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The Adjacency Matrix of The Compatible Action Graph for Finite Cyclic Groups of p-Power Order

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ABSTRACT

Let G and H be two finites p-groups, then $G \otimes H$ is the non-abelian tensor product of G and H. In this paper, the compatible action graph $\Gamma^{p}_{G\otimes H}$ for $G \otimes H$ has been considered when $G = H$ and $G \neq H$ for the two finite p-groups by determining the adjacency matrix for $\Gamma^{p}_{G\otimes H}$ and studied some of its properties.

1. Introduction

The algebraic structures such as groups or rings becomes an important research topic to study the theoretical relationship between graph theory and some algebraic structures. The study of this relationship based on using some tools or properties of graph theory to establish a specific type of graphs. For example, by using the rings, the non-commuting graph of rings is defined by Erfanian et. al in [4]. Some results that concern on the pseudo-Von Neumann regular graph of the Cartesian product of rings have been studied by Khalel and Arif in [8]. Then, Erfanian et. al in [5] provided the generalized of the conjugate graph $\Gamma_{(G,n)^c}$. Furthermore, Zulkarnain et. al in [6] presented the conjugacy class graphs of the non-abelian 3-groups. On the other hand, the adjacency matrix of the conjugate graph for some metacyclic 2-groups has been determined by Alimon et. al in [7]. While, Pranjali and Vats in [12] discussed the adjacency matrix for the zero-Divisor graphs over the finite ring of Gaussian integer. Shahoodh et. al in [1] introduced new graph namely compatible action graph $\Gamma^{p}_{G\otimes H}$ for the nonabelian tensor product for two finites p-groups. Then, the author proved that the graph $\Gamma^{p}_{G\otimes H}$ is connected graph, Bipartite graph when $G \neq H$ and it's not complete graph. In this paper, we determined the adjacency matrix of the graph $\Gamma^{p}_{G\otimes H}$when $G = H$ and $G \neq H$ for such type of groups. The structure of this paper is started with the preliminary results which may needed to introduce the main results section. Then, the results of this study are given in section 3, and in section 4, the conclusions of this paper have been presented.

2. Preliminary Results

In this section, some of the past results that concern on compatible action graph for the finite p-groups are stated. Furthermore, some definitions and fundamental concepts that concern on the group theory and graph theory are also given in this section. These results are needed in the study of this research.

Definition 2.1 [2] A group G is called a cyclic group, if there is some element $a \in G$ such that $G = \{a^n | n \in \mathbb{Z}\}$.

Definition 2.2 [2] Let G be a group. The isomorphism from the group G onto itself is called an automorphism of the group G. The set of all automorphisms of the group G is denoted by $Aut(G)$.

Definition 2.3 [9] Let G and H be two groups; the action of G on H is the mapping $\Gamma: G \longrightarrow Aut(H)$ such that $\Gamma(gg')(h) = \Gamma(g)(\Gamma(g')(h))$ for each $g, g' \in G$ and $h \in H$.

Definition 2.4 [9] Let G and H be two groups which act on each other. The mutual actions are said to be compatible with each other and with the actions of G and H if

$^{(^{g}h)}g' = {}^{g}(^{h}(^{g^{-1}}g'))$ and $^{(^{h}g)}h' = {}^{h}(^{g}(^{h^{-1}}h'))$

for all $g, g' \in G$ and $h, h' \in H$.

Definition 2.5 [10] Let M and N be two groups. The non-abelian tensor product $M \otimes N$ is the group generated by $m \otimes n$, with the actions are compatible and satisfying the following relations:

1. $mm' \otimes n = (mm'm^{-1} \otimes {}^{m}n)(m \otimes n)$
2. $m \otimes nn' = (m \otimes n)({}^{n}m \otimes nn'n^{-1})$

for each $m, m' \in M$ and $n, n' \in N$.

Definition 2.6 [11] A graph G is consists of two sets which are $V(G)$ and $E(G)$, the set of the vertices $V(G)$ of the graph G and the set of the edges $E(G)$ of the graph G that connect these vertices.

Definition 2.7 [11] The adjacency matrix for any graph is the matrix $A(G) = [a_{ij}]$ such that a_{ij} is the number of the edges joining v_i and v_j , where $v_i, v_j \in V(G)$.

Definition 2.8 [1] Let G and H be two finite p-groups where p is an odd prime. Furthermore, let (ρ, ρ') be a pair of the compatible actions for $G \otimes H$ of G and H, where $\rho \in Aut(G)$ and $\rho' \in Aut(H)$. Then, the compatible action graph is

$$\Gamma^{p}_{C_{p^\alpha} \otimes C_{p^\beta}} = \left(V(\Gamma^{p}_{C_{p^\alpha} \otimes C_{p^\beta}}), E(\Gamma^{p}_{C_{p^\alpha} \otimes C_{p^\beta}}) \right) .$$

The set of the vertices of this graph, is the set $V(\Gamma^{p}_{C_{p^\alpha} \otimes C_{p^\beta}})$ that contains of $Aut(G)$ and $Aut(H)$, and the edge set of this graph is the set $E(\Gamma^{p}_{C_{p^\alpha} \otimes C_{p^\beta}})$ that connects these vertices, which is the set of all compatible pairs of actions (ρ, ρ'). Furthermore, two vertices ρ and ρ' are adjacent if they are compatible.

Definition 2.9 [14] The Trace of the square matrix $A = [a_{ij}]$ is the sum of the elements on the main diagonal of A, i.e. $Tr(A) = a_{11} + a_{22} + \cdots + a_{nn}$.

Definition 2.10 [3] Let G and H be two finite cyclic groups of p-power order where $p = 2$, then

$\Gamma_{G \otimes H} = \left(V(\Gamma_{G \otimes H}), E(\Gamma_{G \otimes H}) \right)$ is a compatible action graph for the non-abelian tensor product of G and H. Furthermore, the vertices set of this graph is the set $V(\Gamma_{G \otimes H})$ which contains all the automorphisms of the groups G and H while the edge set is the set $E(\Gamma_{G \otimes H})$ of all compatible pairs of actions (σ, σ') for $G \otimes H$ where $\sigma \in Aut(G)$ and $\sigma' \in Aut(H)$, and two vertices σ and σ' are adjacent if they are compatible on each other.

Proposition 2.1 [1] Let $G \cong C_{p^\alpha}$ and $H \cong C_{p^\beta}$ be two finite p-groups where p is an odd prime and $\alpha, \beta \geq 3$. Then

1. $\left| \Gamma^{p}_{C_{p^\alpha} \otimes C_{p^\beta}} \right| = (p-1)(p^{\alpha-1} + p^{\beta-1})$ where $G \neq H$.
2. $\left| \Gamma^{p}_{C_{p^\alpha} \otimes C_{p^\beta}} \right| = (p-1)p^{\alpha-1}$ where $G = H$.

Proposition 2.2 [3] Let $G = \langle g \rangle \cong C_{2^m}$ and $H = \langle h \rangle \cong C_{2^n}$ be two finite p-groups where $p = 2, m \geq 4$ and $n \geq 3$. Then

1. $\left| \Gamma_{G \otimes H} \right| = 2^{m-1} + 2^{n-1}$ if $m \neq n$
2. $\left| \Gamma_{G \otimes H} \right| = 2^{m-1}$ if $m = n$.

Theorem 2.1 [2] If G is a cyclic group of order p^α with p is an odd prime and $\alpha \in \mathbb{Z}^+$, then $Aut(C_{p^\alpha}) \cong C_{p-1} \times C_{p^{\alpha-1}} \cong C_{(p-1)p^{\alpha-1}}$ and $\left| Aut(C_{p^\alpha}) \right| = \varphi(p^\alpha) = (p-1)p^{\alpha-1}$.

Theorem 2.2 [2] If G is a cyclic group of order $2^n, n \geq 3$. Then $Aut(G) \cong C_2 \times C_{2^{n-1}}$ and $|Aut(G)| = \varphi(2^n) = 2^{n-1}$.

Theorem 2.3 [13] Let $G = \langle g \rangle \cong C_{p^\alpha}$ and $H = \langle h \rangle \cong C_{p^\beta}$ be groups such that $\alpha, \beta \geq 3$. Furthermore, let $\rho \in Aut(G)$ with $|\rho| = p^k$, where $k = 1, 2, \ldots, \alpha - 1$ and $\rho' \in Aut(H)$ with $|\rho'| = p^{k'}$, where $k' = 1, 2, \ldots, \beta - 1$. Then (ρ, ρ') is a compatible pair of actions if and only if $k + k' \leq \min\{\alpha, \beta\}$.

Corollary 2.1 [9] Let G and H be groups. Furthermore, let G act trivially on H. If G is abelian, then for any action of H on G, the mutual actions are compatible.

3. Results and Discussion

This section presents the adjacency matrix of the graph $\Gamma^{p}_{G \otimes H}$ when $G = H$ and $G \neq H$ for such type of groups. The results in this section have been computed with the help of GAP software [15].

Proposition 3.1 Let $G = \langle x \rangle \cong C_{3^3}$ be a finite p-group, where $p = 3$. Then, the adjacency matrix of the graph $\Gamma^{3}_{G \otimes G}$ is:

$$B\left(\Gamma^{3}_{G \otimes G}\right) = \begin{bmatrix}
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}$$

Proof:

Clearly that G has 27 elements, but from Proposition 2.1(ii), there exist 18 vertices in $V\left(\Gamma^{3}_{G \otimes G}\right)$. Therefore, according to Definition 2.8, the vertices of the graph $\Gamma^{3}_{G \otimes G}$ are the automorphisms of the group G. Thus, $B = [b_{ij}]$ for $\Gamma^{3}_{G \otimes G}$ is a square matrix of size 18×18. ■

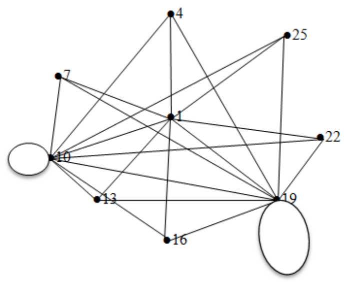

Figure 3.1: The compatible action graph for $\Gamma^{3}_{G \otimes G}$

Proposition 3.2 Let $G = \langle x \rangle \cong C_{5^2}$ be a finite p-group where $p = 5$. Then, the adjacency matrix of the graph $\Gamma^5_{G\otimes G}$ is:

$$B\left(\Gamma^5_{G\otimes G}\right)=\begin{bmatrix}
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}$$

Proof:
According to the definition of the order of the group, G has 25 elements, and from Proposition 2.1(ii), there exist 20 vertices in $V\left(\Gamma^5_{G\otimes G}\right)$. Therefore, by using Definition 2.8, the vertices of the graph $\Gamma^5_{G\otimes G}$ are the automorphisms of the group G. Thus, $B = [b_{ij}]$ for $\Gamma^5_{G\otimes G}$ is a square matrix of size 20×20. ■

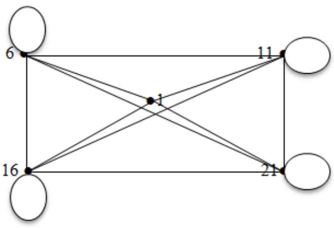

Figure 3.2: The compatible action graph for $\Gamma^5_{G\otimes G}$

In general, for p is an odd prime the adjacency matrix for this graph is given as follows.

Theorem 3.1 Let $G = \langle x \rangle \cong C_{p^\alpha}$ be a finite p-group and $\alpha \geq 2$. Then, the adjacency matrix of the graph $\Gamma^p_{G\otimes G}$ is a square matrix of size $((p-1)p^{\alpha-1}) \times ((p-1)p^{\alpha-1})$.

Proof: Let $G \cong C_{p^\alpha}$, then $|G| = p^\alpha, \alpha \geq 2$. Let ρ_1 and ρ_2 be two vertices of $V\left(\Gamma^p_{G\otimes G}\right)$ where $\rho_1, \rho_2 \in Aut(G)$, then by Definition 2.8, ρ_1 and ρ_2 are adjacent if they are compatible. Thus, by Theorem 2.1, $|Aut(G)| = (p-1)p^{\alpha-1}$ which means there are $(p-1)p^{\alpha-1}$ vertices in $V\left(\Gamma^p_{G\otimes G}\right)$ which can be represented as rows and columns in $B = [b_{ij}]$ for $\Gamma^p_{G\otimes G}$. Thus, we conclude the result as required. ■

Next, the adjacency matrix for this graph when $p = 2$.

Proposition 3.3 Let $G = \langle x \rangle \cong C_{2^5}$ be a finite p-group where $p = 2$. Then, the adjacency matrix of the graph $\Gamma^2_{G\otimes G}$ is:

$$B\left(\Gamma^2_{G\otimes G}\right)=\begin{bmatrix}
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0\\
1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0\\
1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0\\
1&0&0&0&0&0&0&1&1&0&0&0&0&0&0&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0\\
1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0\\
1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0\\
1&0&0&0&0&0&0&1&1&0&0&0&0&0&0&1
\end{bmatrix}$$

proof:
Clearly that $|G| = 32$. By Proposition 2.2(ii), there exist 16 vertices in $V\left(\Gamma^2_{G\otimes G}\right)$. Thus, by referring to Definition 2.10, the vertices of the graph $\Gamma^2_{G\otimes G}$ are the automorphisms of the group G. Therefore, $B = [b_{ij}]$ for $\Gamma^2_{G\otimes G}$ is a square matrix of size 16×16. ■

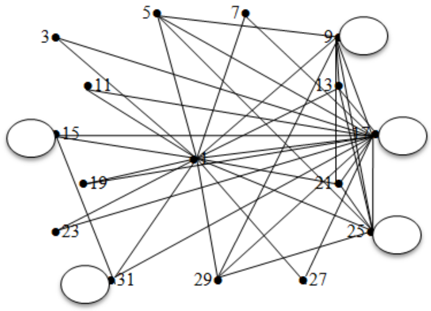

Figure 3.3: The compatible action graph for $\Gamma^2_{G\otimes G}$

In general, when $p = 2$, the adjacency matrix for this graph is presented as follows.

Theorem 3.2 Let $G = \langle x \rangle \cong C_{2^n}$ be a finite cyclic group of 2-power order and $n \geq 2$. Then, the adjacency matrix of the graph $\Gamma^2_{G\otimes G}$ is a square matrix of size $(2^{n-1}) \times (2^{n-1})$.

Proof:
Let $G \cong C_{2^n}$, then $|G| = 2^n, n \geq 2$. Let ρ_1 and ρ_2 be two automorphisms of the set $Aut(G)$, where $\rho_1, \rho_2 \in V\left(\Gamma^2_{G\otimes G}\right)$, then ρ_1 and ρ_2 are adjacent if they are compatible. By Theorem 2.2, $|Aut(G)| = 2^{n-1}$ which means there exist 2^{n-1} vertices in $V\left(\Gamma^2_{G\otimes G}\right)$. Thus, $B = [b_{ij}]$ for $\Gamma^2_{G\otimes G}$ is a square matrix of size $(2^{n-1}) \times (2^{n-1})$. ■

Corollary 3.1 The determinate of the adjacency matrix of the compatible action graph when $G = H$ is zero.

Proof: By Corollary 2.1, the trivial action is always compatible with any other action, then it is an adjacent with any other vertex and the value is one in the first row and the first column of the matrix. Now, let v be any vertex of $V\left(\Gamma^p_{G\otimes G}\right)$, if the order of v is of p-power, then by Theorem 2.3, v is an adjacent with some other vertices of p-power order and the value is one. Otherwise, v is not an adjacent and the value is zero in every row and column of the matrix. Thus, we conclude that the determinate of the matrix is equal to zero. ■

Corollary 3.2 The adjacency matrix of the compatible action graph when $G = H$ is a singular matrix

Proof: It follows from Corollary 3.1. ■

Corollary 3.3 The trace of the adjacency matrix of the compatible action graph when $G = H$ is greater than or equal to one.

Proof: By Definition 2.9, $Tr(A) = a_{11} + a_{22} + \cdots + a_{nn}$. By Corollary 2.1, the trivial action is always compatible with any other action. That is mean, at least there is one compatible pair of actions on the main diagonal of the matrix which is proved the result. ■

Next, the adjacency matrix of the compatible action graph when $G \neq H$.

Proposition 3.4 Let $G = \langle x \rangle \cong C_{3^2}$ and $H = \langle y \rangle \cong C_{3^3}$ be two finite p-groups where $p = 3$. Then, the adjacency matrix of the graph $\Gamma^3_{G \otimes H}$ is:

$$B\left(\Gamma^3_{G\otimes H}\right)=\begin{bmatrix} 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

Proof: Let $G \cong C_{3^2}$, then $|G| = 9$ and let $H \cong C_{3^3}$, then $|G| = 27$. Assume that ρ_1 and ρ_2 be two vertices of $V\left(\Gamma^3_{G\otimes H}\right)$, then ρ_1 and ρ_2 are adjacent if they are compatible. By Theorem 2.1, $|Aut(G)| = 6$ and $|Aut(H)| = 18$. Without loss of generality, if we consider $Aut(G)$ as rows and $Aut(H)$ as columns, then we get a matrix of size 6×18. ■

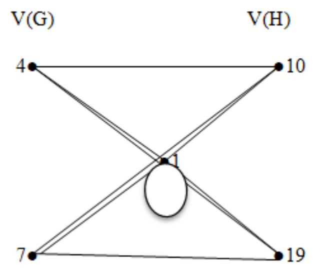

Figure 3.4: The compatible action graph for $\Gamma^3_{G\otimes H}$

Proposition 3.5 Let $G = \langle x \rangle \cong C_{3^3}$ and $H = \langle y \rangle \cong C_{3^2}$ be two finite p-groups where $p = 3$. Then, the adjacency matrix of the graph $\Gamma^3_{G\otimes H}$ is:

$$B\left(\Gamma^3_{G\otimes H}\right)=\begin{bmatrix} 1&1&1&1&1&1\\ 1&0&0&0&0&0\\ 1&0&0&0&0&0\\ 1&0&0&0&0&0\\ 1&0&0&0&0&0\\ 1&0&0&0&0&0\\ 1&0&1&0&1&0\\ 1&0&0&0&0&0\\ 1&0&0&0&0&0\\ 1&0&0&0&0&0\\ 1&0&0&0&0&0\\ 1&0&0&0&0&0\\ 1&0&1&0&1&0\\ 1&0&0&0&0&0\\ 1&0&0&0&0&0\\ 1&0&0&0&0&0\\ 1&0&0&0&0&0\\ 1&0&0&0&0&0 \end{bmatrix}$$

Proof: It follows by Proposition 3.4.

In general, when p is an odd prime and $G \neq H$, the adjacency matrix for this graph is given as follows.

Theorem 3.3: Let $G = \langle x \rangle \cong C_{p^\alpha}$ and $H = \langle y \rangle \cong C_{p^\beta}$ be two finite p-groups where $\alpha, \beta \geq 2$. Then, the adjacency matrix of the graph $\Gamma^p_{G\otimes H}$ is a matrix of size $((p-1)p^{\alpha-1}) \times ((p-1)p^{\beta-1})$.

Proof: Clearly that $|G| = p^\alpha$ and $|H| = p^\beta$ with $\alpha, \beta \geq 2$. By Theorem 2.1, $|Aut(G)| = (p-1)p^{\alpha-1}$ and $|Aut(H)| = (p-1)p^{\beta-1}$. Without loss of generality, let $Aut(G)$ and $Aut(H)$ represented as the rows and columns respectively, then we get a matrix of size $(p-1)p^{\alpha-1} \times (p-1)p^{\beta-1}$ as required. ■

Proposition 3.6 Let $G = \langle x \rangle \cong C_{2^3}$ and $H = \langle y \rangle \cong C_{2^4}$ be two finite p-groups where $p = 2$. Then, the adjacency matrix of the graph $\Gamma^2_{G\otimes H}$ is:

$$B\left(\Gamma^2_{G\otimes H}\right)=\begin{bmatrix} 1&1&1&1&1&1&1&1\\ 1&0&0&1&1&0&0&1\\ 1&1&1&1&1&1&1&1\\ 1&0&0&1&1&0&0&1 \end{bmatrix}$$

Proof: It follows from Theorem 2.2. ■

Proposition 3.7 Let $G = \langle x \rangle \cong C_{2^4}$ and $H = \langle y \rangle \cong C_{2^3}$ be two finite p-groups where $p = 2$. Then, the adjacency matrix of the graph $\Gamma^2_{G\otimes H}$ is:

$$B\left(\Gamma^2_{G\otimes H}\right)=\begin{bmatrix} 1&1&1&1\\ 1&0&1&0\\ 1&0&1&0\\ 1&1&1&1\\ 1&1&1&1\\ 1&0&1&0\\ 1&0&1&0\\ 1&1&1&1 \end{bmatrix}$$

Proof: It follows from Proposition 3.6. ■

In general, when $p = 2$ and $G \neq H$, the adjacency matrix for this graph is given as follows.

Theorem 3.4: Let $G = \langle x \rangle \cong C_{2^m}$ and $H = \langle y \rangle \cong C_{2^n}$ be two finite p-groups where $m, n \geq 2$. Then, the adjacency matrix of the graph $\Gamma^2_{G\otimes H}$ is a matrix of size $(2^{m-1}) \times (2^{n-1})$.

Proof: It follows from Theorem 2.2. ■

4. Conclusions

The adjacency matrix of the graph $\Gamma^{p}_{G\otimes H}$ when $G = H$ and $G \neq H$ for such type of groups has been studied. Meanwhile, two cases have been discussed when $G = H$ which are $p = 2$ and p is an odd prime. In this case, the obtained results shown that, the adjacency matrix of this graph is a square matrix in the two cases. Furthermore, for the case of $G \neq H$, the obtained results shown that the adjacency matrix is not a square matrix. The results can be extended to other type of graph.

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مصفوفة التجاور لبيان التصرفات المتوافقة للزمر *(p-groups)*

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الملخص

في هذا البحث حددنا مصفوفة التجاور لبيان التصرفات المتوافقة للضرب المتواتر غير الابدالي $\Gamma^{p}_{G\otimes H}$ للزمر p-groups عندما تكون $G = H$ و $G \neq H$. في هذا البيان يكون الرأسان متجاوران اذا وفقط اذا كانا متوافقين مع بعضهم البعض.