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On Regular Semi Supra Open set and Regular Semi Supra Continuity

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ABSTRACT

The aim of this paper is to present the concepts of bi^i-supra continuous function, bi^i-semi supra continuous function, bi^i-regular open set and bi^i-regular semi supra open set in bi^i-supra topological space. We studied relations between these functions and check up from some of its characteristics.

1. Introduction

In 1963 Kelly [1] presented the concept of bi - topological space which was called a set "equipped with two topologies a bi - topological space and is denoted by ($\mathfrak{X}$, S$\mathfrak{J}$, P$\mathfrak{J}$) where $\mathfrak{J}_1, \mathfrak{J}_2$ are two topologies defined on $\mathfrak{X}$. In 1983 Mashhoure, M. S. Allam. A. A. Mohamoud F.S. and Khedr, F.H.[3] presented the supra topological space . In 2012 Jamunarani, R. and Jeyanthi, P., [8] present the Regular sets in generalized topological spaces. New concepts after we introduced the new definitions called bi^i-supra topological spaces on the topological space ($\mathfrak{X}$, $\mathfrak{J}$) and comparable between bi^i-supra topological space which is induced from bi-topological spaces $(\mathfrak{X}, \mathfrak{J}_i, \mathfrak{J}_j)$ where i,j ∈ {1,2} such that i ≠ j and our new definitions bi^i-supra topological space.

2. Bi-Supra Topological Space

Definition 2.1:

Let $\mathfrak{X}$ be a non-empty set. Let S$\mathfrak{J}$ is the set of all semi opn subsets of $\mathfrak{X}$ (for short S($\mathfrak{X}$) **[6]** , and Let P$\mathfrak{J}$ be the set of all pre-open subsets of $\mathfrak{X}$ (for short Po($\mathfrak{X}$)) **[3].** then we say about ($\mathfrak{X}$, S$\mathfrak{J}$, P$\mathfrak{J}$) is a bi-supra topological space. where each of ($\mathfrak{X}$, S$\mathfrak{J}$) and ($\mathfrak{X}$, P$\mathfrak{J}$) are supra topological spaces .

Definition 2.2 [3]:

A subfamily $\mathfrak{W}$ of a family of all subset of non empty set $\mathfrak{X}$ is said to be a supra topology on $\mathfrak{X}$ if :

1.,$\varphi \in \mathfrak{J}$.

2. If $\mathfrak{K}_i \in \mathfrak{W}$, ∀ i ∈ I ⟹ ⋃ $\mathfrak{K}_i \in \mathfrak{W}$, where I is any index set. ($\mathfrak{X}$, $\mathfrak{W}$) its called a supra topological spaces. The elements of $\mathfrak{W}$ are called supra opn sets in ($\mathfrak{X}$, $\mathfrak{W}$) and the complement of a supra opn sets is called a supra closed sets.

Definition 2.3:

A subset $\mathfrak{K}$ of a supra topological space ($\mathfrak{X}$, $\mathfrak{W}$) be called:

1. semi supra open set if $\mathfrak{K} \subseteq cl_{\mathfrak{W}}(int_{\mathfrak{W}}(\mathfrak{K}))$, the complement of semi supra opn set is said to be semi supra closed set **[5].**

2. pre supra open set if $\mathfrak{K} \subseteq int_{\mathfrak{W}}(cl_{\mathfrak{W}}(\mathfrak{K}))$, the complement of pre supra open set is said to be pre supra closed set **[1].**

3. $\aleph$-supra open set if $\mathfrak{K} \subseteq int_{\mathfrak{M}}(cl_{\mathfrak{M}}(int_{\mathfrak{M}}(\mathfrak{K})))$, the complement of $\aleph$-supra open set is said to be $\aleph$-supra closed set. **[2]**
4. $\mathfrak{P}$ -supra open if $\mathfrak{K} \subseteq (cl_{\mathfrak{M}}(int_{\mathfrak{M}}(cl_{\mathfrak{M}}(\mathfrak{K}))$, the complement of $\mathfrak{P}$ -supra open set is said to be $\mathfrak{P}$ – supra closed set. **[4]**
5. regular supra open if $\mathfrak{K} = int_{\mathfrak{M}}(cl_{\mathfrak{M}}(\mathfrak{K}))$, the complement of regular supra open set is said to be regular supra closed set **[7].**

3. Regular Semi Supra Open in Bi- Supra Topological Space

Definition 3.1:
Let $(\mathfrak{X}, So(\mathfrak{X}), Po(\mathfrak{X}))$ be a bi^i- supra topological space, a subset $\mathfrak{K}$ of $\mathfrak{X}$ is regular semi supra open set if $\mathfrak{K} = int_{s\mathfrak{M}}(cl_{p\mathfrak{M}}(\mathfrak{K}))$ and denoted by $\mathcal{RSO}(\mathfrak{X})$.

Now we introduce some remarks which are needed in our work

Remark 3.2:
1. Finite union of bi^i – regular supra open sets is not necessary to be a bi^i –regular supra open sets.
2. Finite intersection of bi^i-regular supra open sets is not necessary to be a bi^i –regular supra opn sets.
This will be shown by the example:
Let $\mathfrak{X} = \{ 1, 2 ,3 ,4 \}$
$\mathfrak{M} = \{\varphi, \mathfrak{X},\{1\},\{2\},\{4\},\{1,2\},\{1,3,4\},\{1,2,4\},\{1,2,3\},\{1,,4\},\{2,4\}\}$
$\mathfrak{M}^c = \{\varphi, \mathfrak{X},\{2,3,4\},\{1,3,4\},\{3,4\},\{2\},\{3\},\{4\},\{1,2,3\},\{2,3\},\{1,3\}\}$
bi^i-supra open $= \{\varphi, \mathfrak{X},\{1\},\{3\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,3,4\},\{1,2,4\},\{2,3,4\}\}$
$(bi^i - \text{supra})^c = \{\varphi, \mathfrak{X},\{2,3,4\},\{1,2,4\},\{1,2\},\{1,4\},\{1,2\},\{4\},\{3\},\{2\},\{1\}\}$
bi^i-Regular supra open $= \{ \varphi, \mathfrak{X},\{1\},\{3\},\{1,2,4\},\{2,3,4\}\}$
Note that $\{1\},\{3\}$ are bi^i-regular supra open but $\{1\} \cup \{3\} = \{1,3\}$ is not bi^i-regular supra open
And so $\{2,3,4\},\{1,2,4\}$ are bi^i-regular supra but $\{2,3,4\} \cap \{1,2,4\}=\{2,4\}$ is not bi^i-regular supra open.

Remark 3.3:
1. A finite union of $\mathcal{RSO}(\mathfrak{X})$ is not necessary to be $\mathcal{RSO}(\mathfrak{X})$.
2. A finite intersection of $\mathcal{RSO}(\mathfrak{X})$ is not necessary to be $\mathcal{RSO}(\mathfrak{X})$ open sets
It will be shown in the following example:
Let $\mathfrak{X} =\{1,2,3,4\}$
$\mathfrak{M} =\{\varphi, \mathfrak{X},\{1\},\{2,3\},\{1,2,3\},\{1,3,4\},\{3,4\}\}$
$\mathfrak{M}^c = \{ \varphi, \mathfrak{X},\{2,3,4\},\{1,4\},\{4\},\{2\},\{1,2\}\}$
$So(\mathfrak{X}) = \{\varphi, \mathfrak{X},\{1\},\{2,3\},\{3,4\},\{1,2,3\}\{1,3,4\},\{2,3,4\}\}$
$Po(\mathfrak{X}) = \{\varphi, \mathfrak{X},\{1\},\{3\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\}\}$
bi^i-supra $= \{\varphi, \mathfrak{X},\{1\},\{2,3\},\{3,4\},\{1,2,3\}\{1,3,4\},\{2,3,4\},\{3\},\{1,3\},\{1,2,4\}\}$
$(bi^i - \text{supra})^c = \{\varphi, \mathfrak{X},\{2,3,4\},\{1,4\},\{1,2\},\{4\},\{2\},\{1\},\{1,2,4\},\{2,4\},\{3\}\}$
bi^i-$So(\mathfrak{X}) = \{\varphi, \mathfrak{X},\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\}\}$
bi^i-$Po(\mathfrak{X}) = \{\varphi, \mathfrak{X},\{1\},\{3\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\}\}$
$(bi^i - Po(\mathfrak{X}))^c = \{ \varphi, \mathfrak{X},\{2,3,4\},\{1,2,4\},\{2,4\},\{1,4\},\{1,2\},\{4\},\{3\},\{2\},\{1\}\}$
$bi^i - \mathcal{RSO}(\mathfrak{X}) = \{ \varphi, \mathfrak{X},\{1\},\{3\},\{1,2\},\{1,2,4\},\{2,3,4\}\}$
we note $\{2,3,4\}$ and $\{1,2,4\}$ are $bi^i - \mathcal{RSO}(\mathfrak{X})$ but $\{2,4\}$ is not belong to $bi^i - \mathcal{RSO}(\mathfrak{X})$. As that respect to the union we find $\{1,2\}$ and $\{3\}$ belong to $bi^i - \mathcal{RSO}(\mathfrak{X})$, but $\{1,2,3\}$ is not belong to $bi^i - \mathcal{RSO}(\mathfrak{X})$.

Definition 3.4:
A subset $\mathfrak{K}$ of a space $(\mathfrak{X}, So(\mathfrak{X}), P(\mathfrak{X}))$ is said to be
1. $\aleph$-$So(\mathfrak{X})$-supra open if $\mathfrak{K} \subseteq int_{s\mathfrak{M}}(cl_{s\mathfrak{M}}(int_{s\mathfrak{M}}(\mathfrak{K}))$.
2. $\mathfrak{P}$- $So(\mathfrak{X})$-supra open if $\mathfrak{K} \subseteq cl_{s\mathfrak{M}}(int_{s\mathfrak{M}}(cl_{s\mathfrak{M}}(\mathfrak{K})))$.

Lemma 3.5
Every $\mathcal{RSO}(\mathfrak{X})$ and $\mathfrak{P}$-$So(\mathfrak{X})$-supra open and $\aleph$-$So(\mathfrak{X})$ are independent.

Proof
Directly from definition.

Definition 3.6:
A subset $\mathcal{S}$ of a bi^i- supra topological space is called bi^i-supra clopen set if it is both bi^i-supra open and bi^i- supra closed set in $(\mathfrak{X}, So(\mathfrak{X}), Po(\mathfrak{X}))$.

Corollary 3.7:
Let $\mathfrak{K}$ be a bi^i- regular supra clopen set. Suppose that $\mathcal{F}$ is a bi^i-supra closed set. Then $\mathfrak{K} \cap \mathcal{F}$ is a bi^i-regular semi supra closed sets.

Proof:
We have the $\mathfrak{K} \cap \mathcal{F}$ be supra closed in. Hence $cl_{\mathfrak{M}}$-$(\mathfrak{K} \cap \mathcal{F}) = \mathfrak{K} \cap \mathcal{F}$ in $\mathfrak{K}$. Let $\mathfrak{K} \cap \mathcal{F} \subset \mathcal{U}$, where $\mathcal{U}$ is regular supra open in $\mathfrak{K}$. Hence $\mathfrak{K} \cap \mathcal{F}$ is a regular semi supra closed set in the regular supra closed. Apply Theorem $\mathfrak{B} \subset \mathfrak{K} \subset \mathfrak{X}$.

Theorem 3.8:
suppose that $\mathfrak{B} \subset \mathfrak{K} \subset \mathfrak{X}$, $\mathfrak{B}$ is a bi^i-regular semi supra closed set relative to $\mathfrak{K}$ and that $\mathfrak{K}$ is a bi^i-regular supra clopen subset of $\mathfrak{X}$. Then $\mathfrak{B}$ is bi^i-regular semi supra closed relative to.

Proof
Let $\mathfrak{B} \subset \mathcal{U}$ and let $\mathcal{U}$ be bi^i-regular supra open. We have $\mathfrak{B} \subset (\mathfrak{K} \cap \mathcal{U})$. But $\mathfrak{B}$ is a bi^i-regular supra closed set relative to $\mathfrak{K}$. Hence $bi^i - cl_{\mathfrak{K}}(\mathfrak{B}) \subset (\mathfrak{K} \cap \mathcal{U})$ …1,
Note that $\mathfrak{K} \cap \mathcal{U}$ is regular open in $\mathfrak{K}$. But $bi^i - cl_{\mathfrak{K}}(\mathfrak{B}) = bi^i$-$cl_{\mathfrak{M}}(\mathfrak{B}) \cap \mathfrak{K}$ ….2,
From (1) and (2) it follows that
$(\mathfrak{K} \cap bi^i$-$cl_{\mathfrak{M}}(\mathfrak{B})) \subset (\mathfrak{K} \cap \mathcal{U})$. Consequently $\mathfrak{K} \cap bi^i$-$cl_{\mathfrak{M}}(\mathfrak{B}) \subset \mathcal{U}$.
Hence $\mathfrak{K} \cap (bi^i$-$cl_{\mathfrak{M}}(\mathfrak{B}) \cup ((bi^i - cl_{\mathfrak{M}}(\mathfrak{B}))^c \subset (\mathcal{U} \cup (bi^i - cl_{\mathfrak{M}}(\mathfrak{B}))^c)$.

That is $(\mathfrak{K} \cap \mathfrak{X}) \subset (\mathcal{U} \cup (bi^i - cl_{\mathfrak{W}}(\mathfrak{B}))^c)$
so $\mathfrak{K} \subset (\mathcal{U} \cup (bi^i - cl_{\mathfrak{W}}(\mathfrak{B}))^c)$ = G .. (3)
But then G be a supra open set. since $\mathfrak{K}$ be a bi^i-regular-supra closed in $\mathfrak{X}$,
from (3) we have
$(bi^i - cl_{\mathfrak{W}}(\mathfrak{K}))^c \subset (\mathcal{U} \cup (bi^i - cl_{\mathfrak{W}}(\mathfrak{B}))^c)$ = G …(4)
But $bi^i - cl_{\mathfrak{W}}(\mathfrak{B}) \subset bi^i - cl_{\mathfrak{W}}(\mathfrak{K})$ …(5)
From (4) and (5) we have(cl $\mathfrak{B}$) $\subset$ $(\mathcal{U} \cup (bi^i - cl_{\mathfrak{W}}(\mathfrak{B}))^c)$.
Hence $bi^i - cl_{\mathfrak{W}}(\mathfrak{B}) \subset \mathcal{U}$ because $bi^i - cl_{\mathfrak{W}}(\mathfrak{B}) \cap (bi^i - cl_{\mathfrak{W}}(\mathfrak{B}))^c =$.
This implies that $\mathfrak{B}$ is regular semi supra closed relative to .

lemma 3.9:

If $\mathfrak{K}$ is bi^i-regular open and bi^i-regular semi closed then $\mathfrak{K}$ is bi^i-semi clopen.

4. Continuity in bi^i-Supra Topological Space

Definition 4.1

A function
$f: (\mathfrak{X}, So(\mathfrak{X}), Po(\mathfrak{X})) \rightarrow (\mathcal{Y}, So(\mathcal{Y}), Po(\mathcal{Y}))$is called:

1. bi^i-Semi supra continuous function if the inverse image for any semi- supra open set in $\mathcal{Y}$ is semi-supra open set in $\mathfrak{X}$.
2. bi^i- Regular supra continuous function if the inverse image for any regular - supra open set in $\mathcal{Y}$ is regular- supra open set in $\mathfrak{X}$.
3. bi^i-regular semi supra-continuous function if the inverese image for any regular semi – supra open set in $\mathcal{Y}$ is regular semi- supra open set in $\mathfrak{X}$.

Theorem 4.2:

Let $f: (\mathfrak{X}, So(\mathfrak{X}), Po(\mathfrak{X})) \rightarrow (\mathcal{Y}, So(\mathcal{Y}), Po(\mathcal{Y}))$, be a bi^i-supra function then the following are equivalents:

1. f be a bi^i-$\mathcal{R}$So($\mathfrak{X}$) supra continuous.
2. inverse image for every bi^i-regular supra open subset in $\mathcal{Y}$ is bi^i-regular semi supra open in .
3. inverese image of every bi^i-regular supra closed subset in $\mathcal{Y}$ be a bi^i-regular semi supra closed in $\mathcal{Y}$

proof

suppose 1) to prove 2)
since f is a bi^i- $\mathcal{R}$So($\mathfrak{X}$) supra continuous then inverese image for every bi^i-regular supra opn subset in $\mathcal{Y}$ be bi^i-regular semi supra opn in $\mathcal{Y}$.
Thus, the condition is fulfilled.
suppose 2) to prove 3)
if we take the complement to the two condition we get
the inverese image for every bi^i-regular supra closed subset of $\mathcal{Y}$ be a bi^i-regular semi supra closed of $\mathcal{Y}$.
It is the three condition .
suppose 3) to prove 1)
let $\mathcal{G}$ be a bi^i-regular supra closed subset of $\mathcal{Y}$
$\therefore \mathcal{G}^c$ be a bi^i-regular supra open subset of $\mathcal{Y}$
$\because \mathcal{G}$ be a bi^i-regular semi supra closed subset of $\mathfrak{X}$
$\therefore \mathcal{G}^c$ be a bi^i-regular semi supra open subset of $\mathfrak{X}$.
Thus, the first condition is fulfilled.

Remark 4.3:

Every bi^i-regular supra continuous function is a bi^i-supra regular semi continuous function but the converse is not true.

Proof:

Let f:($\mathfrak{X}$,So($\mathfrak{X}$), Po($\mathfrak{X}$)) → ($\mathcal{Y}$, So($\mathcal{Y}$), Po($\mathcal{Y}$)) be a bi^i-regular supra continuous function and $\mathfrak{K}$ is regular supra open set in $\mathcal{Y}$.
Then there exist $\mathfrak{B}$ in $\mathfrak{X}$ s.t $f(\mathfrak{B}) = \mathfrak{K}$.
$\because f$ is bi^i-regular semi supra continuous
$\therefore f(\mathfrak{B}) = \mathfrak{K}$ and $f^{-1}(\mathfrak{K}) =$.
For that $f^{-1}(\mathfrak{K})$ be a regular supra open set in $\mathfrak{X}$. Hence supra regular semi open set in $\mathfrak{X}$. Hence f is bi^i-regular semi supra continuous function.
The converse for the previous theory is an incorrect, this will be shown by example.

Example 4.4:

Let $\mathfrak{X}$ ={1,2,3}
$\mathfrak{W} = \{\varphi, \mathfrak{X}, \{1\},\{2\},\{1,2\},\{1,3\},\{2,3\}\}$
$\mathcal{Y}$ ={a, b, c}
$\mathfrak{W} = \{\varphi, \mathcal{Y}$,{a},{$b$},{a ,b},{a, c},{b ,c}}
$f(1)$ = a, $f(2)$ = b , $f(3)$ = c .
We find {a, b} is bi^i-regular semi supra continuouss function but not bi^i-regular supra continuous function.

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حول المجموعات الشبه الفوقية المفتوحة المنتظمة والدوال الشبه الفوقية المستمرة في الفضاءات التبولوجية الفوقية

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الملخص

الغرض من هذا البحث هو تقديم المفاهيم .الدالة المستمرة الفوقية و الدالة المستمرة الشبه الفوقية والمجموعة المفتوحة المنتظمة والمجموعة المفتوحة شبه الفوقية المنتظمة في فضاء ثنائي التبولوجي الفوقي من النمط i العلاقات بين هذه الدوال قد درست وتحرينا بعض الخواص لها.