ,d} , μ = { ϕ ,X ,{a},{b, c},{a, b},{a,b,c}}, I ={ ϕ ,{d}}
$\mu \cap I =\{ \phi \}$.then (X, μ, I) is called **μ-codense.**

Theorem 2.10 [17]

Let (X, μ, I) be an ideal supra topological space and I is μ-codense with μ . Then $X = X^{*\mu}$

Proof :

It is obvious that $X^{*\mu} \subseteq X$. For converse,

Suppose $x \in X$ but $x \notin X^{\mu}$.

Then there exist $U_x \in \mu(x)$ such that $U_x \cap X \in I$ that is $U_x \in I$, a contradiction to the fact that $\mu \cap I = \{ \phi \}$, hence $X = X^{*\mu}$.

3- b-I-supra open set.

Definition 3.1

The ideal supra interior of a subset A denoted by $int^{\mu}(A)$ .is the union of ideal supra open sets included in A.

Definition 3.2

The ideal supra closure of a set A denoted by $cl^{*\mu}(A)$ .is the intersection of ideal supra closed sets including A.

Theorem 3.3

Let (X, μ, I) be an ideal supra topological space and $A \subset X$,then

1. $int^{\mu}(A) \subseteq A$;
2. $A \in \mu$ if and only if $int^{\mu}(A) = A$;
3. $cl^{*\mu}(A) \supseteq A$;
4. A is ideal supra closed set if and only if $cl^{*\mu}(A) = A$;
5. $x \in cl^{*\mu}(A)$ if and only if every ideal supra open set U_x containing x , $U_x \cap A \neq \phi$

Proof:

(1) Proof is obvious from the definition of ideal supra interior.

(2) Since arbitrary union of ideal supra open sets is again an ideal supra open set, then the proof is obvious.

(3) Proof is obvious from the definition of ideal supra closure.

(4) If *A* is an ideal supra closed set, then smallest ideal supra closed set containing A is A. Hence $cl^{*\mu}(A) = A$.

(5) Proof: Let $x \in cl^{*\mu}$. if possible suppose that $U_x \cap A = \phi$, where U_x is ideal supra open set containing x.

Then $A \subset (X - U_x)$ and $X - U_x$ is an ideal supra closed set containing A. Therefor $x \in (X - U_x)$,

a contradiction.

Conversely:

Supposed that $U_x \cap A = \phi$, for every ideal supra open set U_x containing x.

If possible Suppose that $x \notin cl^{*\mu}(A)$, then $x \in X - cl^{*\mu}(A)$.

Then there is a $U`_x \in \mu$ such that $U`_x \in (X - cl^{*\mu}(A))$,

i.e $U`_x \in (X - cl^{*\mu}(A)) \subset (X - A)$.

Hence $U`_x \cap A = \phi$, a contradiction . So $x \in cl^{*\mu}(A)$.

Definition 3.4

Let (X, μ, I) be an ideal supra topological space and $x \in \mu$ .Then μ is said to be an ideal supra neighborhood of a point x of X if for some ideal supra open set $U \in \mu$, $x \in U \subset \mu$.

Definition 3.5

1. A subset A of (X, μ, I) is said to be I-supra -dense for short (I- **μ**-dense) if $A^{*\mu} = X$.
2. A subset A of (X, μ, I) is said to be ∗-supra -dense for short (∗- **μ**-dense) if $A \subseteq A^{*\mu}$.
3. A subset A of (X, μ, I) is said to be ∗-closed supra for short (∗-closed **μ**) if $A^{*\mu} \subseteq A$.
4. A subset A of (X, μ, I) is said to be ∗-perfect supra for short (∗- **μ**- perfect) if $A^{*\mu} = A$.

Remark 3.6

There exist an ideal supra topology $\mu^{*}(I)$, finer than μ generated by

$\mathcal{B}(I, \mu) = \{U - I, U \in \mu \text{ and } I \in \mathbf{I}\}$ and $cl^{*\mu}(A) = A \cup (A)^{*\mu}$ defines a kuratowski closure operator for $\mu^{*}(I)$.

Example 3.7

Let X = {a ,b ,c , d } , μ = { ϕ ,X ,{a},{b, c ,d},{a, b}}, I ={ ϕ ,{ b}};

$\mu^{*}(I)$ = { ϕ ,X ,{a},{b ,c , d},{a, b},{a, c , d} },hence μ^{*} is finer than μ .

Definition 3.8

A sub set A of a supra topological space (X, μ) and $A \subset X$ and A is said to

1. supra open if $A \subset int^{\mu}(A)$, and a subset K is called supra closed if it is complement is supra open .
2. α- supra open if $A \subset int^{\mu}(cl^{\mu}(int^{\mu}(A)))$.
3. pre- supra open if $A \subset int^{\mu}(cl^{\mu}(A))$.
4. semi- supra open if $A \subset cl^{\mu}(int^{\mu}(A))$.
5. b- supra open if $A \subset cl^{\mu}(int^{\mu}(A)) \cup int^{\mu}(cl^{\mu}(A))$.[14]

The class of all semi-supra- open (pre- supra-open, α-supra-open, b- supra- open) sets in X will be denoted by SO (X, *μ*), PO(X, *μ*), αO(X, *μ*) and BO(X,) respectively.

Proposition 3.9

For a space(X, μ) the following are equivalent:

1. PO(X, μ) ∪ SO (X, μ) = BO(X, μ)
2. For each subset $A \subseteq X$, $int^{\mu}(cl^{\mu}(A)) \subseteq cl^{\mu}(int^{\mu}(A))$ or $cl^{\mu}(int^{\mu}(A)) \subseteq int^{\mu}(cl^{\mu}(A))$.

Proof: (1) ⇒ (2): Let $A \subseteq X$ and let $B = (A \cap int^{\mu}(cl^{\mu}(A))) \cup cl^{\mu}(int^{\mu}(A))$.

It easily checked that $int^{\mu}(cl^{\mu}(A)) = int^{\mu}(cl^{\mu}(B))$.

Since $A \cap int^{\mu}(cl^{\mu}(A)) \in PO(X, \mu)$ and $cl^{\mu}(int^{\mu}(A)) \in SO(X, \mu)$, we have that $B \in BO(X, \mu)$. By hypothesis, $B \in PO(X, \mu) \cup SO(X, \mu)$.

If $B \in PO(X, \mu)$, then $cl^{\mu}(int^{\mu}(A)) \subseteq B \subseteq int^{\mu}(cl^{\mu}(B)) = int^{\mu}(cl^{\mu}(A))$.

If $B \in SO(X, \mu)$, then $A \cap int^{\mu}(cl^{\mu}(A)) \subseteq B \subseteq cl^{\mu}(int^{\mu}(B)) = cl^{\mu}(int^{\mu}(A))$ and consequently,

$int^{\mu}(cl^{\mu}(A)) \subseteq cl^{\mu}(int^{\mu}(cl^{\mu}(A))) = cl^{\mu}(A \cap int^{\mu}(cl^{\mu}(A))) \subseteq cl^{\mu}(int^{\mu}(A))$.

(2) ⇒ (1): Let $S \in BO(X, \mu)$. If $int^{\mu}(cl^{\mu}(S)) \subseteq cl^{\mu}(int^{\mu}(S))$,

then $S \subseteq cl^{\mu}(int^{\mu}(S))$ and so $S \in SO(X, \mu)$.

If $cl^{\mu}(int^{\mu}(S)) \subseteq int^{\mu}(cl^{\mu}(S))$ and so $S \in PO(X, \mu)$.

Definition 3.10

A subset A of an ideal supra topological space (X, μ, I) is said to be

1. I-supra open if $A\subset int^{\mu}(A^{*\mu})$, and a subset K is called I- supra closed if it is complement is I- supra open .
2. α-I supra open if $A\subset int^{\mu}(cl^{*\mu}(int^{\mu}(A)))$.
3. pre-I supra open if $A\subset int^{\mu}(cl^{*\mu}(A)))$.
4. semi- I supra open if $A\subset (cl^{*\mu}(int^{\mu}(A)))$.
5. b-I- supra open if $A\subset cl^{*\mu}(int^{\mu}(A)) \cup int^{\mu}(cl^{*\mu}(A)))$.

The family of all I-supra opens (resp., b-I- supra open, semi-I- supra open, pre-I- supra open,α-I- supra open) sets of (X, μ) is denoted by IO(X, μ), BIO(X, μ),SIO(X, μ) , PIO (X, μ) , αIO(X, μ).

Example 3.11

Let X= { a , b ,c } , μ = { ϕ, X ,{a},{c},{a ,c}} and I ={ ϕ,{b}}, A = {a , c} ,then A is b-I- supra open set for $int^{\mu}(cl^{*\mu}(A)) \cup cl^{*\mu}(int^{\mu}(A))$
$= int^{\mu}(\{a, c\}^{*\mu} \cup \{a, c\}) \cup cl^{*\mu}(\{a ,c\})$
$= int^{\mu}(X) \cup (\{a , c\}^{*\mu} \cup \{a ,c\}) = X \cup X \cup \{a ,c\} = X \supset A$ and hence A is b-I- open set and A is pre -I- supra open set since $int^{\mu}(cl^{*\mu}(A))$
$= int^{\mu}(\{a , c\}^{*\mu} \cup \{a ,c\})$
$= int^{\mu}(X) = X \supset A.$

Proposition 3.12 [14]

Every semi-supra open set is b-supra open .

Proof:

Let A is a semi-supra open set in (X, μ). Then $A\subset (cl^{\mu}(int^{\mu}(A)))$.
Hence $A\subset cl^{\mu}(int^{\mu}(A))) \cup int^{\mu}(cl^{\mu}(A)))$.then A is b-supra open in (X, μ).
The converse of above preposition need not be true as shown by the following example:

Example 3.13

Let (X, μ) be a supra topological space, where X ={ 1 , 2 ,3 } , μ = { ϕ, X ,{1},{1, 2},{2 ,3}} and A = {1 , 3} , then A is b- supra open set for $int^{\mu}(cl^{\mu}(A)) \cup cl^{\mu}(int^{\mu}(A))$
$= int^{\mu}(X) \cup cl^{\mu}(\{1 \})$
$= (X) \cup \{1 \}) = X \supset \{1, 3\}$ and hence A is b-supra open set and A is not semi-supra open.

Proposition 3.14

Every pre-supra open set is b-supra open .

Proof:

Let A is a pre-supra open set in (X, μ). Then $A\subset int^{\mu}(cl^{\mu}(A)))$.
Hence $A\subset cl^{\mu}(int^{\mu}(A))) \cup int^{\mu}(cl^{\mu}(A)))$. then A is b-supra open in (X, μ).
The converse of above preposition need not be true.

Example 3.15

Let (X, μ) be a supra topological space, where X ={ a , b ,c } , μ = { ϕ, X ,{a},{b},{a, b}} and A = {b} , then A is b- supra open set for $int^{\mu}(cl^{\mu}(A)) \cup cl^{\mu}(int^{\mu}(A))$
$= int^{\mu}(\{b, c\}) \cup cl^{\mu}(\{b \})$
$= \phi \cup \{b, c\} = \{b, c\} \supset \{b\}$ and hence A is b-supra open set and A is not pre-supra open.

Proposition 3.16

Every semi-I-supra open set is b-I-supra open .

Proof:

Let A is a semi-I-supra open set in (X, μ ,I). Then $A\subset (cl^{*\mu}(int^{\mu}(A)))$.
Hence $A\subset cl^{*\mu}(int^{\mu}(A))) \cup int^{\mu}(cl^{*\mu}(A)))$.then A is b-I-supra open in(X, μ).
The converse of above preposition need not be true as shown by the following example:

Example 3.17

Let (X, μ ,I) be a supra topological space, where X ={ a , b ,c } , μ = { ϕ, X ,{a},{a, b},{b ,c}} ,I = { ϕ,{b}},here A = {a, c} then A is b-I-supra open set , but it is not semi-I-supra open.

Example 3.18

Let X = {a ,b ,c ,d} , μ = { ϕ, X,{a ,b},{b},{a ,b, d}} , I = { ϕ,{b}} , A = {b ,d} is b-I-supra open but it is not semi-I-supra open .
Because $int^{\mu}(cl^{*\mu}(A)) \cup cl^{*\mu}(int^{\mu}(A)) = int^{\mu}(\{b ,d\}^{*\mu} \cup \{b ,d\}) \cup cl^{*\mu}(int^{\mu}(\{b, d\})$
$= int^{\mu}(X) \cup (\{b\}^{*\mu} \cup \{b\}) = X \cup \{b\} = X \supset A$ and hence A is b-I- supra open set .
And since $cl^{*\mu}(int^{\mu}(A)) = cl^{*\mu}(int^{\mu}(\{b ,d\}))$
$= \{b\}^{*\mu} \cup \{b\} = \emptyset \cup \{b\} = \{b\} \not\supset A$ and hence A is not semi-I-supra open.

Proposition 3.19

For a subset of an ideal supra topological space, the following condition hold:

a. Every b-I-supra open set is b- supra open

Proof:-

Let A is b-I-supra open then $A\subset int^{\mu}(cl^{*\mu}(A)) \cup cl^{*\mu}(int^{\mu}(A)) =$
$A\subset (int^{\mu}(A))^{*\mu} \cup (int^{\mu}(A)) \cup int^{\mu}(A^{*\mu} \cup A)$
$= cl^{\mu}(int^{\mu}(A)) \cup (int^{\mu}(A)) \cup int^{\mu}(cl^{\mu}(A) \cup A)$
$\subset cl^{\mu}(int^{\mu}(A)) \cup int^{\mu}(cl^{\mu}(A))$.

b. Every pre-I-supra open is b-I- supra open set

Proof:-

Let A is pre-I- supra open set, then we have $A\subset int(cl^{*}(A))$
Hence $A\subset int^{\mu}(cl^{*\mu}(A)) \cup cl^{*\mu}(int^{\mu}(A))$, then A is b-I- supra open set.

c. Every semi-I- supra open is b-I- supra open set

Proof:-

Let A is semi-I- supra open set, then we have $A\subset cl^{*\mu}(int^{\mu}(A))$
Hence $A\subset int^{\mu}(cl^{*\mu}(A)) \cup cl^{*\mu}(int^{\mu}(A))$, then A is b-I-open set.

d. SIO (X, μ) ∪ PIO(X, μ) ⊂ BIO(X, μ).

Proof: - The proof is obvious.

Example 3.20

Let X=(1,2,3,4} be the ideal supra topological space by setting μ = {X, ∅,{2},{3,4}{2,3,4}}, I={∅,{3},{4},{3,4}} ,then A={1,2} is b-I-supra open set but it is not pre-I-supra open.

Proposition 3.21

Let S be a b-I-supra open set such that $int^{\mu}(S) = \emptyset$ then S is pre-I-supra open set.

Proof: - Since $S\subset int^{\mu}(cl^{*\mu}(S)) \cup cl^{*\mu}(int^{\mu}(S)) = int^{\mu}(cl^{*\mu}(\emptyset)) \cup cl^{*\mu}(int^{\mu}(\emptyset)) = int^{\mu}(cl^{*\mu}(S))$ .then S is pre-I-supra open.

Lemma 3.22

Let (X, T, I) an ideal topological space and A,B subsets of X. Then, the following properties hold:

a.If $A \subset B$, then $A^{*\mu} \subset B^{*\mu}$,[17]

b. If U∈T , then $U \cap A^{*\mu} \subset (U \cap A)^{*\mu}$,

c. $A^{*\mu} = cl^{\mu}(A^{*}) \subset cl^{\mu}(A)$,
d. $(A \cup B)^{*\mu} = A^{*\mu} \cup B^{*\mu}$,
e. $(A \cap B)^{*\mu} \subset A^{*\mu} \cap B^{*\mu}$. [17]

Proposition 3.23

Let (X, μ, I) an ideal topological space and let A, U subsets of X. If A is a b-I-supra open set and $U \in \mu$, then $A \cap U$ is a b-I-supra open set.

Proof: - by assumption

$A \subset int^{\mu}(cl^{*\mu}(A)) \cup cl^{*\mu}(int^{\mu}(A))$ and $U \subset int^{\mu}(U)$

$A \cap U \subset [int^{\mu}(cl^{*\mu}(A)) \cup cl^{*\mu}(int^{\mu}(A))] \cap int^{\mu}(U)$

$\subset (int^{\mu}(cl^{*\mu}(A)) \cap int^{\mu}(U)) \cup (cl^{*\mu}(int^{\mu}(A)) \cap int^{\mu}(U))$

$= int^{\mu}((A^{*\mu} \cap U)) \cup (A \cap U)) \cup ((int^{\mu}(A))^{*\mu} \cap int^{\mu}(U)) \cup int^{\mu}(A) \cap int^{\mu}(U)$

$\subset ((int^{\mu}(A \cap U))^{*\mu} \cup (A \cap U)) \cup ((int^{\mu} A \cap U))^{*\mu} \cup (int^{\mu}(A \cap U)))$

$= int^{\mu}(cl^{*\mu}(A \cap U)) \cup cl^{*\mu}(int^{\mu}(A \cap U))$

Thus $A \cap U$ is b-I-supra open set.

Proposition 3.24

Every α- I supra open is semi-I-supra open .

Proof: -

Let A is α- I supra open in (X, μ, I) therefore. $A \subset int^{\mu}(cl^{*\mu}(int^{\mu}(A)))$.it is obvious that $int^{\mu}(cl^{*\mu}(int^{\mu}(A))) \subset cl^{*\mu}(int^{\mu}(A))$.Hence $A \subset cl^{*\mu}(int^{\mu}(A))$.

The converse claim in the proposition is not usually true.

Example 3.25

Let (X, μ, I) be an ideal supra topological space where

X = {a ,b ,c ,d} , μ = { ϕ, X,{a}{a ,b},{b}} , I = { ϕ,{c}} , A = {b ,c} is a

Semi-I-supra open but it is not α-I-supra open .

Proposition 3.26

Every α- I supra open is pre-I-supra open .

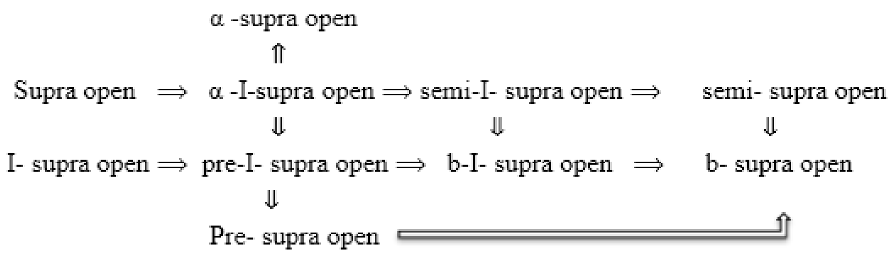
Proof: -

Let A is α- I supra open in (X, μ, I) therefore. $A \subset int^{\mu}(cl^{*\mu}(int^{\mu}(A)))$.it is obvious that $int^{\mu}(cl^{*\mu}(int^{\mu}(A))) \subset int^{\mu}(cl^{*\mu}((A))$ .Hence $A \subset int^{\mu}(cl^{*\mu}((A))$.

Now

From above propositions and examples we have the following diagram in which the converses of the implications need not be true.

Diagram 3.27



Proposition 3.28

Let (X, μ, I) an ideal supra topological space and let A, B subsets of X.

a. If $U_{\alpha} \in BIO(X, \mu)$ for each $\alpha \in \Delta$ then $\cup\{ U_{\alpha} : \alpha \in \Delta\} \in BIO(X, \mu)$,

b. If $A \in BIO(X, \mu)$ and $B \in \mu$, then $A \cap B \in BIO(X, \mu)$.

Proof:- a)

Since $U_{\alpha} \in BIO(X, \mu)$,

We have $U_{\alpha} \subset int^{\mu}(cl^{*\mu}(U_{\alpha})) \cup cl^{*\mu}(int^{\mu}(U_{\alpha}))\ \forall\ \alpha \in \Delta$, then by (Lemma 3.22)

$\cup_{\alpha \in \Delta} U_{\alpha} \subset \cup_{\alpha \in \Delta} [int^{\mu}(cl^{*\mu}(U_{\alpha})) \cup cl^{*\mu}(int^{\mu}(U_{\alpha}))]$

$\subset \cup_{\alpha \in \Delta} \{[int^{\mu}(U_{\alpha}) \cup ((int^{\mu}(U_{\alpha}))^{*\mu}] \cup (int^{\mu}(U_{\alpha}) \cup (U_{\alpha})^{*\mu})]\}$

$\subset [int^{\mu}(\cup_{\alpha \in \Delta} U_{\alpha}) \cup (\cup_{\alpha \in \Delta}(int^{\mu}(U_{\alpha}))^{*\mu}] \cup (int^{\mu}((\cup_{\alpha \in \Delta} U_{\alpha}) \cup (\cup_{\alpha \in \Delta}(U_{\alpha})^{*\mu}))]$

$\subset [int^{\mu}(\cup_{\alpha \in \Delta} U_{\alpha}) \cup ((int^{\mu}(\cup_{\alpha \in \Delta} U_{\alpha}))^{*\mu}] \cup [(int^{\mu}((\cup_{\alpha \in \Delta} U_{\alpha}) \cup (\cup_{\alpha \in \Delta}(U_{\alpha})^{*\mu}))]$

$\subset [int^{\mu}(cl^{*\mu}(\cup_{\alpha \in \Delta} U_{\alpha})] \cup [cl^{*\mu}(int^{\mu}(\cup_{\alpha \in \Delta} U_{\alpha})]$

Hence $\cup_{\alpha \in \Delta} U_{\alpha}$ is b-I-supra open.

Proof:- b) The proof is similar to that of(proposition 3.23).

Definition 3.29

A subset A of an ideal supra topological space (X, μ, I) is said to be b-I-supra closed if its complement is b-I-supra open.

Example 3.30

Let (X, μ, I) is ideal supra topological space and

X = {a , b , c}, μ = { ϕ, X,{a}{a ,b},{b ,c}} , μ^{c} = { X, ϕ, {b ,c}{c},{a}} ,

I = { ϕ,{b}} ,and let A = {a } , then A is b-I-supra closed.

Theorem 3.31

If a subset A of an ideal supra topological space (X, μ, I) is b-I-supra closed, then

$[int^{\mu}(cl^{*\mu}(A)) \cap cl^{*\mu}(int^{\mu}(A))] \subset A$

Proof:-

Since A is b-I-supra closed, $X\text{-}A \in BIO(X, \mu)$ and since $\mu^{*}(I)$ is finer than μ, we have

$X\text{-}A \subset cl^{*\mu}(int^{\mu}(X\text{-}A)) \cup int^{\mu}(cl^{*\mu}(X\text{-}A)) \subset cl^{\mu}(int^{\mu}(X\text{-}A)) \cup int^{\mu}(cl^{\mu}(X\text{-}A))$

$=[X\text{-}[int^{\mu}(cl^{\mu}(A)]] \cup [X\text{-}[cl^{\mu}(int^{\mu}(A))]] \subset [X\text{-}[int^{\mu}(cl^{*\mu}(A))]] \cup [X\text{-}[cl^{*\mu}(int^{\mu}(A))]]$

$=X\text{-}[[int^{\mu}(cl^{*\mu}(A))]] \cap [cl^{*\mu}(int^{\mu}(A))]]$

$=[int^{\mu}(cl^{*\mu}(A))] \cap [cl^{*\mu}(int^{\mu}(A))] \subset A$.

Corollary 3.32

Let A be a subset of an ideal supra topological space (X, μ, I) such that

$X\text{-}[int^{\mu}(cl^{*\mu}(A)] = cl^{*\mu}(int^{\mu}(X\text{-}A))$ and $X\text{-}[cl^{*\mu}(int^{\mu}(A))] = int^{\mu}(cl^{*\mu}(X\text{-}A))$ then A is b-I-supra closed if and if $[int^{\mu}(cl^{*\mu}(A))] \cap [cl^{*\mu}(int^{\mu}(A))] \subset A$

Proposition 3.33

1. Arbitrary union of b-I- supra open set is always b-I- supra open set.
2. Finite intersection of b-I- supra open sets may fail to be b-I- supra open set.
3. X is b-I- supra open set.

Proof:

1. Let A and B be two b-I- supra open sets.
Then, $A \subset int^{\mu} (cl^{*\mu} (A)) \cup cl^{*\mu} (int^{\mu} (A))$, and
$B \subset int^{\mu} (cl^{*\mu} (B)) \cup cl^{*\mu} (int^{\mu} (B))$. Then
$A\cup B \subset int^{\mu} (cl^{*\mu} (A\cup B)) \cup cl^{*\mu} (int^{\mu} (A\cup B))$,therefore
$A\cup B$ is b-I- supra open set.
2- In the example
$X = \{a , b , c\}$, $\mu = \{ \phi, X,\{a\}\{a ,b\},\{b ,c\}\}$, $I = \{ \phi,\{b\}\}$, $A = \{a ,c\}$,$B= \{b ,c\}$
are b-I- supra open sets ,but the intersection $\{c\}$ is not b-I- supra open set.
3- The proof is obvious.

Proposition 3.34

1. Arbitrary intersection of b-I- supra closed sets is always b-I- supra closed set.
2. Finite union of b-I- supra closed set may fail to be b-I- supra closed set.

Example 3.35

Let (X, μ, I) is ideal supra topological space and $X = \{a , b , c\}$, $\mu = \{ \phi, X,\{a\}\{b ,c\},\{a ,c\}\}$, $\mu^c = \{ X, \phi, \{b ,c\}\{a\},\{b\}\}$,
$I = \{ \phi, \{b\}\}$, and let $A = \{a \}$, $B = \{b\}$ is b-I-supra closed sets but their union $\{a , b\}$ is not b-I-supra closed set.

Definition 3.36

• The b-I-supra closure of a set A , denoted by $cl_b^{*\mu}(A)$ .Is the intersection of b-I- supra closed set including A.

• The b-I-supra interior of a set A , denoted by $int^{\mu}_b (A)$.Is the union of b-I- supra open sets included in A.

Proposition 3.37

1. $A \subseteq cl_b^{*\mu}(A)$; and $A = cl_b^{*\mu}(A)$ iff A is a b-I-supra closed set,
2. $int^{\mu}_b (A) \subseteq A$; and $int^{\mu}_b (A) = A$ iff A is a b-I-supra open set;
3. $X - int^{\mu}_b (A) = cl_b^{*\mu}(X-A)$;
4. $X - cl_b^{*\mu} (A) = int^{\mu}_b (X-A)$.

Proposition 3.38

a. $int^{\mu}_b (A) \cup int^{\mu}_b (B) \subseteq int^{\mu}_b (A\cup B)$;
b. $cl_b^{*\mu} (A\cap B) \subseteq cl_b^{*\mu} (A) \cap cl_b^{*\mu} (B)$.

Proof:

a. let $A \subseteq A\cup B \wedge B \subseteq A\cup B$ [by def. of the union]
implies $int^{\mu}_b (A) \subseteq int^{\mu}_b (A\cup B) \wedge int^{\mu}_b (B)\subseteq int^{\mu}_b (A\cup B)$.
implies $int^{\mu}_b (A) \cup int^{\mu}_b (B) \subseteq int^{\mu}_b (A\cup B)$
b. Let $A\cap B \subseteq A$ [by def. of the intersection]
implies $cl_b^{*\mu} (A\cap B) \subseteq cl_b^{*\mu} (A) \wedge cl_b^{*\mu} (A\cap B) \subseteq cl_b^{*\mu} (B)$
implies $cl_b^{*\mu} (A\cap B) \subseteq cl_b^{*\mu} (A) \cap cl_b^{*\mu} (B)$.

Proposition 3.39

For an ideal supra topological space (X, μ, I) and $A\subseteq X$ we have:

a.If $I = \phi$, then A is b-I- supra open if and only if A is b- supra open.
b. If $I = P(X)$, then A is b-I- supra open if and only if $A\in \mu$.
c.If $I = N$, then A is b-I- supra open if and only if A is b- supra open.

Proof:-

a- **Necessity** is easy as diagram for **Sufficiency** note that in case of the minimal ideal $A^{*\mu} = cl (A)$.
b- **Necessity :** If A is a b-I- supra open set ,
then $A \subset int^{\mu} (cl^{*\mu} (A)) \cup cl^{*\mu} (int^{\mu} (A))$
$= int^{\mu} (A^{*\mu} \cup A) \cup (int^{\mu} (A))^{*\mu} \cup (int^{\mu} (A)) = int (\phi \cup A) \cup (\phi \cup (int^{\mu} (A))$
$= int^{\mu} (A) \cup int^{\mu} (A) = int^{\mu} (A)$. Hence A is supra open.
Sufficiency it is easy**.**
c- **Necessity:** it is easy and we need to show only sufficiency.
Note that the local function of A with respect to N and μ we have: $A^{*\mu}(N) = cl^{\mu} (int^{\mu} (cl^{\mu} (A)))$.
Hence A is b-I- supra open set if and only if
$A \subset int^{\mu} (cl^{\mu} (int^{\mu} (cl^{\mu} (A))) \cup A) \cup cl^{\mu} (int^{\mu} (cl^{\mu} (int^{\mu} (A))) \cup A$.
Suppose that A is b-I- supra open. Since always
$int^{\mu} (cl^{\mu} (A)) \cup cl^{\mu} (int^{\mu} (A)) \subset A\cup cl^{\mu} (int^{\mu} (cl^{\mu} (A))) \cup cl^{\mu} (int^{\mu} (A))$,
then $A\subset int^{\mu} (A\cup cl^{\mu} (int^{\mu} (cl^{\mu} (A)))) \cup cl^{\mu} (int^{\mu} (A))$
$= int^{\mu} (A\cup A^{*\mu} (N)) \cup cl^{\mu} (int^{\mu} (A))$. Hence A is b-I-supra open set.

Lemma 3.40

Let (X, μ, I) be an ideal supra topological space and let $A\subseteq X$ then
$U\in \mu \Longrightarrow U\cap A^{*\mu} = U\cap (U\cap A)^{*\mu} \subseteq (U\cap A)^{*\mu}$.

Proposition 3.41

The intersection of α-I-supra open set and is b-I-supra open set is b-I- supra open set.

Proposition 3.42

Each b-I-open and μ^*-closed is semi- I-supra closed.

Proof:

Let A is b-I- supra open and μ^*-closed set then
$A\subset int^{\mu} (cl^{*\mu} (A)) \cup cl^{*\mu} (int^{\mu} (A))$
$= int^{\mu} (A) \cup [int^{\mu} (A) \cup (int^{\mu} (A))^{*\mu}] = int^{\mu} (A) \cup (int^{\mu} (A))^{*\mu}$
$= cl^{*\mu} (int (A))$.

Proposition 3.43

Let A, B be a subset of a space (X, μ, I) such that A is b-I- supra open and B is b-I- supra closed in X .Then there exist a b-I- supra open set H and b-I- supra closed set K such that
$A\cap B\subset K$ and $H\subset A\cup B$.

4- b-I-supra continuous mapping

Definition 4.1

a. A function $f: (X, \mu) \rightarrow (Y,\sigma)$ is called b-supra continuous if the inverse image of each supra open set in Y is b- supra open set in X.[14]
b. A function $f: (X, \mu) \rightarrow (Y,\sigma)$ is called pre- supra continuous if the inverse image of each supra open set in Y is pre- supra open set in X.

c. A function f: (X, μ,I) →(Y,σ) is called pre-I- supra continuous if the inverse image of each supra open set in Y is pre-I- supra open set in X.
d. A function f: (X, μ) →(Y,σ) is called semi- supra continuous if the inverse image of each supra open set in Y is semi- supra open set in X.
e. A function f: (X, μ, I) →(Y,σ) is called semi-I- supra continuous if the inverse image of each supra open set in Y is semi-I- supra open set in X.
f. A function f: (X, μ) →(Y,σ) is called α- supra continuous if the inverse image of each supra open set in Y is α- supra open set in X.
g. A function f: (X, μ, I) →(Y,σ) is called α -I- supra continuous if the inverse image of each supra open set in Y is α -I- supra open set in X.
h. A function f: (X, μ,I) →(Y,σ) is called b-I- supra continuous if the inverse image of each supra open set in Y is b -I - supra open set in X.

The following examples show that b-I- supra continuous function do not need to be pre-I- supra continuous and semi-I- supra continuous and b- supra continuous function does not need to be b-I- supra continuous.

Example 4.2

Let X=Y ={a ,b ,c ,d} be the supra topological space by setting μ = σ ={{a},{d},{a ,d},∅,X} and I ={∅,{c}} on X Define a function f: (X, μ, I) →(Y, σ) as f (a) =f(c) =d and f (b) = f(d) =b ,then f is b-I- supra continuous but it is not pre- supra continuous.

Since A= {a, c} is b-I-supra open A ⊂ $int^{\mu}(cl^{*\mu}(A)) \cup cl^{*\mu}(i\,\phi\, t^{\mu}(A))$

$= int^{\mu}(\{a, c\}^{*\mu} \cup \{a, c\}) \cup cl^{*\mu}(\{a\}) = int^{\mu}(\{a, b, c\} \cup \{a, c\}) \cup \{a\}^{*\mu} \cup \{a\}$

$= int^{\mu}(\{a, b, c\}) \cup \{a ,b, c\} \cup \{a\} = \{a\} \cup \{a ,b, c\} = \{a ,b, c\} \supset A.$

Hence f is b-I- supra continuous.

And A is not pre-I-supra open $A \subset int(cl^{*}(A)) = int^{\mu}(\{a, c\}^{*\mu} \cup \{a, c\})$

$= int^{\mu}(\{a, b, c\}) = \{a\} \not\supset A.$

Example 4.3

Let (X, μ) be the real line with the indiscrete supra topology and (Y, σ) the real line with the usual topology, then the identity function f: (X, μ, p(x)) → (Y, σ) is b-continuous but not b-I-supra continuous.

Example 4.4

Let X =Y = {a, b, c} be the supra topological space by setting

μ = σ = {∅, X, {a, b}} and I = {∅ , {c}} , define a function f: (X, μ, I) →(Y, σ) as follows :

f(a) = a , f(b) =c,

And f (c) =b, then f is b-I- supra continuous but not semi -I- supra continuous.

Remark 4.5

If a function f: (X, μ, I) →(Y, σ) is semi-I- supra continuous (pre-I- supra continuous), then f is b-I- supra continuous.

Proposition 4.6

If the function f: (X, μ, I) →(Y, σ) is b-I- supra continuous, then f is b- supra continuous.

Proof:

Let A is b-I- supra open set, then we have

$A \subset int^{\mu}(cl^{*\mu}(A)) \cup cl^{*\mu}(int^{\mu}(A))$

$A \subset (int^{\mu}(A))^{*\mu} \cup (int^{\mu}(A)) \cup int^{\mu}(A^{*\mu} \cup A)$

$= cl^{\mu}(int^{\mu}(A)) \cup (int^{\mu}(A)) \cup int^{\mu}(cl^{\mu}(A) \cup A) \subset cl^{\mu}(int^{\mu}(A)) \cup int^{\mu}(cl^{\mu}(A)).$

Hence A is b-supra open set , and hence f is b- supra continuous.

Proposition 4.7

Every supra continuous function f: (X, μ, I) →(Y, σ) is pre-I- supra continuous.

The converse is not true in general as shown in the following example.

Example 4.8

Consider first the classical Dirichlet function f: ℝ → ℝ

$$f(x) = \begin{cases} 1, & x \in Q \\ 0, & x \notin Q \end{cases}$$

Let F be the ideal of all finite subsets of ℝ. The Dirichlet function f: (ℝ, μ, F) → (ℝ, μ) is pre-I- supra continuous, since every point of ℝ belongs to the μ- local function of the rationales with respect to F and T as well as to the μ- local function of the irrationals. Hence f is even I- supra continuous. But on the other hand The Dirichlet function is not supra continuous at any point of its domain.

Proposition 4.9

Every I- supra continuous function f: (X, μ, I) →(Y, σ) is pre-I- supra continuous.

The converse is again not true in general as shown in the following example

Example 4.10

Let X ={1 , 2, 3, 4} , μ ={ ϕ ,X,{1,3},{4},{1,3, 4}},and let σ ={ ϕ ,Y,{1,3, 4}},

I={ ϕ ,{3,}{4},{3,4}} . Set A = {1, 3, 4}.then the identity function

f: (X, μ, I) →(Y, σ) is pre-I- supra continuous but not I-supra continuous.

Proposition 4.11

Every pre- I- supra continuous function f: (X, μ, I) →(Y, σ) is pre- supra continuous.

Example 4.12

A pre-supra continuous function need not be pre-I- supra continuous .

Let (X, μ) be the real line with the indiscrete supra topology and (Y, σ) the real line with the usual topology. The identity function f: (X, μ, P(X)) →(Y, σ) is pre-supra continuous but not pre-I-supra continuous.

Proposition 4.13 A function f: (X, μ, I) →(Y, σ) is satisfy following:

1- Every pre- I- supra continuous function is b-I- supra continuous.

2- Every b- I- supra continuous function is b-supra continuous.

3- Every pre-supra continuous function b-supra continuous.

4- Every α- I-supra continuous function is pre-I- supra continuous.

5- Every α- I-supra continuous function is α-supra continuous.

6- Every α- I-supra continuous function is semi-I-supra continuous.
7- Every semi- I-supra continuous function f: (X, μ, I) →(Y, σ) is semi-supra continuous.
8- Every semi-supra continuous function f: (X, μ,) →(Y, σ) is b-supra continuous.

Remark 4.14
The following diagram holds for a function f: (X, μ, I) →(Y, σ):

α – μ- continuous
⇑
μ-continuous ⟹ α –I- μ- continuous⟹ semi-I- μ-continuous ⟹semi-μ-continuous
⇓ ⇓ ⇓
I- μ- continuous ⟹ pre-I- μ- continuous ⟹ b-I- μ- continuous ⟹ b-μ- continuous
⇓
Pre- μ- continuous ⟹

Definition 4.15
A function f: (X, μ) →(Y, σ) is said to be b-supra irresolute if f^{-1} (V) is b-supra open in (X, μ) for every b-supra open set V of (Y, σ).

Example 4.16
Let X ={1,2,3} = Y , μ ={ ϕ ,X,{1,2}},and let σ ={ ϕ ,Y,{1}{1,2}}, then the identity function f: (X, μ) →(Y, σ) is b- supra irresolute .

Definition 4.17
A function f: (X, μ, I) →(Y, σ, J) is said to

- be I-supra irresolute if f^{-1} (V) is I-supra open in (X, μ, I) for every I-supra open set V of (Y, σ, J).
- Semi -I-supra irresolute, if f^{-1} (V) is semi- I-supra open in (X, μ, I) for every semi-I-supra open set V of (Y, σ, J).
- α-I- supra irresolute, if f^{-1} (V) is α - I-supra open in (X, μ, I) for every α -I-supra open set V of (Y, σ, J).
- Pre -I-supra irresolute, if f^{-1} (V) is pre- I-supra open in (X, μ, I) for every pre-I- supra open set V of (Y, σ, J).
- b -I-supra irresolute, if f^{-1} (V) is b- I-supra open in (X, μ, I) for every b-I-supra open set V of (Y, σ, J).

Definition 4.18
Let N be a subset of a space (X, μ, I) and let x∈ X, then N is called b-I-supra neighborhood of x if there exists a b-I-supra open set U containing x such that U⊂N.

Proposition 4.19
Let f: (X, μ, I) →(Y, σ, J) and g: (Y, σ, J) → (Z, ν) be two functions where I and J are ideals on X and Y. respectively. Then, gof is b-I-supra continuous if f is b-I supra continuous and g is supra continuous.

Proof:
Let W⊂Z is supra open in ν since g is supra continuous then g^{-1} (w) is supra open in Y,
Since f is supra continuous then $f^{-1}(g^{-1}$ (w)) is supra open in X. Then (gof) $^{-1}$ (w) is b-I-supra open in X .Then gof is b-I-supra continuous

Proposition 4.20
Let f: (X, μ, I) →(Y, σ, J) and g: (Y, σ, J) → (Z, ν) be two functions where I and J are ideals on X and Y. respectively. Then:

1. gof is pre-I-supra continuous, if f is pre-I –supra continuous and g is supra continuous.
2. gof is pre- supra continuous, if f is pre-I- supra continuous and g is supra continuous.
3. gof is semi-I- supra continuous, if f is semi-I-supra continuous and g is supra continuous.
4. gof is semi- supra continuous, if f is semi-I- supra continuous and g is supra continuous
5. gof is α-I- supra continuous, if f is α-I- supra continuous and g is supra continuous.
6. gof is α - supra continuous, if f is α-I- supra continuous and g is supra continuous.

Proof: Obvious

Lemma 4.21
For any functions f: (X, μ, I) →(Y, σ), f (I) is an ideal on Y.

Definition 4.22
A functions f: (X, μ, I) →(Y, σ, J) is called b-I- supra open (resp., b-I- supra closed) if for each U∈ μ (resp., supra closed set F) f (U) (resp., f (F)) is b-J- supra open (resp., b-J- supra closed).

Remark 4.23
Every b-I- supra open (resp., b-I- supra closed) function is b- supra open (resp., b- supra closed) and the converses are false in general.

Example 4.24
Let X = {1,2 ,3}, μ_1 ={ ϕ ,X,{2 ,3}}, μ_2 ={ ϕ ,X,{1}{2},{1,2}}, and I = { ϕ ,{3}}. Then the identity function f: (X, μ_1) →(X , μ_2 ,I) is b- supra open but not b-I- supra open.

Example 4.25
Let X = {a ,b ,c}, μ_1 ={ ϕ ,X,{a}}, μ_2 ={ ϕ ,X,{3}{2},{2,3}}, and I = { ϕ ,{3}}. Define a function f: (X, μ_1) →(X , μ_2 ,I) as follows: f(a) = a , f(b) = f(c) = b .
Then, f is b- supra closed but not b-I- supra closed.

Definition 4.27
a.A functions f: (X, μ, I) →(Y, σ, J) is called semi-I-supra open (resp., semi-I- supra closed) if for each U∈ μ (resp., supra closed set F) f (U)
(resp., f (F)) is semi-J- supra open (resp., semi-J-supra closed).
b. A functions f: (X, μ, I) →(Y, σ, J) is called pre-I-supra open

(resp., pre-I- supra closed) if for each U∈ μ (resp., supra closed set F) f (U) (resp., f (F)) is pre-J- supra open (resp., pre-J- supra closed).

c.A functions f: (X, μ, I) →(Y, σ, J) is called α-I- supra open (resp., α -I- supra closed) if for each U∈ μ (resp., supra closed set F) f (U)

(resp., f (F)) is α -J- supra open (resp., α -J- supra closed).

Remark 4.28

a.Every semi-I- supra open (resp., semi-I- supra closed) function is

b-I- supra open (resp., b-I- supra closed) ;

b. Every pre-I- supra open (resp., pre-I- supra closed) function is

b-I- supra open (resp., b-I- supra closed).

Theorem 4.29

A functions f: (X, μ, I) →(Y, σ, J) is called b-I- supra open if and only if for each x ∈ X and each supra neighborhood U of x , there exists V∈ BJO(Y, σ) containing f (x) such that V⊂ f(U).

Proof:

Suppose that f is a b-I- supra open function for each x ∈ X and each supra neighborhood U of x,

There exists U_0 ∈ μ such that x∈ U_0 ⊂ U.

since f is b-I- supra open, V =f (U_0) ∈ BJO(Y, σ) and f(x) ∈ V⊂ f(U).

Conversely, let U be supra open set of (X, μ). For each x ∈ U,

There exists V_x ∈ BIO(X, μ) such that f(x) ∈ V_x⊂ f (U).

Therefore we obtain f(U) = ∪ { V_x : x ∈ U} and hence by ,

f(U) ∈ BJO(Y, σ).

This shows that f is b-I- supra open.

Theorem 4.30

Let f: (X, μ, I) →(Y, σ, J) be b-I- supra open (resp., b-I- supra closed). If W is any subset of Y and F is a supra closed (resp., supra open) set of X containing

f^{-1}(W), then there exists a b-I- supra closed (resp., b-I- supra open) subset H of Y containing W such that f^{-1}(H) ⊂ F.

Proof

Suppose that f is a b-I- supra open function. Let W is any subset of Y and F is a supra closed subset of X containing f^{-1}(W).

Then X-F is supra open and since f is a b-I- supra open,

f(X-F) is b-I- supra open.

Hence H = Y- f(X-F) is b-I- supra closed.

It follows from f^{-1}(W) ⊂ F that W ⊂H .

moreover, we obtain f^{-1}(H) ⊂ F. for a b-I- supra closed function.

Theorem 4.31

For any bijective function: (X, μ) →(Y, σ, J), the following are equivalent:

1. f^{-1} : (Y, σ, J) →(X, μ) is b-I- supra continuous;
2. f is b-I- supra open;
3. f is b-I- supra closed.

Proof . It is straightforward.

Definition 4.32

A functions f: (X, μ, I) →(Y, σ, J) is called ∗-I- supra continuous if the pre image of every supra open set in (Y, σ) is ∗- supra dense in itself.

Proposition 4.33

For a subset A ⊂ (X, μ, I), if the condition (int^{μ} (A) $^{\mu}$∗)∗ $^{\mu}$ ⊂ int^{μ} (∗$^{\mu}$) holds, then the following are equivalent:

1. A is I- supra open;
2. A is b- I- supra open and∗- supra dense in itself.

Proof: (1) ⟹ (2)

Let A be an I- supra open subset of (X, μ, I). Then A ⊂ int^{μ} (A∗$^{\mu}$) ⊂ A∗ $^{\mu}$, which shows that A is ∗- supra dense in itself.

Since A is I- supra open, then A is pre-I- supra open and so

A ⊂ int^{μ} (cl∗$^{\mu}$ (A))

⊂ int^{μ} (cl∗$^{\mu}$ (A)) ∪ cl∗$^{\mu}$ (int^{μ} (A)).

Thus A is b-I- supra open.

(2) ⟹ (1)

Let A is b- I- supra open and∗- supra dense in itself. then since

(int^{μ} (A)∗ $^{\mu}$)∗ $^{\mu}$ ⊂ int^{μ} (A∗ $^{\mu}$),

A⊂ int^{μ} (cl∗ $^{\mu}$ (A)) ∪ cl∗ $^{\mu}$ (int^{μ} (A))

= int^{μ} (A∪A∗ $^{\mu}$) (int^{μ} (A)∪ (int^{μ} (A)∗ $^{\mu}$) ⊂ int^{μ} (A∗) int^{μ} (A))∪(int^{μ} (A))∗ $^{\mu}$

= int^{μ} (A)∗ $^{\mu}$ ∪ (int^{μ} (A))∗ $^{\mu}$ = int^{μ} (A∗ $^{\mu}$) .

Proposition 4.34

If a functions f: (X, μ, I) →(Y, σ, J) is called -I- supra continuous and if

(int^{μ} (A)∗$^{\mu}$)∗ $^{\mu}$ ⊂ int^{μ} (A∗$^{\mu}$) for each subset A of X , then f is b-I- supra continuous and ∗-I- supra continuous.

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حول مثاليات الفضاءات التبولوجية الفوقية

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الملخص

في هذا البحث قدمنا من خلال مفهوم مثاليات الفضاءات التبولوجية الفوقية قدمنا صفوف من المفاهيم (المجموعة الكثيفة الفوقية من النمط I , المجموعة الكثيفة الفوقية من النمط *, المجموعة المغلقة الفوقية من النمط * والمجموعة التامة الفوقية من النمط *) في الثاليات التبولوجية الفوقية . واخيرا درسنا وتحرينا بعض خصائص تلك المفاهيم بالإضافة الى ذلك قدمنا بعض الدوال المستمرة و خصائصها.