ON bi- INTUITIONSTIC TOPOLOGICAL SPACE

Taha H. Jassim[1] , Zenah T. Abdulqader[2] , Hiba O. Mousa[2]
[1] *Department of Mathematics, college of computer science and Mathematics, University of Tikrit , Tikrit , Iraq*
[2] *Department of Mathematics , College of Education for Women , Tikrit University , Tikrit , Iraq*

Abstract:
In this paper we introduce a new definition is called bi- intuitionistic topological space and from this concept we present some kinds of closed set (semi –closed, pre-closed ,β-closed ,α-closed) setes in bi- intuitionistic topological space, define generalized closed set (sg-closed ,gs-closed ,gp-closed, gα-closed , αg- closed , gβ-closed) sets in bi - intuitionistic topological space and give relationship among them , we introduce the definition of T_{gs}-space in bi- intuitionistic topological space and from this consept we get new results .

1- Introduction and terminologies :

A triple (X, T_i, T_j) (i ≠j) where X≠Ø and T_i , T_j are topologies on X is called a bi – topological space in 1963 ,Kelly[3] , The concept of fuzzy set introduced for first time in1965 by Zadeh, 1996 Coker [4] introduced the concept of intuitionistic set and intuitionistic topology as a special case of intuitionistic fuzzy topological spaces. Now we define the bi- intuitionistic topological space if X ≠ Ø , T_i , T_j are tow intuitionistic topologies on X then (X ,T_i , T_j) are bi- intuitionistic topological space(bi- ITS for short).

Now we recall the definition of an intuitionistic set and intuitionistic topology and some basic properties which are needed .

(1-1) definition:[5],[6]

Let X be a non-empty set. An intuitionistic set A (IS, for short) is an object having the form $A = \langle X, A_1, A_2 \rangle$ where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \emptyset$. The set A_1 is called set of members of A, while A_2 is called set of nonmember of A.

(1-2) definition:[6],[7]

Let A and B be two IS having the form $A = \langle x, A_1, A_2 \rangle$ and $B = \langle x, B_1, B_2 \rangle$ respectively, then,

a) $A \subseteq B \Leftrightarrow A_1 \subseteq B_1 \;\&\; A_2 \supseteq B_2$

b) $A = B \Leftrightarrow A \subseteq B \;\&\; B \subseteq A$

c) $\overline{A} = \langle x, A_2, A_1 \rangle$

d) $A \cap B = \langle x, A_1 \cap B_1, A_2 \cup B_2 \rangle$

e) $A \cup B = \langle x, A_1 \cup B_1, A_2 \cap B_2 \rangle$

f) $\widetilde{X} = \langle x, X, \emptyset \rangle$

g) $\widetilde{\emptyset} = \langle x, \emptyset, X \rangle$.

(1-3) definition[4]:

Let X and Y be two non-empty sets and $f: X \to Y$ be a function.

a) If $B = \langle y, B_1, B_2 \rangle$ is an IS in Y , then the preimage (inverse image) of B under f is denoted by $f^{-1}(B)$ is an IS in X and defined by $f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2) \rangle$.

b) If $A = \langle x, A_1, A_2 \rangle$ is an IS in X, then the image of A under f is denoted by f(A)is IS in Y defined by $f(A) = \langle y, f(A_1), \underline{f}(A_2) \rangle$ where $\underline{f}(A2) = (f(A^c2)^c)$, A any sub set of X.

(1-4) definition:[6]

An intuitionistic topology (IT for short) on a nonempty set X is a family T of IS's in X containing $\widetilde{\emptyset}$, $\widetilde{X}$, and closed under finite intersection and arbitrary union. In this case the pair (X, IT)is called an intuitionistic topological spaces ,(ITS for short), and any IS in T is known as an intuitionistic open set (IOS, for short) in X, the complement of IOS is called intuitionistic closed set (ICS, for short) in X .

(1-5) definition [2],[7]:

Let (X, IT) be ITS , and let $A = \langle x, A_1, A_2 \rangle$ be IS in X. then the intuitionistic interior of ISA (int A ,for short) and intuitionistic closure of ISA (cl A , for short) are defined by

int A=∪ {G ∈ T : G ⊆ A }

cl A =∩ {F : A ⊆ F , $\overline{F}$ ∈ T }

(1-6) definition: [1]

a) Let $P_\sim$ be an IP in X and $A = \langle x, A_1, A_2 \rangle$ be an IS in X. $P_\sim$ is said to be contained in A (for short $P_\sim \in A$, if $p \in A_1$).

b) Let $P_\approx$ be VIP in X and $A = \langle x, A_1, A_2 \rangle$ be an IS in X. $P_\approx$ is said to be contained in A, ($P_\approx \in A$, for short if, $p \notin A_2$).

Now we introduce a new definitions which is needed in our work .

(2-1) definition:

We say that (X, IT_i ,IT_j) bi- intuitionistic topological space if for each of (X,IT_i) and (X,IT_j)is intuitionistic topological space on X .

(2-2) definition:

Let (X, IT_i ,IT_j) bi-ITS and G be a sub set of X then G is said to be (i, j)- intuitionistic open set((i, j)IOS for short) if G = A ∪ B where A ∈ IT_i and B ∈ IT_j the complement of (i, j)-open set is (i, j)-intuitionistic closed set ((i, j)ICSfor short).

(2-3)Example:

Let $X = \{1,2,3\}$ and $IT_i = \{\widetilde{\emptyset}, \widetilde{X}, A, B, C\}$ where $A = \langle X, \{3\}, \{1,2\} \rangle, B = \langle X, \{1\}, \{3\} \rangle, C = \langle X, \{1,3\}, \emptyset \rangle$.and $IT_j = \{\widetilde{\emptyset}, \widetilde{X}, D, E\}$ where $D = \langle X, \{1\}, \{2\} \rangle, E = \langle X, \{1\}, \{2,3\} \rangle$

(i, j)- open set = $\{\widetilde{\emptyset}, \widetilde{X}, A, B, C, D, E, F, G\}$ where $F = \langle X, \{1,3\}, \{2\} \rangle$,$G = \langle X, \{1\}, \emptyset \rangle$.

(i, j)- closed set=$\{\emptyset, \widetilde{X}, \overline{A}, \overline{B}, \overline{C}, \overline{D}, \overline{E}, \overline{F}, \overline{G}\}$

(2-4) definition

Let (X, IT_i ,IT_j) bi-ITS and $A = \langle x, A_1, A_2 \rangle$ is IS in X. then the intuitionistic interior and intuitionistic closure of A are denoted by (i, j)int(A) and(i, j) cl(A) respectively and defined as a union of all (i, j)- IOS of X that contained in A and the intersection of all (i, j) -ICS in X that contain A respactively .

(2-5)Remark

Let (X, IT_i ,IT_j) bi-ITS, and $A = \langle x, A_1, A_2\rangle$ be IS in X. Then $(i,j)cl(\overline{A}) = \overline{(i,j)int(A)}$ & $(i,j)\ int(\overline{A}) = \overline{(i,j)\ cl(A)}$.

Now we give the definition of semi, pre ,semi pre (β) , pre semi (α) in bi ITS,s .

(2-6) definitions

Let be(X, IT_i ,IT_j) bi- ITS, and $A = \langle x, A_1, A_2\rangle$ be IS in X. Then A is called:

1) (i, j) intuitionistic semi-open set ((i, j)ISOS, for short) if $A \subseteq IT_j\ cl(IT_i int(A))$

2(i, j)intuitionisticα-ope set ((i, j)(IαOS, for short)if $A \subseteq IT_i int\left(IT_j cl\left(IT_i int(A)\right)\right)$.

3)(i, j) intuitionistic pre-open set ((i, j)(IPOS, for short)if $A \subseteq IT_i int\left(IT_j cl(A)\right)$

4) (i, j)intuitionistic β-open set (i, j)(IβOS, for short)if $A \subseteq IT_j cl\left(IT_i int\left(IT_j cl(A)\right)\right)$.

The complement of (i, j)ISOS (resp. (i, j) IαOS, (i, j)IPOS, and(i, j) IβOS) is called(i, j) intuitionistic semi- closed set (resp. (i, j) intuitionistic α-closed, (i, j)intuitionistic pre-closed, , and (i, j) intuitionistic β-closed) set in X. ((i, j) ISCS, (i, j) IαCS, (i, j) IPCS, and (i, j) IβCS, for short).

(2-7)Theorem :

Let (X, IT_i ,IT_j) bi-ITS, and $A = \langle x, A_1, A_2\rangle$ IS in X. then

i.A is (i, j)ICS then A is(i, j) IαCS , (i, j)ISCS, (i, j) IPCS AND(i, j) IβCS .

ii.A is (i, j) IαOS then A is (i, j)ISOS,((i, j) IPOS, (i, j) IβOS.)

iii.A is (i, j) ISOS then A is(i, j) IβOS.

iv.A is (i, j) IPOS then A is (i, j)IβOS.

Proof: [clear from definition]

(2-8)Example:

Let $X = \{1,2,3\}$ and $IT_i = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle X, \{3\}, \{1,2\}\rangle, B = \langle X, \{1\}, \{3\}\rangle, C = \langle X, \{1,3\}, \emptyset\rangle$.and

$IT_j = \{\tilde{\emptyset}, \tilde{X}, D, E\}$ where $D = \langle X, \{1\}, \{2\}\rangle, E = \langle X, \{1\}, \{2,3\}\rangle$,

ISCX={$\tilde{\emptyset}$, $\tilde{X}$,A,B,E,K_1,K_2,K_3,K_4,K_5 }.

IPCX={$\tilde{\emptyset}$, $\tilde{X}$,A, E,K_1,K_3,K_4,K_5,K_6,K_7,K_8,K_9,K_{10},K_{11},K_{12},K_{13},K_{14},K_{15}, K_{16},K_{17}}

IαCX={$\tilde{\emptyset}$, $\tilde{X}$,A,E ,K_1,K_3,K_4}

IβCX={$\tilde{\emptyset}$, $\tilde{X}$,A,B,C,E,K_1,K_2 ,K_3,K_4,K_5,K_6,K_7,K_8,K_9,K_{10},K_{11},K_{12},K_{13},K_{14},K_{15} ,K_{16},K_{17}}

Where $K_1=\langle X, \{3\}, \{1\}\rangle$, $K_2=\langle X, \{1,2\}, \{3\}\rangle$, $K_3=\langle X, \emptyset, \{1\}\rangle$, $K_4=\langle X, \emptyset, \{1,2\}\rangle$, $K_5=\langle X, \emptyset, \{1,3\}\rangle$, $K_6=\langle X, \{2\}, \{1\}\rangle$, $K_7=\langle X, \{2\}, \{3\}\rangle$, $K_8=\langle X, \{2\}, \{1,3\}\rangle$,

$K_9=\langle X, \{2\}, \emptyset\rangle$, $_0=\langle X, \{3\}, \{2\}\rangle$, $K_{11}=\langle X, \{3\}, \emptyset\rangle$, $K_{12}=\langle X, \{2,3\}, \{1\}\rangle$ $K_{13}=\langle X, \{2,3\}, \emptyset\rangle$, $K_{14}=\langle X, \emptyset, \{2\}\rangle$, $K_{15}=\langle X, \emptyset, \{2\}\rangle$, $K_{16}=\langle X, \emptyset, \{2,3\}\rangle$ $K_{17}=\langle X, \emptyset, \emptyset\rangle$

(2-9)Remark

Let (X, IT_i ,IT_j) bi-ITS, and $A = \langle x, A_1, A_2\rangle$ be IS in X .then

(i, j)ISOS and (i, j)IPOS is indepented) from example (2-3) $K_2=\langle X, \{1,2\}, \{3\}\rangle$ is (i, j)ISOS but not (i, j)IPOS and $K_{15}=\langle X, \emptyset, \{2\}\rangle$ is (i, j)IPOS but not (i, j)ISOS .

(2-10) definition

Let (X, IT_i ,IT_j) bi-ITS, and $A = \langle x, A_1, A_2\rangle$ be IS in X. Then the intersection of all (i, j) ISCS (resp (i, j)IαCS, (i, j)IPCS and(i, j) IβCS) in X that containing A is called the semi-closure (resp. (i, j)α-closure , (i, j) pre-closure, and (i, j) β-closure)of A and denoted by (i, j) scl(A) (resp. (i, j) αcl(A), (i, j) pcl(A), and(i, j) βcl(A)) .

<u>**Note**</u> It is well-known that:

$$(i,j)scl(A) = A \cup IT_j int\left(IT_i cl(A)\right), (\text{ resp. } (i,j)\alpha cl(A) = A \cup IT_i cl\ IT_j int\left(IT_i cl(A)\right),$$

$$(i,j)pcl(A) = A \cup IT_i cl\ IT_j int(A), (i,j)\beta cl(A) = A \cup IT_j int\left(IT_i cl\ IT_j int\ (A)\right)$$

(2-11) definition

Let (X, IT_i, IT_j) be bi- ITS, and $A = \langle x, A_1, A_2\rangle$ be IS in X. Then the union of all(i, j) ISOS (resp. (i, j)IαOS, (i, j) IPOS and(i, j) IβOS) in X that contained A is called the(i, j) semi-interior (resp. (i, j) α-interior , (i, j)pre-interior and(i, j) β-interior) of A and denoted by (i, j)sint(A) (resp. (i, j) αint(A), (i, j)pint(A) and(i, j) βint(A)) .

<u>**Note**</u> It is well-known that

$$(i,j)sint(A) = A \cap IT_j cl\ IT_i int(A), (\text{ resp. } (i,j)\alpha cl(A) = A \cap IT_i int\ IT_j cl\ IT_i int(A),$$

$$(i,j)pint(A) = A \cap IT_i int\ IT_j cl(A), (i,j)sint(A) = A \cap IT_j cl\ IT_i int IT_j cl(A).$$

(2-13)Proposition

Let (X, IT_i ,IT_j) be bi- ITS , and $A = \langle x, A_1, A_2\rangle$ be IS in X. Then A is(i, j) IαOS in X if and only if it is both (i, j)ISOS and(i, j) IPOS in X.

<u>**Proof:**</u> [clear from definition].

3-Generalized closed set in bi- intuitionistic topological spaces

(3-1) definitions

Let (X, IT_i ,IT_j) be bi- ITS, an IS Ã in X is called:

1) (i, j) Generalized closed (briefly, (i, j) g-closed), if $IT_j cl(A) \subseteq U$, whenever $A \subseteq U$ and U is IT_i-ISOS.

2) (i, j) Semi-generalized closed (briefly, (i, j) sg-closed), if IT_j- scl(A) ⊆ U, whenever A ⊆ U and U is IT_i- ISOS,

3) (i, j)Generalized semi-closed (briefly, (i, j) gs-closed), if IT_j- Scl(A) ⊆ U, whenever A ⊆ U and U is IT_i- IOS,

4) (i, j)Generalized α-closed (briefly, (i, j) gα-closed), if $IT_j-\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is IT_i- IαOS,

5) (i, j)α-generalized closed (briefly, (i, j) αg-closed), if $IT_j-\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is IT_i IOS,

6) (i, j)Generalized β-closed (briefly, (i, j) gβ-closed), if $IT_j-\beta cl(A) \subseteq U$, whenever $A \subseteq U$ and U is IT_i- IOS,
7) (i, j)Generalized pre-closed (briefly, (i, j) gp-closed), if IT_j- $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is IT_i -IOS.

An IS A in X is(i, j) g-open (resp. (i, j) sg-open, (i, j) gs-open, (i, j) gα-open, (i, j) αg-open, (i, j) gβ-open, and (i, j)gp-open), if the $\overline{A}$is(i, j) g-closed (resp. (i, j) sg-closed, (i, j) gs-closed, (i, j) gα-closed, (i, j)αg-closed, (i, j) gβ-closed and(i, j) gp-closed).

(3-2)Theorem:

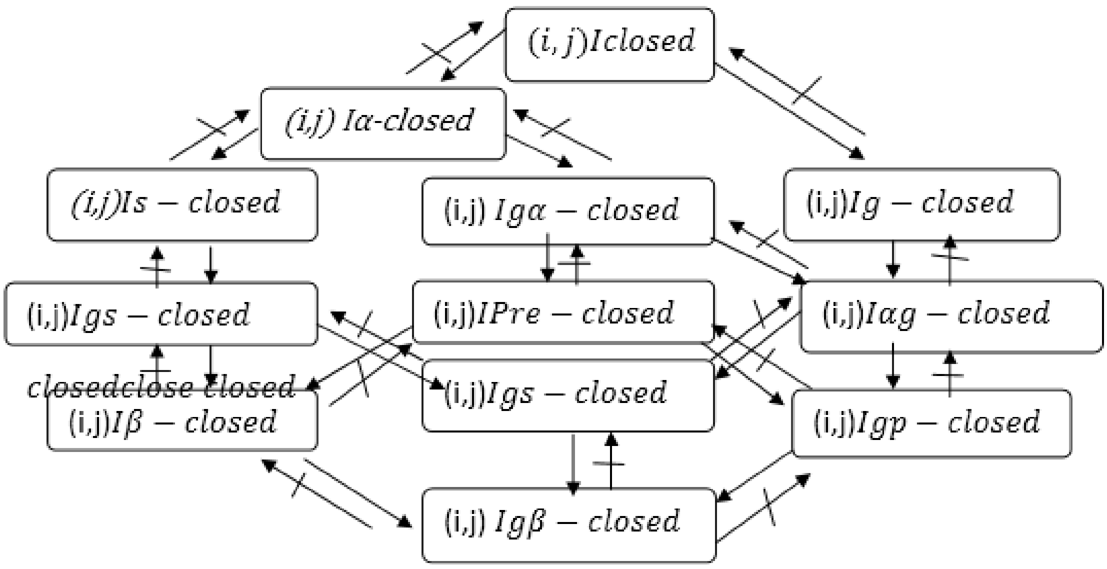
Let (X, IT_i ,IT_j) bi- ITS. An intuitionistic subset A of X is (i, j)g-open if and only if, for each (i, j)ICS F in X such that $F \subseteq (i, j)int(A)$ whenever $F \subseteq A$

Proof

⇒ Suppose that A is(i, j) g-open set in X, and let F be any closed set such that $F\subseteq A$, so by definition$\tilde{\overline{A}}$ is (i, j)g-closed set in X. Therefore, for each(i, j) IOS U say $U = \overline{F}$ in $\underline{X}$,$\overline{A} \subseteq \overline{F}$, then(i, j) $cl(\overline{A}) \subseteq \overline{F}$, so $\overline{\overline{F}} = F \subseteq (i, j)cl(\overline{A}) = (i, j)int(A)$ by Remark (2-5).
⇐ suppose that for each(i, j) ICS $F \subseteq A$ then $F \subseteq (i, j)int(A)$, we have to prove that A is (i, j)g-open, i.e. we have to prove that $\overline{A}$ is(i, j) g-closed, let U be any IOS in X such that $\overline{A} \subseteq U$, we have to prove that(i, j) $cl(\overline{A}) \subseteq U$. For if, since U is (i, j) IOS, then $\overline{U}$ is (i, j)ICS and $\overline{U}$ A , so by hypothesis $\overline{U} \subseteq (i, j)int(\overline{A})$. Therefore(i, j), $\overline{Int(A)} = (i, j)cl(\overline{A}) \subseteq \overline{\overline{U}} = U$. By Remark (2-5) we get that $\overline{A}$ is(i, j) g-closed.

(3-3)Theorem

Let(X,IT_i, IT_{ji})bi- ITS. Then the following implications in the diagram are true but not reversible.



Proof

The method of prove this theorem is to take one implication and prove truth one and give a counter example for the other at the end of the proof.

1) (i,j)Closed⇒)(i, j)g-closed, but the converse is not true.

We have to prove that, if A is(i,j)- closed set then A is(i,j) g-closed. For if, since A is(i,j)- closed, then(i, j) cl(A) = A. Now, for each (i,j)- IOS U , $A \subseteq U$. We have(i, j) $cl(A) = A \subseteq U$.

2) (i, j) IClosed⇒ (i, j)Iα-closed, but the converse is not true.

We have to prove that, if A is (i,j)- closed, then A is (i , j)Iα-closed. For if, since A is (i , j)closed, then $IT_i cl(A) = A$, so IT_j int $IT_i cl(A) \subseteq IT_i cl$ (A), therefore $IT_i cl\ IT_j int IT_i\ cl(A) \subseteq IT_i clA = A$. But $\alpha cl(A) = A \cup IT_i\ cl(IT_j\ int(IT_i\ cl(A)\ )\ \subseteq A \cup IT_i cl(A) = IT_i cl(A) = A$, Therefore(i , j) $\alpha cl(A) \subseteq A$, and we have from definition of $A \subseteq (i\ , j)\alpha cl(A)$, so we get that(i , j) $\alpha cl(A) = A$. i.e. A is(i , j)I α-closed..

3) (i, j)Ig-closed ⇒ (i, j)Iαg-closed and the converse is not true.

We have to prove that, if a is(i , j)Ig-closed, then A is (i , j) Iαg-closed. For if, since A is Ig-closed, so for each U∈ IT_i, $A \subseteq U$, then$IT_i cl(A) \subseteq U$. Since$IT_i cl(A) \subseteq U$, then $IT_j int IT_i cl(A) \subseteq A \subseteq U$, so$IT_j int IT_i cl(A) \subseteq IT_i cl IT_j int IT_i cl(A) \subseteq IT_i cl(A) \subseteq U$, so (i, j)$\alpha cl(A) \subseteq IT_i cl(A) \subseteq U$. That is,(i,j)$\alpha cl(A) \subseteq U$. Therefore A Iαg-closed.

4) (i, j) Iαg-closed⇒ (i, j)Igp-closed and the converse is not true

We have to prove that, if A is(i, j)αg-closed, then A is (i, j) gp-closed. For if, since A is Iαg-closed, so for each U∈ IT_i , $A \subseteq U$, then (i, j)$\alpha cl(A) \subseteq U$.

$(i,j)\alpha cl(A) = A \cup IT_i clIT_j intIT_i cl(A)$, $(i,j)pcl(A) = A \cup IT_i clIT_j int(A) \subseteq A \cup IT_i clIT_j intIT_i cl(A) = (i,j)\alpha cl(A)$ Since $IT_i cl(A) \subseteq U$, then $IT_j intIT_i cl(A) \subseteq A \subseteq U$, so$IT_j\ intIT_i cl(A) \subseteq IT_i clIT_j intIT_i cl(A) \subseteq IT_i cl(A) \subseteq U$, so (i,j)pcl(A) $\subseteq (i,j)\alpha cl(A) \subseteq U$. That is, $(i,j)pcl(A) \subseteq U$. Therefore A (i,j)Igp-closed.

6) (i,j)Igs-closed⇒ (i,j)Igβ-closed and the converse is not true.

We have to prove that, if A is (i,j)I gs-closed, then A is(i,j)Igβ-closed. For if, since A is(i,j)Igs-closed, so for each U∈ IT_i , A ⊆ U, then $(i,j)scl(A) \subseteq U$.

Since $(i,j)scl(A) = A \cup IT_j intIT_i cl(A) \subseteq U$, and since $IT_j intIT_i cl(A) \subseteq IT_j intIT_i cl\ IT_j int(A) \subseteq U$ $(i,j)\beta cl(A) \subseteq U$.

Therefore A is (i,j) Igβ-closed.

7)- (i,j) Iβ-closed⇒ (i,j)Igβ-closed and the converse is not true.

We have to prove that, if A is(i,j)Iβ-closed, then A is(i,j) Igβ-closed. For if,

since A is (i,j)Iβ-closed, $IT_j intIT_i clIT_j int(A) \subseteq A$. Let U be any IOS in IT_i such that $A \subseteq U$ Since $(i,j)\beta cl(A) = A \cup IT_j intIT_i clIT_j int(A) = A \subseteq U$.

Therefore A(i,j)Igβ-closed.

8) (i,j)Iαg-closed⇒ (i,j)Igs-closed and the converse is not true in general.

We have to prove that, if A is (i,j) Iαg-closed, then A is(i,j)I gs-closed. For if,

since A is(i,j) Iαg-closed, so for each U ∈ IT_i ,A ⊆ U, then(i,j) αcl(A) ⊆ U.

$(i,j)scl(A) = A \cup IT_j intIT_i cl(A) \subseteq A \cup IT_i\ clIT_j intIT_i cl(A) = (i,j)\alpha cl(A) \subseteq U$

Since (i,j) $scl(A) \subseteq (i,j)\alpha cl(A) \subseteq U$, That is, $(i,j)scl(A) \subseteq U$.

Therefore, A is (i,j) Igs-closed.

9) (i,j)Igp-closed⇒ (i,j)Igβ-closed and the converse is not true in general.

We have to prove that, if A is (i,j)Igp-closed, then A is (i,j)Igβ-closed. For if,

since A is⇒ (i,j)Igp-closed, so for each U ∈ IT_i ,A ⊆ U, then(i,j) pcl(A) ⊆ U

sine U ∈ IT_i, IT_j int A ⊆ A, then

$\beta cl(A) = A \cup IT_j intIT_i clIT_j int(A) \subseteq A \cup IT_i clIT_j int(A) \subseteq A \cup IT_i clIT_j int(A) = A \subseteq U$,so(i,j)Iβcl(A) ⊆ U.

That is, A is (i,j)Igβ-closed.

10)(i,j)Iα-Closed⇒ (i,j)Is-closed, but the converse is not true

Since $IT_j\ intIT_i cl(A) \subseteq IT_i clIT_j intIT_i cl(A) \subseteq A$,so the result follows

11(i,j)I p-Closed⇒ (i,j)Iβ-closed, but the converse is not true

Since $IT_i clIT_j int(A) \subseteq IT_j intIT_i clIT_j intc(A) \subseteq A$, so the result follows.

12)(i,j)Iα-closed⇒ (i,j)Igα-closed and the converse is not true in general.

We have to prove that, if A is (i,j)Iα-closed, then A is (i,j)Igα-closed. For if,

since A is (i,j)Iα-closed, then$IT_i\ clIT_j intIT_i clA \subseteq A$. Let $A \subseteq U$, where U is any IT_i α-open , Since $(i,j)\alpha cl(A) = A \cup IT_i clIT_j intIT_i cl(A) \subseteq A \subseteq U$.

Therefore A is(i,j)Igα-closed.

13) (i,j)Is-closed⇒ (i,j)Isg-closed and the converse is not true in general.

We have to prove that, if A is (i,j)Is-closed, then A is (i,j)Isg-closed. For if,

since A is (i,j)Is-closed , then $IT_j intIT_i clA \subseteq A$. Let $A \subseteq U$, where U is any IT_is-open , Since $(i,j)scl(A) = A \cup IT_j intIT_i cl(A) \subseteq A \subseteq U$.

Therefore A is (i,j)Isg-closed.

14) (i,j)Igα-closed⇒ (i,j)Ipre-closed and the converse is not true.

We have to prove that, if A is (i,j)Igα-closed, then A is(i,j)Ipre-closed. For if,

since A is (i,j)Igα-closed , then , if for each U is IT_iαOS , $A \subseteq U$ then $(i,j)I\alpha clA \subseteq U$. $(i,j)\alpha cl(A) = A \cup IT_i cIT_j lintIT_i cl(A) \subseteq U$ and since $IT_i clIT_j int(A) \subseteq IT_i clIT_j intIT_i cl(A) \subseteq U$. $IT_i\ clIT_j int\ (A) \subseteq A$.

Therefore A is (i,j)Ipre-closed

15) (i,j)Isg-closed⇒(i,j)I β-closed, and the converse is not true in general

We have to prove that, if A is(i,j)Isg-closed, then A is (i,j)Iβ-closed. For if, since A is (i,j)Isg-closed then , if for each U∈ IT_iSOS, $A \subseteq U$ then $(i,j)sclA \subseteq U$. $(i,j)scl(A) = A \cup IT_j intIT_i cl(A) \subseteq U$ and since $IT_j intIT_i clIT_j int(A) \subseteq IT_j intIT_i cl(A) \subseteq U$ ($IT_j intA \subseteq A$) then $IT_i intIT_i clIT_j int\ (A) \subseteq A$. Therefore A (i,j)Iβ-closed.

16) (i,j)Igα-closed⇒ (i,j)Iαg-closed, and the converse is not true in general.

We have to prove that, if A is(i,j)I gα-closed, then A is(i,j)I αg-closed. For if,

since A is (i,j)Igα-closed, then, if for each U ∈ IT_iαOS, $A \subseteq U$ then $(i,j)\alpha clA \subseteq U$. Since $(i,j)\alpha cl(A) \subseteq IT_i clA \subseteq U$ for each IOS G , A⊆ G(G is(i,j)IαOS)

Therefore, $(i,j)\alpha cl(A) \subseteq IT_i clA \subseteq G$, A(i,j)I αg-closed

17) (i,j)I sg-closed⇒ (i,j)Igs-closed, an the converse is not true in general.

We have to prove that, if A is (i,j)Isg-closed, then A is (i,j)Igs-closed. For if,

since A is (i,j)Isg-closed , then , if for each U∈ ISOS , $A \subseteq U$ then(i,j)sclA ⊆ U. Since $(i,j)scl(A) \subseteq IT_i clA \subseteq U$ for each(i,j)IOS G , A ⊆ G(G is (i,j)ISOS)

Therefore, $(i,j)scl(A) \subseteq IT_i clA \subseteq G$, A is (i ,j)Isg-closed.

The following example shows that;

1) (i,j) I αg-closed $\not\rightarrow$ (i,j) I gα-closed

2) (i,j)Igp-closed ,(i,j)Ip-closed , (i,j)Igs-closed ,(i,j)Igβ-closed and (i,j)I β-closed $\not\rightarrow$ (i,j) gα-closed

3) (i,j)I gs-closed , (i,j) I β-closed and (i,j) Igβ-closed $\not\rightarrow$,(i,j)Isg-closed .

4) ,(i,j) Ig-closed $\not\rightarrow$ (i,j)I closed

(3-4)Example:

Let $X = \{a, b, c\}$ and $IT_i = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle X, \{a\}, \{b, c\}\rangle, B = \langle X, \{c\}, \{a, b\}\rangle, C = \langle X, \{a, c\}, \{b\}\rangle$ and $IT_j = \{\tilde{\emptyset}, \tilde{X}, D, E\}$ where $D = \langle X, \{a\}, \{b\}\rangle$, $E = \langle X, \{a, c\}, \emptyset\rangle$

$I\alpha O(X) = \{\tilde{\emptyset}, \tilde{X}, A, B, C, D, E, G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9, G_{10}, G_{11}, G_{12}, G_{13}\} = ISO(X)$. where $G_1 = \langle X, \{a\}, \{C\}\rangle$, $G_2 = \langle X, \{a\}, \emptyset\rangle$, $G_3 = \langle X, \{b\}, \{a\}\rangle$, $G_4 = \langle X, \{b\}, \{c\}\rangle$, $G_5 = \langle X, \{b\}, \{a, c\}\rangle$, $G_6 = \langle X, \{b\}, \emptyset\rangle$, $G_7 = \langle X, \{c\}, \{a\}\rangle$, $G_8 = \langle X, \{c\}, \{b\}\rangle$, $G_9 = \langle X, \{c\}, \emptyset\rangle$, $G_{10} = \langle X, \{a, b\}, \{c\}\rangle$, $G_{11} = \langle X, \{a, b\}, \emptyset\rangle$, $G_{12} = \langle X, \{b, c\}, \{a\}\rangle$, $G_{13} = \langle X, \{b, c\}, \emptyset\rangle$.

1) Let $L = \langle X, \{b\}, \emptyset\rangle \subseteq U = X$. then L is (i, j)Iαg-closed set because

$(i, j)\ \alpha cl(L) = A \cup ITi - cl(ITj - int(ITi - cl(L)))$
$= (i, j)\ \alpha cl(L) = A \cup ITi - cl(ITj - int(X))$
$= L \cup X = X \subseteq U$.

But is not (i, j)Igα-closed set because the only (i, j) IαOX in X containing L is G_{11}, G_{13} But (i, j)Igα-cl $= X \nsubseteq G_{11}, G_{13}$.

2) L is (i, j)Igp-closed ((i, j)Ip-closed , (i, j)Igs-closed ,(i, j)Igβ-closed and (i, j)Iβ-closed) because :
(i, j)pcl(L)=L $\subseteq$ U, (i, j)Ip-closed = IT_i c l(IT_j-int(L))$\subseteq$ L = IT_i-cl($\emptyset$) =$\emptyset \subseteq$ L.
, (i, j) gscl(L) =X $\subseteq$ U and ,(i, j)gβcl(L)=L $\subseteq$ U . but are not (i, j)gα-closed set because by (1).

3)L is (i, j) gs-closed set since (i, j) gscl(L)=L $\cup$ ITj int(ITi cl(L))
(i, j) gscl(L)=L $\cup$ X = X $\subseteq$ U .and L is (i, j)I β-closed , ,(i, j)Igβ-closed
Because (i, j)I β-closed : ITj int(ITi clITj int(L)) $\subseteq$ L so : ITj int(ITi cl $\emptyset$) =$\emptyset \subseteq$ L , (i, j)Igβcl(L) =L $\subseteq$ U .But not (i, j) Isg –closed because the only ISOX in X containing L is G_{11}, G_{13} But (i, j) gscl(L) = X $\nsubseteq G_{11}, G_{13}$

4) since L =$\langle X, \{b\}, \emptyset\rangle \subseteq$ U=X then cl(L)=X $\subseteq$ U therefore L is (i, j)Ig-closed set but not closed set L $\notin$ IT_i-closed set.

The following examples show that;

1) (i, j)Igβ-closed $\not\Rightarrow$ (i, j) Isg –closed, (i, j) I gs – closed ,and (i, j) Igα –closed .
2) (i, j)Iβ-closed $\not\Rightarrow$ (i, j) Isg –closed
3) (i, j) Ip -closed $\not\Rightarrow$ (i, j) Igα –closed
4) (i, j) Igp -closed $\not\Rightarrow$ (i, j) Igα -closed

(3-5) Example:

Let $X = \{a, b, c\}$ and $IT_i = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle X, \{a, b\}, \emptyset\rangle$, $B\langle X, \{a\}, \{c\}\rangle$ and $IT_j = \{\tilde{\emptyset}, \tilde{X}, C, D\}$ where $C = \langle X, \emptyset, \{b\}\rangle$, $D = \langle X, \{a\}, \{b\}\rangle$.

Let $H = \langle X, \emptyset, \{c\}\rangle \subseteq U = \langle X, \{a\}, \{c\}\rangle$

1) H is (i, j)Igβ-closed set because (i, j)Igβ-cl(H)= H $\subseteq$ U.But H is not (i, j)Isg –closed, (i, j) I gs –closed because (i, j) I gscl(H)= X $\nsubseteq$ U , and not (i, j)Igα –closed because (i, j) I gαcl(H)= X $\nsubseteq$ U

2) H is (i, j)Iβ-closed : IT_jint(IT_icl IT_jint(H)) $\subseteq$ H then
IT_jint (IT_i cl($\emptyset$)) = $\emptyset \subseteq$ H since (i, j) I gαcl(H)= X $\nsubseteq$ U then H is not (i, j) Isg –closed.

3) since (i, j) Ip -closed set (IT_i cl IT_jint(H)) $\subseteq$ H thenIT_ic l$\emptyset$ =$\emptyset \subseteq$ H so H is (i, j) Ip -closed set but H is not (i, j) Igα –closed because (i, j) I gαcl(H)= X $\nsubseteq$ U

4) H is (i, j) Igp –closed because (i, j) Ipcl(H)=H$\cup$ IT_icl (IT_j int (H))
=H$\cup$ IT_icl$\emptyset$ =H $\subseteq$ U but is not (i, j) Igα –closed because (i, j) I αcl(H)= X $\nsubseteq$ U

(3-6) Example:

From example (2-3) let M = $\langle X, \{1\}, \{2\}\rangle \subseteq$ U =X then

1) M is (i, j) Igp –closed set because (i, j) I pcl(M)= M $\cup$ IT_icl (IT_jint (M))
=M $\cup$ IT_icl(D) = M $\cup$ X= X $\subseteq$ U, But M $\notin$ IPCX .
And (i, j) I βcl(M)= M $\cup$ IT_jint (IT_icl (IT_jint (M)) = M $\cup$ X = X $\subseteq$ U so M is (i, j) Igβ –closed set but M $\notin$ IβCX..

2) Let F= $\langle X, \{1\}, \emptyset\rangle \subseteq$ U= X then (i, j) I Scl(F)= F$\cup$ IT_jint(IT_icl (F)) = F $\cup$ X = X $\subseteq$ U so F is (i, j) Igs –closed set but F $\notin$ ISCX IT_icl(ITj And (i, j) I αcl(F)= F $\cup$ IT_jint(IT_icl (F)) = F $\cup$ X =X so F is (i, j) Igα –closed But F $\notin$ IαCX..

Now we introduce the definition of T_{gs}-space in bi-ITS.

(3-7)definition:

(X , IT_i , IT_j) bi-ITS is said to be Tgs-space, if every (i,j)Igs-closed set in X is (i,j)Isg-closed set in X.

(3-8)proposition:

A subset A of (X , IT_i , IT_j) bi-ITS is (i,j)Igα- closed if and only if $X_1 \cap (i, j)\alpha cl(A) \subseteq A$, where
$X_1 = \{P_\sim = \langle x, \{p\}, \{p\}^c\rangle \tilde{\in} X : P_\sim \text{is no where dense in } \tilde{X}\}$

Proof: The same method of proof in ITS see [1]

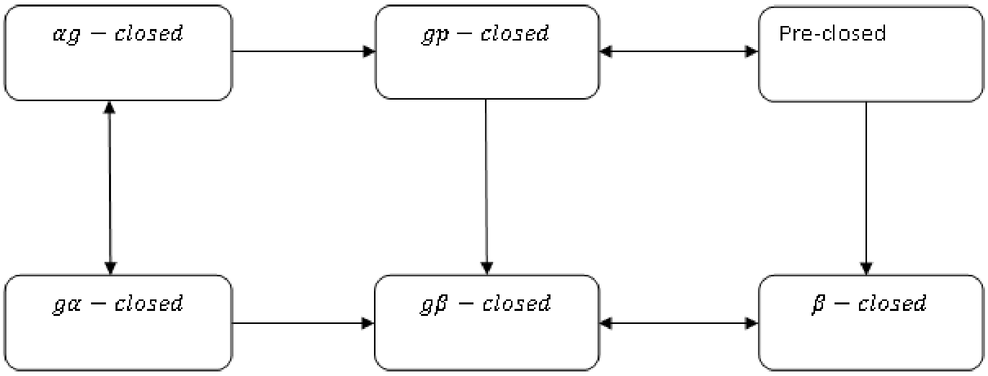
(3-9)theorem:

For (X , IT_i , IT_j) bi-ITS , the following statements are equivalent.

1. (X , IT_i , IT_j) is Tgs-space,
2. $P_\sim$is either (i,j)Ipre-open or (i,j)I closed for each $P_\sim \in \tilde{X}$.
3. Every(i,j)I αg-closed in X is (i,j)Igα-closed.
4. Every gp-closed set in X is pre-closed.
5. Every gβ-closed set in X is β-closed in X.
6. Every gp-closed in X is β-closed.

Proof: by similar way on the Tgs space in ITS see [1].

Now we get tow equavilant relation and one new implication in theorem (3-9) .



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حول الفضاء ثنائي التبولوجي الحدسي

طه حميد جاسم [1] ، زينة طه عبد القادر [2] ، هبة عمر موسى [2]

[1] قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

[2] قسم الرياضيات ، كلية التربية للبنات ، جامعة تكريت ، تكريت ، العراق

الملخص

في هذا البحث نقدم تعريفا جديدا يسمى الفضاء ثنائي التبولوجي الحدسي وعن هذا المفهوم نقدم بعض أنواع المجموعات المغلقة (المجموعة شبه مغلقة، المجموعة قبل المغلقة، المجموعة β المغلقة، المجموعة مغلقة α) في الفضاء ثنائي التبولوجي الحدسي. وتعريف المجموعات المغلقة المعممة (sg–مغلقة، gs المغلقة، gp المغلقة، gα المغلقة، αg المغلقة، gβ مغلقة) في الفضاء ثنائي التبولوجي الحدسي. وإعطاء العلاقة فيما بينها وكذلك قدمنا تعريف الفضاء T_{gs} ومن خلال هذا المفهوم توصلنا الى علاقات جديدة.