The Distribution of Zeros of the Solutions of Linear Homogeneous Differential Equations of the Sixth Order Using Semi-critical Intervals

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Abstract

In this pap, we investigate the distribution of zeros for the solution of linear Homogeneous Differential Equations (LHDE) in the semi-critical intervals ,for the boundary value problems. The method used in this paper is different from [1-3] in which the authors used geometric approach to distribute the zeros of the solutions of LHDE. We used an analytic approach. Moreover we stated the relation between semi-critical intervals for the boundary value problems .

Keywords: linear Differential Equations, distribution of zeros for the solution, boundary value problems, semi-oscillatory interval, semi-critical interval, fundamental normal solution.

1. Introduction

The studies of the distribution of the solution zeros of the LHDE begun in 1960s. Geometric approaches, for LHDE of orders three and four, are commonly used in the literature since they depend on the set of fundamental solutions [1,2]. In [3] the authors considered LHDE of order N. However, their approach is very complicated and requires additional conditions. The authors in [3-9] proposed a different concept to prove the distribution of zeros in which they used the set of the normal fundamental solutions (NFS). Using NFS has the advantage of providing analytical methodology which resulted in important conclusions and easiness to prove them. This study, is an extension of [4-9], where the authors conducted their research paper on fifth order LHDE. In this paper we consider LHDE of sixth order with a boundary conditions of three points (m = 3).

2. Definitions and Characteristics

Consider the following boundary value problem

$$x^{(6)} + \sum_{j=0}^{5} g_j(x)x^{(j)} = 0 \qquad (1)$$

$$x(t_i) = \acute{x}(t_i) = \cdots = x^{(p_i)}(t_i) = 0 \qquad (2)$$

where $\alpha \le t_1 < t_2 < \cdots < t_m < \infty$, m is the number of points $[t_i, i = 1, \dots, m]$, p_i is the number of conditions at the points $t_i, i = 1,2, \dots, m$, $g_j(x)$ are continuous on $[\propto, \beta)$

Problem (1) and (2) is called $\ll (p_1p_2 \dots p_m) - \text{problem} \gg$.

When the point t_1 is fixed, the family of non-trivial solution of the problem $\ll (p_1p_2 \dots p_m) - \text{problem} \gg$ is denoted by $W_{p_1p_2\dots p_m}(t, t_1)$.

Definition 2-1 [5]**:** The interval $[\propto, \beta)$, $(a < \alpha < \beta < \infty)$ is called semi-oscillatory, if any non-trivial solution for equation (1) has no more than five zeros [including multiplicity] in $[\propto, \beta)$. The largest semi-oscillatory interval that begins at the point α is denoted by $[\alpha, r(\alpha))$.

Definition 2-2[6]: The interval $[\propto, \gamma)$ where $\ll (p_1p_2 \dots p_m) - \text{problem} \gg$ has a unique solution is called semi-critical, and the largest semi- critical intervals that begins at the point α is denoted by $[\alpha, r_{p_1p_2\dots p_m}(\alpha))$.

In this research paper, we discuss non- trivial solution of boundary-value problem (1) and (2) in the semi-critical intervals, especially $\ll (231) - \text{problem} \gg$ where the solution has zero of multiplicity 2 at $t = t_1$, and a zero of multiplicity 3 at $t = t_2$, and a zero of multiplicity 1 at $t = t_3$, where $\alpha \le t_1 < t_2 < r_{231}(\alpha) < t_3$.

The first zero after $t = t_2$ is denoted by $r_{231}(\alpha, t_1, t_2)$, it's clearly that

$r_{231}(\alpha) \le r_{231}(\alpha, t_1, t_2)$ and $r_{231}(\alpha) = \inf\ r_{231}(\alpha, t_1, t_2)$. (3)

Generally, the first zero after $t = t_{m-1}$ is denoted by $r_{p_1p_2\dots p_m}(\alpha, t_1, t_2, \dots, t_{m-1})$ for which

$r_{p_1p_2\dots p_m}(\alpha) = \inf r_{p_1p_2\dots p_m}(\alpha, t_1, t_2, \dots, t_{m-1})$ (4)

where $p_1 + p_2 + \cdots + p_m = 6$.

Lemma 2-1 [7]: The function $r_{p_1p_2\dots p_m}(\alpha, t_1, t_2, \dots, t_{m-1})$ (where t_1 is fixed) is continuos from the right (the right limit exists) at the points $t_2, t_3, \dots, t_{m-1}$ in the set $R_{m-1}[\alpha, \infty)$. i.e.

$$\lim_{\substack{t_2 \to t_2^0 \\ t_3 \to t_3^0 \\ \vdots \\ t_{m-1} \to t_{m-1}^0}} r_{p_1p_2\dots p_m}(\alpha, t_1, t_2, \dots, t_{m-1}) = r_{p_1p_2\dots p_m}(\alpha, t_1, t_2^0, t_3^0 \dots, t_{m-1}^0).$$

Lemma 2-2 [7]: The set of fundamental normal solution for equation (1) (i.e. $\{u_j(t, t_1), j = 0,1, \dots, m-1\}$) with respect to t_1 has the following forms

$$u_j^{(i)}(t, t_1) = (t - t_1)^{j-i}\psi_{ij}(t, t_1), \quad i, j = 0,1, \dots, m-1. \qquad (5)$$

where

$$\psi_{ij}(t_1, t_1) = \begin{cases} \frac{u_j^{(i)}(t_1, t_1)}{(j-i)!} & j \ge i \\ 0 & j < i \end{cases} \qquad (6)$$

3. Main Results

In this section we present two theorems for the distribution of zeros for the solutions of LHDEs in the semi-critical intervals.

Theorem 3-1: In the interval $[\alpha, r_{231}(\alpha))$, any non-trivial solution (for the equation (1)) that has a zero at t_1 of multiplicity five cannot have a simple zero to the right of t_1, i.e. $r_{231}(\alpha) \le r_{51}(\alpha)$, when $t_2 \to t_1$.

Proof: First of all we show that the family of non-trivial solution for $\ll (231) - \text{problem} \gg$ at the fixed point t_1 contains at least one solution that becomes a non-trivial solution for $\ll (51) - \text{problem} \gg$ when $t_2 \to t_1$.

$$\lim_{t_2 \to t_1} W_{231}(t, t_1) = W_{51}(t, t_1)$$

From Vallee Poisinee theorem, for each $t_1 \in [\alpha, r(\alpha))$,there exists a semi-oscillatory interval $[t_1, r(t_1))$ [10]. Choose $\varepsilon > 0$, such that $[t_1, t_1 + \varepsilon) \subset [t_1, r(t_1))$.

And let $\{u_0(t,t_1), u_1(t,t_1), \dots, u_5(t,t_1)\}$ be a set of the normal fundamental solutions for (1) with respect to t_1 , i.e.

$$u_j^{(i)}(t_1, t_1) = \begin{cases} 0 , & i \ne j \\ 1 , & i = j \end{cases}$$

Thus, the family of non-trivial solution for the equation (1) can be written as:

$$W(t, t_1) = \sum_{j=p_1}^{5} c_j u_j(t, t_1) \qquad (7)$$

where c_j is an arbitrary constants.

From the boundary condition for $\ll (231) - \text{problem} \gg$ we get the following homogeneous system .

$$\sum_{j=p_1}^{5} c_j u_j^{(k_i)}(t_i, t_1) = 0 \qquad (8)$$

where

$$k_i = 0,1, \dots, p_i - 1 \ ; \ i = 2,3 \ ; \ \sum_{i=1}^{m} p_i = 6$$

A necessary and sufficient condition for the system (8) to have a non-trivial solution (for unknown c_js) is :

$$D(t_1, t_2) = \det.\left(u_j^{(k_i)}(t_i, t_1) : j = 2,3,4,5 \ ; \ k_i = 0,1, \dots, p_i - 1 \ ; \ i = 2,3\right) = 0$$

The rank of the matrix of system (8) is equal to 3 and it's different from zero.

that is

$$\Delta(t_1, t_2) = \begin{vmatrix} u_3(t_2,t_1) & u_4(t_2,t_1) & u_5(t_2,t_1) \\ u_3'(t_2,t_1) & u_4'(t_2,t_1) & u_5'(t_2,t_1) \\ u_3''(t_2,t_1) & u_4''(t_2,t_1) & u_5''(t_2,t_1) \end{vmatrix} \ne 0$$

where $\alpha \le t_1 < t_2 < t_1 + \varepsilon$.

In fact, if $\Delta(t_1, t_2) = 0$, then the homogeneous system has a non-trivial solution $\bar{c}_3, \bar{c}_4$ and $\bar{c}_5$ in $[t_1, t_1 + \varepsilon)$.Thus, the nontrivial solution for the equation (1): $W(t, t_1) = \bar{c}_3 u_3(t, t_1) + \bar{c}_4 u_4(t, t_1) + \bar{c}_5 u_5(t, t_1)$ has six zeros in the $[t_1, t_1 + \varepsilon)$ where $[t_1, t_1 + \varepsilon) \subset [t_1, r(t_1))$. Three of six zeros are at the point t_1 and the other three zeros at the point t_2, this contradicts the concept of semi-oscillatory interval.

In the system (8) ,the first three equation constitute a system of nonhomogeneous equation

$$c_3 u_3(t_2,t_1) + c_4 u_4(t_2,t_1) + c_5 u_5(t_2,t_1) = -c_2 u_2(t_2,t_1)$$
$$c_3 u_3'(t_2,t_1) + c_4 u_4'(t_2,t_1) + c_5 u_3'(t_2,t_1) = -c_2 u_2'(t_2,t_1)$$
$$c_3 u_3''(t_2,t_1) + c_4 u_4''(t_2,t_1) + c_5 u_5''(t_2,t_1) = -c_2 u_2''(t_2,t_1)$$

Using Grammar-method, we find the values of c_3, c_4 and c_5. Note that c_2 is a free parameter depends on t_1 and t_2 that is $c_2 = c_2(t_1, t_2)$ then the general of non-trivial solution for $\ll (231) - \text{problem} \gg$ depends on c_2. i.e.

$$W_{231}(t, t_1) = c_2 u_2(t, t_1) + \frac{\Delta_3(t_1,t_2)}{\Delta(t_1,t_2)} u_3(t, t_1) + \frac{\Delta_4(t_1,t_2)}{\Delta(t_1,t_2)} u_4(t, t_1) + \frac{\Delta_5(t_1,t_2)}{\Delta(t_1,t_2)} u_5(t, t_1) \qquad (9)$$

where $\Delta_i(t_1, t_2)$, $i = 3,4,5$ can be obtained from $\Delta(t_1, t_2)$ replacing $(-c_2 u_2(t,t_1) \quad -c_2 u_2'(t,t_1) \quad -c_2 u_2''(t,t_1))^T$ by first, second and third columns respectively.

From the equations (5) and (9) we find

$$W_{231}(t, t_1) = -c_2 \left(-u_2(t, t_1) + \frac{1}{(t_2 - t_1)} \frac{\alpha(t_1,t_2)}{\delta(t_1,t_2)} u_3(t,t_1) + \frac{1}{(t_2 - t_1)^2} \frac{\beta(t_1,t_2)}{\delta(t_1,t_2)} u_4(t,t_1) + \frac{1}{(t_2 - t_1)^3} \frac{\gamma(t_1,t_2)}{\delta(t_1,t_2)} u_5(t,t_1) \right)$$

where

$$\delta(t_1,t_2) = \psi_{03}(t_2,t_1) \begin{vmatrix} \psi_{14}(t_2,t_1) & \psi_{15}(t_2,t_1) \\ \psi_{24}(t_2,t_1) & \psi_{25}(t_2,t_1) \end{vmatrix} - \psi_{13}(t_2,t_1) \begin{vmatrix} \psi_{04}(t_2,t_1) & \psi_{05}(t_2,t_1) \\ \psi_{24}(t_2,t_1) & \psi_{25}(t_2,t_1) \end{vmatrix} + \psi_{23}(t_2,t_1) \begin{vmatrix} \psi_{04}(t_2,t_1) & \psi_{05}(t_2,t_1) \\ \psi_{14}(t_2,t_1) & \psi_{15}(t_2,t_1) \end{vmatrix}.$$

$$\alpha(t_1,t_2) = \psi_{02}(t_2,t_1) \begin{vmatrix} \psi_{14}(t_2,t_1) & \psi_{15}(t_2,t_1) \\ \psi_{24}(t_2,t_1) & \psi_{25}(t_2,t_1) \end{vmatrix} - \psi_{12}(t_2,t_1) \begin{vmatrix} \psi_{04}(t_2,t_1) & \psi_{05}(t_2,t_1) \\ \psi_{24}(t_2,t_1) & \psi_{25}(t_2,t_1) \end{vmatrix} + \psi_{22}(t_2,t_1) \begin{vmatrix} \psi_{04}(t_2,t_1) & \psi_{05}(t_2,t_1) \\ \psi_{14}(t_2,t_1) & \psi_{15}(t_2,t_1) \end{vmatrix}.$$

$$\beta(t_1,t_2) = \psi_{03}(t_2,t_1) \begin{vmatrix} \psi_{12}(t_2,t_1) & \psi_{15}(t_2,t_1) \\ \psi_{22}(t_2,t_1) & \psi_{25}(t_2,t_1) \end{vmatrix} - \psi_{13}(t_2,t_1) \begin{vmatrix} \psi_{02}(t_2,t_1) & \psi_{05}(t_2,t_1) \\ \psi_{22}(t_2,t_1) & \psi_{25}(t_2,t_1) \end{vmatrix} + \psi_{23}(t_2,t_1) \begin{vmatrix} \psi_{02}(t_2,t_1) & \psi_{05}(t_2,t_1) \\ \psi_{12}(t_2,t_1) & \psi_{15}(t_2,t_1) \end{vmatrix}.$$

$$\gamma(t_1,t_2) = \psi_{03}(t_2,t_1) \begin{vmatrix} \psi_{14}(t_2,t_1) & \psi_{12}(t_2,t_1) \\ \psi_{24}(t_2,t_1) & \psi_{22}(t_2,t_1) \end{vmatrix} - \psi_{13}(t_2,t_1) \begin{vmatrix} \psi_{04}(t_2,t_1) & \psi_{02}(t_2,t_1) \\ \psi_{24}(t_2,t_1) & \psi_{22}(t_2,t_1) \end{vmatrix} + \psi_{23}(t_2,t_1) \begin{vmatrix} \psi_{04}(t_2,t_1) & \psi_{02}(t_2,t_1) \\ \psi_{14}(t_2,t_1) & \psi_{12}(t_2,t_1) \end{vmatrix}.$$

Since c_2 is an arbitrary constant, we assume that $c_2(t_2,t_1) = -(t_2 - t_1)^3$. By taking the limit of both sides when $t_2 \to t_1$ we obtain ,

$$\lim_{t_2\to t_1} W_{231}(t,t_1) = c(t_1)u_5(t,t_1) \quad (10)$$

Where $c(t_1) = \frac{\gamma(t_1,t_1)}{\delta(t_1,t_1)}$

And from equation (6) we find $\gamma(t_1,t_1) = \frac{1}{144}$, $\delta(t_1,t_1) = \frac{1}{8640}$. By substituting in equation (10), we find

$$\lim_{t_2\to t_1} W_{231}(t,t_1) = 60u_5(t,t_1) \quad (11)$$

Thus we proved that the family of non-trivial solution $\ll (231) - \text{problem} \gg$ contains a solution that becomes a solution for $\ll (51) - \text{problem} \gg$ when $t_2 \to t_1$.

By the lemma (1) The function $r_{231}(\alpha,t_1,t_2)$ is continuos from the right, then we get the following inequality

$$\inf_{\alpha\le t_1<t_2<r_{231}(\alpha)} r_{231}(\alpha,t_1,t_2) \le \inf_{\alpha\le t_1<r_{231}(\alpha)} r_{231}(\alpha,t_1) \quad (12)$$

where

$$\lim_{t_2\to t_1} r_{231}(\alpha,t_1,t_2) = r_{231}(\alpha,t_1)$$

From equations (3) and (11), we find

$$\inf_{\alpha\le t_1<t_2<r_{231}(\alpha)} r_{231}(\alpha,t_1,t_2) = r_{231}(\alpha) \quad (13)$$

$$\inf_{\alpha\le t_1<r_{231}(\alpha)} r_{231}(\alpha,t_1) = r_{51}(\alpha) \quad (14)$$

And from (12), (13) and (14), we get $r_{231}(\alpha) \le r_{51}(\alpha)$ ■

Theorem 3-2: In the interval $[\alpha, r_{321}(\alpha))$,any non-trivial solution (for the equation (1)) that has a zero at t_1 of multiplicity five cannot have a simple zero to the right of t_1 i.e. $r_{321}(\alpha) \le r_{51}(\alpha)$, when $t_2 \to t_1$.

Proof: From Vallee Poisinee theorem, for each $t_1 \in [\alpha, r(\alpha))$, there exists a semi-oscillatory interval $[t_1, r(t_1))$ [10]. Choose $\varepsilon > 0$, such that $[t_1, t_1 + \varepsilon) \subset [t_1, r(t_1))$.

And let $\{u_0(t,t_1), u_1(t,t_1), \dots, u_5(t,t_1)\}$ be a set of the normal fundamental solutions for (1) with respect t_1 .

Thus, the family of non-trivial solution for the equation (1) can be written as :

$$W(t,t_1) = \sum_{j=p_1}^{5} c_j u_j(t,t_1) \quad (15)$$

where c_j is an arbitrary constants.

From the boundary condition for $\ll (321) - \text{problem} \gg$ we get the following homogeneous system.

$$\sum_{j=p_1}^{5} c_j u_j^{(k_i)}(t_i,t_1) = 0 \quad (16)$$

where

$$k_i = 0,1,\dots,p_i - 1; \quad i = 2,3\,; \quad \sum_{I=1}^{m} p_i = 6$$

A necessary and sufficient condition for the system (16) to have a non-trivial solution (for unknown c_j's) is:

$$D(t_1,t_2) = \det.\left(u_j^{(k_i)}(t_i,t_1): j = 3,4,5\,;\ k_i = 0,1,\dots,p_i - 1\,;\ i = 2,3\right) = 0$$

The rank of the matrix of system (16) is equal to 2 and it's different from zero.

That is

$$\Delta(t_1,t_2) = \begin{vmatrix} u_4(t_2,t_1) & u_5(t_2,t_1) \\ \acute{u}_4(t_2,t_1) & \acute{u}_5(t_2,t_1) \end{vmatrix} \ne 0$$

where $\alpha \le t_1 < t_2 < t_1 + \varepsilon$.

In fact, if $\Delta(t_1,t_2) = 0$ then the homogeneous system has a non-trivial solution $\bar{c}_4$ and $\bar{c}_5$ in $[t_1, t_1 + \varepsilon)$. Thus, the nontrivial solution for the equation (1), $W(t,t_1) = \bar{c}_4 u_4(t,t_1) + \bar{c}_5 u_5(t,t_1)$ has six zeros in the semi-oscillatory interval $[t_1, t_1 + \varepsilon)$ where $[t_1, t_1 + \varepsilon) \subset [t_1, r(t_1))$ four of six zeros are at the point t_1 and the other two zeros at the point t_2, this contradicts the concept of semi-oscillatory interval.

In the system (16), the first two equations constitute a system of nonhomogeneous equation

$$c_4u_4(t_2,t_1) + c_5u_5(t_2,t_1) = -c_3u_3(t_2,t_1)$$
$$c_4\acute{u}_3(t_2,t_1) + c_5\acute{u}_3(t_2,t_1) = -c_3\acute{u}_3(t_2,t_1)$$

Using Grammar-method, we find the values of c_4 and c_5. Note that c_3 is a free parameter depends on t_1 and t_2 that is $c_3 = c_3(t_1,t_2)$ then the family of non-trivial solution for $\ll (321) - \text{problem} \gg$ depends on c_3. i.e.

$$W_{321}(t,t_1) = c_3u_3(t,t_1) + \frac{\Delta_4(t_1,t_2)}{\Delta(t_1,t_2)}u_4(t,t_1) + \frac{\Delta_5(t_1,t_2)}{\Delta(t_1,t_2)}u_5(t,t_1) \quad (17)$$

where $\Delta_i(t_1,t_2)$, $i = 4,5$ can be obtained from $\Delta(t_1,t_2)$ replacing $(-c_3u_2(t,t_1) \quad -c_2\acute{u}_3(t,t_1))^T$ by first and second columns respectively.

From the equations (5) and (17) we find

$$W_{231}(t,t_1) = -c_3\left(-u_3(t,t_1) + \frac{1}{t_2-t_1}\frac{\alpha(t_1,t_2)}{\delta(t_1,t_2)}u_4(t,t_1) + \frac{1}{(t_2-t_1)^2}\frac{\beta(t_1,t_2)}{\delta(t_1,t_2)}u_4(t,t_1)\right)$$

where

$$\delta(t_1,t_2) = \psi_{04}(t_2,t_1)\psi_{15}(t_2,t_1) - \psi_{14}(t_2,t_1)\psi_{05}(t_2,t_1)$$
$$\alpha(t_1,t_2) = \psi_{03}(t_2,t_1)\psi_{15}(t_2,t_1) - \psi_{13}(t_2,t_1)\psi_{05}(t_2,t_1)$$
$$\beta(t_1,t_2) = \psi_{04}(t_2,t_1)\psi_{13}(t_2,t_1) - \psi_{14}(t_2,t_1)\psi_{03}(t_2,t_1)$$

Since c_3 is an arbitrary constant, we assume that $c_3(t_2,t_1) = (t_2 - t_1)^2$. By taking the limit of both sides when $t_2 \to t_1$ we obtain,

$$\lim_{t_2\to t_1} W_{321}(t,t_1) = c(t_1)u_5(t,t_1) \quad (18)$$

where $c(t_1) = -\frac{\beta(t_1,t_1)}{\delta(t_1,t_1)}$.

And from equation (6) we find $\beta(t_1,t_1) = -\frac{1}{144}$ and $\delta(t_1,t_1) = \frac{1}{2880}$ By substituting in equation (18), we can find

$$\lim_{t_2\to t_1} W_{321}(t,t_1) = 20u_5(t,t_1) \quad (19)$$

Thus we proved that the family of non-trivial solution $\ll (321) - \text{problem} \gg$ contains a solution that becomes a solution for $\ll (51) - \text{problem} \gg$ when $t_2 \to t_1$.

By the lemma (1) The function $r_{321}(\alpha,t_1,t_2)$ is continuos from the right, then we get the following inequality

$$\underbrace{\inf}_{\alpha\le t_1<t_2<r_{321}(\alpha)} r_{321}(\alpha,t_1,t_2) \le \underbrace{\inf}_{\alpha\le t_1<r_{321}(\alpha)} r_{321}(\alpha,t_1) \quad (20)$$

where

$$\lim_{t_2\to t_1} r_{321}(\alpha,t_1,t_2) = r_{321}(\alpha,t_1)$$

From equations (4) and (19), we find

$$\underbrace{\inf}_{\alpha\le t_1<t_2<r_{321}(\alpha)} r_{321}(\alpha,t_1,t_2) = r_{321}(\alpha) \quad (21)$$

$$\underbrace{\inf}_{\alpha\le t_1<r_{321}(\alpha)} r_{321}(\alpha,t_1) = r_{51}(\alpha) \quad (22)$$

And from (20),(21)and (22), we get $r_{321}(\alpha) \le r_{51}(\alpha)$ ■

Future Work:

This study may be extended in two different ways, once by considering the case where $r_{141}(\alpha) \le r_{51}(\alpha)$ and $r_{411}(\alpha) \le r_{51}(\alpha)$ and by considering boundary conditions with higher number of points.

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توزيع اصفار الحلول للمعادلات التفاضلية الخطية المتجانسة من الرتبة السادسة باستخدام الفترات دون الحرجة

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قسم الرياضيات ، كلية التربية للعلوم الصرفة ، جامعة كركوك ، كركوك ، العراق

الملخص

في هذا البحث درسنا توزيع الاصفار لحلول المعادلات التفاضلية الخطية المتجانسة في الفترات شبه الحرجة لمسائل القيم الحدودية . الطريقة المتبعة في هذا البحث يختلف عن [1−3] والذي استخدم المؤلفون النهج الهندسي لتوزيع اصفار الحلول للمعادلات التفاضلية الخطية المتجانسة. اما نحن فقد استخدمنا المنهج التحليلي لتوزيع اصفار الحلول للمعادلات التفاضلية من الرتبة السادسة. بالإضافة على ذلك بينا العلاقة بين الفترات شبه الحرجة لمسائل القيم الحدودية.