Semi Ideal on Supra Topological Space

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Abstract

In this paper we introduces the ideal on supra topological space and we shall discuss the properties of this space. In this space we introduces two operators ( )$*^{\mu}$ and ψ_{μ} . A generalized set has also been introduced in this space ( )$*^{\mu}$ and ψ_{μ} .

A generalized set has also been introduced in this space with the help of ψ_{μ} operator.

1. Introduction

The concept of ideal in topological space was first introduced by Kuratowski [5] and Vaidyanathswamy[13]. They also have defined local function in ideal topological space. Further Hamlett and Jankovic in [3] and [4] studied the properties of ideal topological spaces and they have introduced another operator called ψ operator. They have also obtained a new topology from original ideal topological space. Using the local function, they defined a Kuratowski Closure operator in new topological space.

Further, they showed that interior operator of the new topological space can be obtained by ψ -operator. Modak and Bandyopadhyay [9] in 2007 have defined generalized open sets using ψ operator. More recently AI-Omri and Noiri[1] have defined the ideal m-space and introduced two operators as like similar to the local function and ψ operator.

Different types of generalized open sets like semi-open[6], preopen[7], semi per open[2], α-open[11] already are there in literature and these generalized sets have a common property which is closed under arbitrary union. Mashhour et al[8] put all of the sets in a pocket and defined a generalized space which is supra topological space. In this space we have introduced ideal and defined two set operators, μ-local function and $S\psi_{\mu}$ operator. Further we have discussed the properties of these two operators. Finally we have introduced Sμ-codense ideal, and $S\psi_{\mu}$-C set with the help of $S\psi\pi$ operator and discussed the properties of such notions.

2. Preliminaries

In this section we introduced some definitions and results which are relevant of this paper.

Definition 2.1.[5] A nonempty collection **I** of subsets of X is called an ideal on X if :

(i). A ∈ **I** and B ⊂ A implies B ∈ **I** (heredity) ;

(ii). A ∈ **I** and B ∈ **I** implies A ∪ B ∈ **I** (finite additivity).

Definition 2.2.[8] A sub family μ of the power set ℘(X) of a nonempty set X is called a supra topology on X if μ satisfies the following conditions:

1. μ contains ϕ and X,
2. μ is closed under the arbitrary union.

The pair (X, μ) is called a supra topological space. In this respect, the member of μ is called supra open set in (X, μ). The complement of supra open set is called supra closed set.

Definition 2.3.[12] Let (X, μ) be a supra topological space and A⊂ X. Then supra interior and supra closure of A in (X, μ) defined as ∪{ U : U ⊆A, U∈ μ} and ∩{ F : A⊆ F, X – F ∈ μ} respectively.

The supra interior and supra closure of A in (X, μ) are denoted as Int^{μ}(A) and Cl^{μ}(A)[13] respectively. From definition, Int^{μ}(A) is a supra open set and Cl^{μ}(A) is a supra closed set.

Definition 2.4. Let (X, μ) be a supra topological space and M ⊂ X. Then M is said to a supra neighbourhood of a point x of X if for some supra open set U ∈ μ, x ∈ U ⊂ M.

Theorem 2.5. Let (X, μ) be a supra topological space and A⊂ X. Then

(i). $Int\mu$(A) ⊆ A.

(ii). A ∈μ if and only if Int^{μ}(A) = A.

(iii). Cl^{μ}(A) ⊇ A.

(iv). A is a supra closed set if and only if Cl^{μ}(A) = A.

(v). x ∈ Cl^{μ}(A) if and only if every supra open set U_x containing x, U_x∩A ≠ ϕ.

Proof.(i). Proof is obvious from the definition of supra interior.

(ii). Since arbitrary union of supra open sets is again a supra open set, then proof is obvious.

(iii). Proof is obvious from the definition of supra closure.

(iv). If A is a supra closed set, then smallest supra closed set containing A is A. Hence Cl^{μ}(A) = A.

(v). Let x ∈ Cl^{μ}(A). If possible suppose that U_x∩A = ϕ, where U_x is a supra open set containing x. Then A ⊂ (X - U_x) and X - U_x is a supra closed set containing A. Therefore x ∈ (X - U_x), a contradiction. Conversely supposed that U_x∩A ≠ ϕ, for every supra open set U_x containing x. If possible suppose that x ∉ Cl^{μ}(A), then x ∈ X - Cl^{μ}(A). Then there is a U'_x ∈ μ such that U'_x ⊂ (X - Cl^{μ}(A)), i.e., U'_x ⊂ (X -Cl^{μ}(A)) ⊂ (X – A). Hence $U'_x \cap A = \phi$, a contradiction. So x ∈ Cl^{μ}(A).

Theorem2.6. Let (X, μ) be a supra topological space and A⊂ X. Then Int^{μ}(A) = X- Cl^{μ} (X – A).

Proof. Let x ∈ Int^{μ} (A). Then there is U ∈ μ, such that x ∈ U ⊂ A. Hence x ∉ X – U, i.e., x ∉ Cl^{μ} (X – U), since X – U is a supra closed set. So x ∉ Cl^{μ}(X – A) (from Definition2.3., Cl^{μ} (X – A) ⊂ Cl^{μ} (X – U)) and hence x ∈ X – Cl^{μ}(X – A). Conversely suppose that x ∈ X – Cl^{μ}(X – A). So x ∉ Cl^{μ}(X – A), then there is a supra open set U_x containing x, such that U_x ∩(X – A) = ϕ. So U_x ⊂ A. Therefore x ∈ Int^{μ}(A). Hence the result.

3. ( )*μ operator

In [3] and [4] Hamlett and Jankovic have considered the local function in ideal topological space and they have obtained a new topology. In this section we shall introduce similar type of local function in semi supra topological space. Before starting the discussion we shall consider the following concept.

A semi supra topological space (X, Sμ) with an semi ideal **SI** on X is called an Semi ideal supra topological space and denoted as (X, Sμ, **SI**).

At first we define following:

Definition 3.1.Let (X, Sμ, **SI**) be a semi ideal supra topological space. A set operator ( )$*^{\mu}$: ℘(X) → ℘(X), is called the μ-local function of **SI** on X with respect to Sμ, is defined as: $(A)^{*\mu}$(**I**, μ) = { x ∈ X: U ∩A ∉ **I** , for every U ∈ μ(x) }, where μ(x) = { U∈ Sμ: x ∈ U}.

This is simply called μ-local function and simply denoted as $A^{*\mu}$.

We have discussed the properties of μ-local function in following theorem:

Theorem3.2. Let (X, Sμ, **SI**) be an semi ideal supra topological space, and let A, B, A1, A2, ----- Ai,---- be subsets of X. Then

(i). $\phi^{*\mu} = \phi$.

(ii). A ⊂ B implies $A^{*\mu} \subset B^{*\mu}$.

(iii). for another semi ideal **SJ** ⊇ **SI** on X, $A^{*\mu}$ (**SJ**) ⊂ $A^{*\mu}$ (**SI**).

(iv). $A^{*\mu} \subset Cl^{\mu}$(A).

(v). $A^{*\mu}$ is an semi supra closed set.

(vi). $(A^{*\mu})^{*\mu} \subset A^{*\mu}$.

(vii). $A^{*\mu} \cup B^{*\mu} \subset (A \cup B)^{*\mu}$.

(viii). $\cup_i A^{*\mu} \subset (\cup_i A_i)^{*\mu}$.

(ix). $(A \cap B)^{*\mu} \subset A^{*\mu} \cap B^{*\mu}$.

(x). for V∈μ, V ∩ (V ∩ A) $^{*\mu}$ ⊂ V∩$A^{*\mu}$.

(xi). for I∈**SI**, $(A \cup SI)^{*\mu} = A^{*\mu} = (A - SI)^{*\mu}$.

Proof. (i). Proof is obvious from the definition of μ-local function.

(ii). Let x ∈ $A^{*\mu}$. Then for every U ∈μ(x), U ∩A ∉ **SI**. Since U ∩ A⊂ U∩B, then U∩ B ∉ **SI**. This implies that x ∈ $B^{*\mu}$.

(iii). Let x ∈ $A^{*\mu}$(S**J**). Then for every U ∈ μ(x), U ∩A ∉ **SJ** . This implies that U ∩ A ∉ **SI** , so x ∈ $A^{*\mu}$(**SI**). Hence $A^{*\mu}$(**SJ**) ⊂ $A^{*\mu}$(**SI**).

(iv). Let x ∈ $A^{*\mu}$. Then for every U∈ μ(x), U ∩ A ∉ **SI**. This implies that U ∩A ≠ ϕ. Hence x ∈ Cl^{μ}(A).

(v). From definition of semi supra neighbourhood, each semi supra neighbourhood M of x contains a U∈μ(x). Now if A∩M∈**I** then for A ∩U ⊂ A∩M, A ∩U ∈**SI** .

It follows that X - $A^{*\mu}$ is the union of semi supra open sets. We know that the arbitrary union of semi supra open sets is a semi supra open set. So X - $A^{*\mu}$ is an semi supra open set and hence $A^{*\mu}$ is an semi supra closed set.

(vi). From (iv), $(A^{*\mu})^{*\mu}$ ⊂ Cl $(A^{*\mu})$ = $A^{*\mu}$, since $A^{*\mu}$ is an semi supra closed set.

(vii). We know that A ⊂ (A∪B) and B ⊂ (A∪B). Then from (ii), $A^{*\mu} \subset (A \cup B)^{*\mu}$and $B^{*\mu} \subset (A \cup B)^{*\mu}$. Hence $A^{*\mu} \cup B^{*\mu} \subset (A \cup B)^{*\mu}$.

(viii). Proof is obvious from (vii).

(ix). We know that A∩B ⊂A and A∩ B ⊂B, then from (ii), $(A \cap B)^{*\mu} \subset A^{*\mu}$ and $(A \cap B)^{*\mu} \subset B^{*\mu}$. Hence $(A \cap B)^{*\mu} \subset A^{*\mu} \cap B^{*\mu}$.

(x). Since V∩A ⊂ A, then $(V \cap A)^{*\mu} \subset A^{*\mu}$. So V∩$(V \cap A)^{*\mu}$ ⊂ V∩ $A^{*\mu}$.

(xi). Since A ⊂ (A∪SI), then

$A^{*\mu} \subset (A \cup SI)^{*\mu}$--------------(i).

Let x ∈ $(A \cup SI)^{*\mu}$. Then for every U ∈ μ(x), U ∩ (A∪SI) ∉ ***SI***. This implies that U ∩ A ∉ ***SI*** (If possible suppose that U ∩A∈ **SI**. Again U∩ SI ⊂ SI implies U∩ SI∈ **SI**

and hence U ∩(A∪SI) ∈ **SI** , a contradiction). Hence x ∈ $A^{*\mu}$ and

$(A \cup SI)^{*\mu} \subset A^{*\mu}$ -------------------(ii).

From (i) and (ii) we have

$(A \cup SI)^{*\mu} = A^{*\mu}$ -------------------(iii).

Since (A - SI) ⊂ A, then

$(A - SI)^{*\mu} \subset A^{*\mu}$ -------------------(iv).

For reverse inclusion, let x ∈ $A^{*\mu}$. We claim that x ∈ $(A - SI)^{*\mu}$, if not, then there is U∈ μ(x), U ∩(A - SI) ∈ **SI**. Given that I∈**SI**, then SI ∪(U∩(A - SI))∈**SI**. This implies that SI ∪ (U∩A) ∈ **SI**. So, U∩A∈**SI**, a contradiction to the fact that x∈ $A^{*\mu}$. Hence

$A^{*\mu} \subset (A - SI)^{*\mu}$ --------------(v).

From (iv) and (v) we have $A^{*\mu} = (A - SI)^{*\mu}$ -----(vi).

Again from (iii) and (vi), we get $(A \cup SI)^{*\mu} = A^{*\mu} = (A - SI)^{*\mu}$.

Following example shows that $A^{*\mu} \cup B^{*\mu}$ = (A ∪ B)$^{*\mu}$ does not hold in general.

Example3.3. Let X = {a, b, c }, μ = {ϕ,X,{a},{b},{a,b}, {a,c}, {b,c},{c}}.

SI = {ϕ, {c}}. Then semi supra open sets containing 'a' are: X,{a},{a,b}, {a,c}; semi supra open sets containing

'b' are: X,{b},{a,b}, {b,c} ; semi supra open sets containing

'c' are: X,{a,c},{b,c}, Consider A = {a,c} and B = {b,c}, then $A^{*\mu}$= {a} and

$B^{*\mu}$ = {b}. Now (A ∪ B)$^{*\mu}$= $\{a,b\}^{*\mu}$= {a,b,c }. Hence $A^{*\mu}$∪ $A^{*\mu}$ ≠(A ∪ B)*μ .

4. ψ_{μ} operator

In this section we introduce another set operator $S\psi_{\mu}$ in (X, Sμ, **SI**). This operator is, as like similar of ψ operator [10],[3] in ideal topological space.

Definition of $S\psi_{\mu}$operator is as follows:

Definition4.1. Let (X, Sμ, **SI**) be a semi ideal supra topological space. An operator ψ_{μ} : ℘(X) → μ is defined as follows for every A∈℘(X), $S\psi_{\mu}$(A) ={x ∈ X: there exists a U∈ μ(x) such that U – A ∈ **SI**}.

We observe that ψ_{μ}(A) = X – (X – A)*μ.

The behaviors of the operator $S\psi_{\mu}$ has been discussed in the following theorem:

Theorem4.2. Let (X,Sμ, **SI**) be a semi ideal supra topological space.

(i). If A⊆X, then $S\psi_{\mu}$(A) ⊃ Int^{μ}(A).
(ii). If A⊆X, then $S\psi_{\mu}$(A) is semi supra open.
(iii). If A⊆ B, then $S\psi_{\mu}$(A) ⊆ $S\psi_{\mu}$(B).
(iv). If A, B∈ ℘(X), then $S\psi_{\mu}$(A)∪ $S\psi_{\mu}$(B) ⊂ $S\psi_{\mu}$(A∪B).
(v). If A, B∈ ℘(X), then $S\psi_{\mu}$(A∩B) ⊂ $S\psi_{\mu}$(A)∩ $S\psi_{\mu}$(B).
(vi). If U ∈μ, then U ⊂ $S\psi_{\mu}$(U).
(vii). If A⊆ X, then $S\psi_{\mu}$(A) ⊂ $S\psi_{\mu}(\psi_{\mu}$(A)).
(viii). If A⊆ X, then $S\psi_{\mu}$(A) = $S\psi_{\mu}(S\psi_{\mu}$(A)) if and only if $(X - A)^{*\mu}$ $\left((X - A)^{*\mu}\right)^{*\mu}$.
(ix). If A∈ SI, then $S\psi_{\mu}$(A) = X – $X^{*\mu}$.
(x). If A⊆X, I∈**SI**, then $S\psi_{\mu}$(A - SI) = $S\psi_{\mu}$(A).
(xi). If A⊆X, I∈**SI**, then $S\psi_{\mu}$(A∪SI) = $S\psi_{\mu}$ (A).
(xii). If (A – B)∪(B – A) ∈ **SI**, then $S\psi_{\mu}$(A) = $S\psi_{\mu}$(B).

Proof.(i). From definition of $S\psi_{\mu}$ operator, $S\psi_{\mu}$(A) = X – (X – A). Then $S\psi_{\mu}$(A) = X – $(X - A)^{*\mu}$ ⊃ X - Cl^{μ}(X – A), from Theorem3.1(iv). Hence $S\psi_{\mu}$ (A) ⊃ Int^{μ}(A) (using Theorem2.7.).

(ii). Since $(X - A)^{*\mu}$ is a semi supra closed set (from Theorem3.2(v)), then X - (X – A)*μ is a semi supra open set. Hence ψ_{μ}(A) is semi supra open.

(iii). Given that A ⊆ B, then (X – A) ⊇ (X – B). Then from Theorem3.2(ii), $(X - A)^{*\mu}$ ⊇ $(X - B)^{*\mu}$ and hence $S\psi_{\mu}$(A) ⊆ $S\psi_{\mu}$(B).

(iv). Proof is obvious from above property.

(v). Since A ∩ B ⊂ A and A∩ B ⊂ B, then from (iii), $S\psi_{\mu}$(A∩B) ⊂ $S\psi_{\mu}$(A)∩ $S\psi_{\mu}$(B).

(vi). Let U∈ Sμ . Then (X – U) is a semi supra closed set and hence Cl^{μ}(X – U) = (X – U) . This implies that $(X - U)^{*\mu}$ ⊂ Cl^{μ}(X – U) = (X – U). Hence U ⊂ X – $(X - U)^{*\mu}$, so U ⊂ $S\psi_{\mu}$(U).

(vii). From (ii), ψ_{μ}(A) ∈μ . Again from (vi), $S\psi_{\mu}$ (A) ⊂ $\psi_{\mu}(\psi_{\mu}$(A)).

(viii). Let $S\psi_{\mu}$(A) = $S\psi_{\mu}(S\psi_{\mu}$(A)). Then X - $(X - A)^{*\mu}$ = $S\psi_{\mu}$ (X - (X – A)$^{*\mu}$)= X - (X -(X – $(X - A)^{*\mu}$) $^{*\mu}$) = X – (($(X - A)^{*\mu}$) $^{*\mu}$ .This implies that $(X - A)^{*\mu}$= $\left((X - A)^{*\mu}\right)^{*\mu}$. Conversely suppose that $(X - A)^{*\mu}$= $\left((X - A)^{*\mu}\right)^{*\mu}$ hold .

Then X - $(X - A)^{*\mu}$ = X -$\left((X - A)^{*\mu}\right)^{*\mu}$ and X - $(X - A)^{*\mu}$ = X - (X - $(X - \left(X - A\right)^{*\mu}))^{*\mu}$ = X – (X – $S\psi_{\mu}$(A)) $^{*\mu}$. Hence $S\psi_{\mu}$(A) = $S\psi_{\mu}(S\psi_{\mu}$(A)).

(ix). We know that $S\psi_{\mu}$(A) = X – $\left(X - A\right)^{*\mu}$ = X - $X^{*\mu}$ (from Theorem3.2.(xi)).

(x). We know that X- (X – (A – SI)) $^{*\mu}$ = X – ((X – A) ∪ SI) $^{*\mu}$= X – $(X - A)^{*\mu}$ from Theorem3.2.(xi)). So I_1(A - SI) = $S\psi_{\mu}$(A).

(xi). We know that X – $(X - A \cup SI)^{*\mu}$= X – ((X – A) – SI) $^{*\mu}$= X – $(X - A)^{*\mu}$ using the Theorem3.1.(xi)). Thus $S\psi_{\mu}$(A∪SI) = $S\psi_{\mu}$(A).

(xii). Given that (A – B)∪(B – A) ∈**SI**, and let A – B = SI_1, B – A =SI_2. We observe that SI_1 and SI_2∈**SI** by heredity. Also observe that B = (A – SI_1)∪ SI_2. Thus $S\psi_{\mu}$(A) = $S\psi_{\mu}$(A – SI_1) = $S\psi_{\mu}$((A – SI_1) ∪ SI_2) = ψ_{μ}(B).

We know that U ⊂ $S\psi_{\mu}$ (U), for U∈ Sμ. But we give an example of a set A which is not semi supra open set but satisfies A ⊆ $S\psi_{\mu}$(A).

Example4.3. Let X = {a, b, c }, μ = $\{\phi, X, \{a\}, \{a, c\}, \{b, c\}\}$, **SI** = {$\phi$, {c}}. Then for A = $\{a, b\}$, $S\psi_{\mu}$ (A) = X – $\{c\}^{*\mu}$= X – ϕ = X . Here A ⊆ $S\psi_{\mu}$(A), but A is not a semi supra open set.

In the following example we shall show that $S\psi_{\mu}$(A∩B) = $S\psi_{\mu}$(A)∩ $S\psi_{\mu}$(B) does not hold in general.

Example 4.4. Consider the Example3.3. Here we consider A = {a,c} and B = {a,b}, then $S\psi_{\mu}$(A) = X – $\{a, c\}^{*\mu}$ = $X - \{a\}$ = $\{b, c\}$ and $S\psi_{\mu}$ (B) = X – $\{\{b, c\}\}^{*\mu}$ = X – $\{b\}$ = Here we $\{a, c\}$. are not able to define an interior operator with the help of $S\psi_{\mu}$ operator because $S\psi_{\mu}$ (A∩B) ≠ $S\psi_{\mu}$(A)∩ $S\psi_{\mu}$(B) in general.

5. Sμ-codense semi Ideal

In this section we introduce concept in semi ideal supra topological space.

Definition5.1. A semi ideal **SI** in a space (X, Sμ, **SI**) is called Sμ-codense ideal if Sμ∩**SI** ={ϕ}.

Following theorems are related to Sμ-codense ideal.

Theorem5.2. Let (X, Sμ, **SI**) be a semi ideal supra topological space and **SI** is Sμ-codense with Sμ. Then X = $X^{*\mu}$.

Proof. It is obvious that X ⊆ X. For converse, suppose x∈ X but x ∉ $X^{*\mu}$. Then there exists U_x ∈ μ(x) such that U_x ∩X ∈ **SI**. That is U_x∈ **SI**, a contradiction to the fact that Sμ∩**SI** = { ϕ }. Hence X = $X^{*\mu}$.

Theorem5.3. Let (X, Sμ, **SI**) be a semi ideal supra topological space. Then following conditions are equivalent:
(i). Sμ ∩ **SI** = { ϕ }.
(ii). $S\psi_{\mu}(\phi) = \phi$.
(iii). if I ∈SI , then $S\psi_{\mu}$(I) = ϕ.

Proof. (i) ⇒(ii). Given that Sμ ∩ **SI** = { ϕ }, then ψπ(ϕ) = X – (X - ϕ)*μ = X - $X^{*\mu}$= ϕ (by Theorem5.2.).

(ii) ⇒(iii). $S\psi_{\mu}$(I) = X – $\left(X - I\right)^{*\mu}$= X – $X^{*\mu}$ (by Theorem3.2.(xi)) = X – $X^{*\mu}$= ϕ

by Theorem5.2.).

(iii) ⇒(i). Suppose that A ∈ Sμ∩**SI**, then A ∈ **SI** and by (iii), $S\psi_{\mu}$ (A) = ϕ . Since A ∈ Sμ, then by Theorem4.2.(vi) we have A ⊆ $S\psi_{\mu}$(A) = ϕ. Hence Sμ∩**SI** = {ϕ}.

6. ψμ - C sets

Modak and Bandyopadhyay in [9] have introduced a generalized set with the help of $S\psi_{\mu}$ - operator in semi ideal topological space (X, Sμ, **SI**). In this section we shall introduce a set with the help of $S\psi_{\mu}$ in (X, Sμ, **SI**) space. Further we shall discuss the properties of this type of sets.

Definition6.1. Let (X, Sμ, S**I**) be a semi ideal supra topological space. A subset A of X is called a $S\psi_{\mu}$- C set if A ⊆ Cl^{μ} ($S\psi_{\mu}$(A)).

The collection of all $S\psi_{\mu}$- C sets in (X, Sμ, S**I**) is denoted by $S\psi_{\mu}$(X, Sμ).

Theorem 6.2. Let (X, Sμ, S**I**) be an ideal supra topological space. If A∈ Sμ, then A ∈ $S\psi_{\mu}$(X, Sμ).

Proof. From Theorem4.2.(vi) it follows that Sμ ⊆ $S\psi_{\mu}$(X, Sμ).

Now we give an example which shows that the reverse implication is not true.

Example 6.3. Consider the Example4.1. and we get A ∈ $S\psi_{\mu}$(X, Sμ) but A ∉ Sμ.

We give an example which shows that any supra closed in (X, Sμ, **SI**) may not be a $S\psi_{\mu}$- C set.

In the following example, by C(Sμ) we denote the family of all semi supra closed sets in (X, Sμ).

Example6.4. We consider the Example3.3. Here A = $\{b\}$ ∈ C(Sμ). Then $S\psi_{\mu}$(A) = X –$\{a, c\}^{*\mu}$ = X – X = ϕ. Therefore A ∈ C(Sμ) but A ∉ $S\psi_{\mu}$(X, Sμ).

Theorem 6.5. Let {Aα : α ∈ Δ} be a collection of nonempty $S\psi_{\mu}$ - C sets in semi ideal supra topological space (X, Sμ, **SI**), then $\cup_{\alpha\in\Delta} A_{\alpha}$ ∈ $S\psi_{\mu}$ (X, Sμ).

Proof. For each $\alpha \in \Delta$, A_{α} ⊆ $Cl^{\mu}(S\psi_{\mu}(A_{\alpha}))$ ⊆ Clμ($S\psi_{\mu}(\cup_{\alpha\in\Delta} A_{\alpha})$). This implies that $\cup_{\alpha\in\Delta} A_{\alpha}$ ⊆ Clμ($S\psi_{\mu}$ $(\cup_{\alpha\in\Delta} A_{\alpha})$) . Thus $\cup_{\alpha\in\Delta} A_{\alpha}$ ∈ $S\psi_{\mu}$ (X, Sμ).

The following example shows that the intersection of two $S\psi_{\mu}$ - C sets in (X, Sμ, **SI**) may not be a $S\psi_{\mu}$- C.

Example 6.6. Consider the Example4.4. Here we consider A = $\{b, c\}$ and B = $\{a, c\}$, then $S\psi_{\mu}$(A) = X – {a,c}*μ = X – {a} = {b,c,d} and $S\psi_{\mu}$ (A) = X – $\{b, c\}^{*\mu}$ = X – {b} ={a,c}. So A, B ∈ $S\psi_{\mu}$ (X, Sμ), but $S\psi_{\mu}(\{c\})$ = X – {a,b,c} = ϕ, and {b}∉ $S\psi_{\mu}$(X, Sμ).

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شبة المثالي للفضاء التبولوجي الفوقي

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الملخص

في هذا البحث قدمنا المثالي في الفضاء التبولوجي الفوقي وناقش خصائص هذا الفضاء. في هذا الفضاء نقدم مؤثرين هما $( )^{*\mu}$ و ψ_{μ} المجموعة المعممة قدمت في هذا الفضاء. المجموعة المعممة قدمت ايضاً في هذا الفضاء بوساطة المؤثر ψ_{μ}.