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Nearly Quasi 2-Absorbing submodule

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Abstract

All rings in this note are commutative rings with identity, and all R-modules are left unitary. "A proper submodule E of an R-module X is called nearly quasi prime submodule, if whenever abx ∈ E, with a, b ∈ R, x ∈ X, implies that, either ax ∈ E + $J(X)$ or bx ∈ E + $J(X)$", where $J(X)$ is the Jacobsen radical of X. this led us to introduce the concept a nearly quasi 2-absorbing submodule as a generalization of nearly quasi prime submodules and 2-absorbing submodules, where a proper submodule E of an R-module X is called nearly quasi 2-absorbing submodule of X, if whenever abcx ∈ E, where a, b, c ∈ R, x ∈ X implies that either abx ∈ E + $J(X)$ or acx ∈ E + $J(X)$ or bcx ∈ E + $J(X)$. We study the basic properties of nearly quasi 2-absorbing. Moreover, the relations of nearly quasi 2-absorbing submodule with other classes of modules are established. Also, characterization, and examples are given.

1 – Introduction

Al-Mothafar and Abdul-Al kalik in 2016 introduce the concept of nearly quasi prime submodules as generalization of prime (quasi prime) submodules, where "a proper submodule E of an R-module X is called nearly quasi prime, if whenever abx ∈ E, with a, b ∈ R, x ∈ X, implies that, either ax ∈ E + $J(X)$, bx ∈ E + $J(X)$ [1]", and "a proper submodule E of an R-module X is called quasi prime if whenever abx ∈ E with a, b ∈ R, x ∈ X, implies that, either ax ∈ E or bx ∈ E [2]", and a proper submodule E of an R-module X is called prime if whenever ax ∈ E, a ∈ R , x∈ X, implies that x ∈ E or a ∈ [E : X] [3],where [E : X] = { r ∈ R: rX ⊆ E}. Many basic properties of nearly quasi-2-absorbing submodules are given, also many characterizations of this concept are establish see proposition (2.6) and corollary (2.7). Also, we show that the intersection of two nearly quasi-2-absorbing submodules need not to be nearly quasi-2-absorbing, but under certain condition it satisfy see Remark (2.8), proposition (2.11).

2 - Nearly quasi-2-absorbing submodules

Recall that a proper submodule E of an R-module X is 2-absorbing if whenever abx ∈ E with a , b ∈ R , x ∈ X , implies that either ax ∈ E or bx ∈ E or ab ∈ [E : X][4]. We introduce in this section the definition of nearly quasi 2-absorbing submodules as generalization of prime , quasi prime , nearly quasi prime submodules, and 2-absorbing submodules.

Definition (2 . 1)

A proper submodule E of an R-module X is said to be nearly quasi 2-absorbing , if whenever rstx ∈ E, r, s, t ∈ R, x ∈ X, implies that rsx∈ E + $J(X)$ or rtx ∈ E+ $J(X)$ or stx ∈ E + $J(X)$.

And a proper ideal I of a ring R is called nearly quasi 2-absorbing ideal if I is nearly quasi-2-absorbing submodule , of R-module R.

Remarks and Examples (2.2)

1– Every prime submodule of an R-module X is nearly quasi-2-absorbing, while the converse is not true in general.

Proof

Assume that E is a prime submodule of an R-module X , and rstx ∈ E , r , s ,t ∈ R , x ∈ X , that is rs(tx) ∈ E, suppose that tx ∉ E and stx∉ E+ $J(X)$, rstx ∉ E+ $J(X)$. since E is a prime and tx ∉ E, it follows that rs ∈[E : X] , that is rsx ∈ E ⊆ E + $J(X)$ for each x ∈ X, hence rsx ∈ E + $J(X)$.

For the converse consider the following example:- let X = Z , R = Z and E = 6Z is a submodule of X, 6Z is nearly quasi-2-absorbing submodule of X but 6Z is not prime submodule of X.

2– it is clear that every nearly quasi prime submodule of X is nearly quasi-2-absorbing , while the converse need not to be true.

For the converse consider the following example:-

Let X = Z⊕Z , R = Z , E = 10Z⊕(0) it is clear that E is nearly quasi-2-absorbing submodule of X, but E is not nearly quasi prime submodule of X, since 2 . 5

(1,0) ∈ E 2 , 5 ∈ Z , (1, 0) ∈ X , but 2 . (1,0) ∉ ∈ E + $\jmath$(X) and 5. (1,0) ∉ E + $\jmath$(X) .

3 – It is clear that every 2-absorbing submodule is nearly quasi-2-absobing. For the converse consider the following example:-

Let X = Z⊕Z , R = Z , E = (0) ⊕ 35Z , E is nearly quasi-2-absorbing but not 2-absorbing , since 5 . 7 (0,1) ∈ E , 5 (0,1) ∉E and 7 (0,1) ∉E and 5 .7 ∉ [E : X] = (0).

4 – It is clear that every quasi prime submodule of X is nearly quasi-2-absobing , but the converse is not true in general.

For the converse consider the following example:- let X = $Z_8 \oplus Z$ and R = Z , E = <$\bar{4}$> ⊕ Z , it is clear that E is nearly quasi-2-absorbing , but not quasi prime , since 2 . 2 ($\bar{1}$,1) ∈ E = <$\bar{4}$> ⊕ Z , but 2 ($\bar{1}$,1) ∉E.

Proposition (2 . 3)

If E and K are two submodules of an R-module X with E ⊈ K , and E is nearly quasi-2-absorbing submodule of X such that $\jmath$ (X) ⊆ $\jmath$ (K) , then E is a nearly quasi-2-absorbing submodule of K.

Proof

Let rstx ∈ E , r , s , t ∈ R , x ∈ K ⊆ X , then either rsx ∈ E + $\jmath$(X) or rtx ∈ E + $\jmath$(X) or stx ∈ E + $\jmath$(X) . But $\jmath$ (X) ⊆ $\jmath$ (K), hence either rsx ∈ E + $\jmath$(K) or rtx ∈ E + $\jmath$(K) or stx ∈ E + $\jmath$(K).

Proposition (2 . 4)

If H is nearly quasi-2-absorbing submodule of X and $\jmath$(X) ⊆ H, then H is a weakly quasi-2-absorbing in X.

Proof

It is clear

Proposition (2 . 5)

Let H be a proper submodule of an R-module X , if [H+$\jmath$(X):(x)] is quasi prime ideal of R for each x ∈ X , then H is nearly quasi-2-absorbing in X**.**

Proof

Let rstx ∈ H , r , s , t ∈ R , x ∈ X , and rsx ∉ H + $\jmath$ (X), implies that rstx ∈ H + $\jmath$ (X), hence rst ∈[H+$\jmath$(X):(x)] , it follows that either rt ∈[H+$\jmath$(X):(x)] or st ∈[H+$\jmath$(X):(x)] , that is either rtx ∈ H+ $\jmath$ (X) or stx ∈ H+ $\jmath$ (X).

The following proposition are characterization of nearly quasi 2-absorbing submodules.

Proposition (2.6)

A submodule H of an R-module X is nearly quasi 2-absorbing in X if and only if for every ideals I_1 , I_2 , I_3 of R , and submodules F of X , with $I_1 I_2 I_3$F ⊆ H , implies that either $I_1 I_2$F ⊆ H + $\jmath$(X) or $I_1 I_3$F ⊆ H + $\jmath$(X) or $I_2 I_3$F ⊆ H + $\jmath$(X).

Proof

⇒

Assume that $I_1 I_2 I_3$F ⊆ H, where $I_1 , I_2 , I_3$are ideals and F is a submodule of X, and suppose that $I_1 I_2$F ⊈H + $\jmath$(X) or $I_1 I_3$F ⊈ H+ $\jmath$(X) or $I_2 I_3$F ⊈H + $\jmath$(X). So there exists x_1 , x_2 , x_3 ∈ F and $a_1 \in I_1$, $a_2 \in I_2$, $a_3 \in I_3$ such that $a_1 a_2 x_1$ ∉ H + $\jmath$(X) and $a_1 a_3 x_2$ ∉ H + $\jmath$(X) and $a_2 a_3 x_3$ ∉ H + $\jmath$(X). Since H is a nearly quasi 2-absorbing submodule of X and $a_1 a_2 a_3 x_1$ ∈ H and $a_1 a_2 x_1$ ∉ H + $\jmath$(X), then we have $a_1 a_3 x_1$ ∈ H+ $\jmath$(X) or $a_2 a_3 x_1$ ∈ H+ $\jmath$(X) . Also $a_1 a_2 a_3 x_2$ ∈ H and $a_1 a_3 x_2$ ∉ H + $\jmath$(X) , implies that either $a_1 a_2 x_2$ ∈ H + $\jmath$(X) or $a_2 a_3 x_2$ ∈ H + $\jmath$(X). Also $a_1 a_2 a_3 x_3$ ∈ H, and $a_2 a_3 x_3$ ∉ H + $\jmath$(X), implies that either $a_1 a_2 x_3$ ∈ H + $\jmath$(X) or $a_1 a_3 x_3$ ∈ H + $\jmath$(X). Thus either $I_1 I_2$F ⊆ H + $\jmath$(X) or $I_1 I_3$F ⊆ H + $\jmath$(X) or $I_2 I_3$F ⊆ H + $\jmath$(X).

⇐

Assume that $a_1 a_2 a_3 x$ ∈ H , a_1 , a_2 , a_3 ∈ R , x ∈ X , then $(a_1)(a_2)(a_3)(x)$ ⊆ H, so either $(a_1)(a_2)(x)$ ⊆ H+$\jmath$(X) or $(a_1)(a_3)(x)$ ⊆ H+$\jmath$(X) or $(a_2)(a_3)(x)$ ⊆ H+$\jmath$(X) , hence either $a_1 a_2 x$ ∈ H+$\jmath$(X) or $a_1 a_3 x$ ∈ H+$\jmath$(X) or $a_2 a_3 x$ ∈ H+$\jmath$(X).

Proposition (2.7)

A submodule H of an R-module X is nearly quasi 2-absorbing in X if and only if for each submodule F of X and for each a_1 , a_2 , a_3 ∈ R , such that $a_1 a_2 a_3$F ⊆ H , implies that either $a_1 a_2$F ⊆ H + $\jmath$(X) or $a_1 a_3$F ⊆ H + $\jmath$(X) or $a_2 a_3$F ⊆ H + $\jmath$(X).

Proof

Direct

Remark (2 . 8)

The intersection of two nearly quasi 2-absorbing submodules of an R-module X need not to be nearly quasi 2-absobing submodule of X, as the following example shows that:-

Let X = Z and R = Z , H = 9Z , K = 2Z , H and K are nearly quasi 2-absorbing submodules of X , but H ∩ K = 18Z is not nearly quasi 2-absorbing in X , since 2 . 3 . 3 . 1 ∈ 18Z , but 2 . 3 . 1 = 6 ∉ 18Z + $\jmath$(X) and 3 . 3 . 1 = 9 ∉ 18Z + $\jmath$(X). Hence 18Z is not nearly quasi 2-absorbing submodule of X.

" Recall that a ring R is a good ring if $\jmath$(R) X = X , where X is an R-module [5]".

"Lemma (2 . 9) [5]

If R is a good ring and N is a submodule of an R-module X, then $\jmath$(X) ∩ N = (N)."

"Lemma (2 . 10) [5, lemma 2 . 3 . 15]

Let X be an R-module , and A , B and C are submodules of X with B ⊈ C . Then (A + B) ∩C = (A ∩ C) + B = (A ∩ C) + (B ∩ C)".

Proposition (2 . 11)

let X be an R-module over a good ring R , and H is nearly quasi 2-absorbing submodule of X and F is a submodule of X with $\jmath$(X) ⊆ F and F ⊈ H , then F ∩ H is nearly quasi 2-absorbing submodule of F.

proof

since F ⊈ H, then F ∩ H is a proper submodule of F , Now let rstx ∈ F ∩ H, where r, s, t ∈ R, x ∈ F, implies that rstx ∈ F and rstx ∈ H, but H is a nearly quasi 2-absorbing in X, then either rsx ∈ H + $\jmath$(X) or rtx ∈ H + $\jmath$(X) or stx ∈ H + $\jmath$(X), since x ∈ F , implies that rsx ∈(H + $\jmath$(X)) ∩ F or rtx ∈(H + $\jmath$(X)) ∩ F or stx ∈(H + $\jmath$(X)) ∩ F. But $\jmath$(X) ⊆ F, so by lemma (2 . 10) , we have either rsx ∈ (H ∩ F) + ($\jmath$(X) ∩ F) or rtx ∈(H ∩ F) + ($\jmath$(X) ∩ F) or stx ∈(H ∩ F) + ($\jmath$(X) ∩ F). But R is a good ring then by lemma (2 . 9) , we have $\jmath$(X) ∩ F = $\jmath$(F). Thus either rsx ∈ (H ∩ F) + $\jmath$(F) or rtx ∈(H ∩ F) + $\jmath$(F) or stx ∈ (H ∩ F) + $\jmath$(F).

Corollary (2 . 12)

let X be an R-module over a good ring R , and H , F are submodules of X with F ⊈ H if H is nearly quasi 2-absorbing in X , and F is a maximal submodule of X, then H ∩ F is nearly quasi 2-absorbing submodule of F.

Proof

Since F is maximal submodule of X, then $J(X) \subseteq F$. Hence the proof follows by proposition (2 . 11)

The following proposition shows that the intersection of two nearly quasi 2-absorbing submodules is nearly quasi 2-absorbing under certain conditions.

Proposition (2 . 13)

If H and F are two nearly quasi-2-absorbing submodules of an R-module X , with $H \subseteq J(X)$ or $F \subseteq J(X)$, then H ∩ F is nearly quasi-2-absorbing submodule of X.

Proof

Assume that rstx ∈ H ∩ F, r, s, t ∈ R and x ∈ X it follows that rstx ∈ H and rstx ∈ F, but both H and F are nearly quasi 2-absobing submodules of X. So, either rsx ∈ H + J(X) = J(X) or rtx ∈ H + J(X) = J(X) or stx ∈ H + J(X) = J(X) and rsx ∈ F + J(X) = J(X) or rtx ∈ F + J(X) = J(X) or stx ∈ F + J(X) = J(X). Thus either rsx ∈ H ∩ F + J(X) or rtx ∈ H ∩ F + J(X) or stx ∈ H ∩ F + J(X) (because H ∩ F ⊆ J(X)). Thus H ∩ F is nearly quasi 2-absorbing of X.

Proposition (2 . 14)

Let H and F be two submodules of an R-module X, with F ⊈ H and J(X) = J(F). If H is nearly quasi 2-absorbing submodule of X, then H ∩ F is nearly quasi 2-absorbing of F.

Proof

It is clear that F ∩ H is a proper submodule of F. Let rstx ∈ F ∩ H , r , s , t ∈ R , x ∈ F , so rstx ∈ F and rstx ∈ H. but H is a nearly quasi 2-absorbing in X , and x ∈ F ⊆ X, then either rsx ∈ H + J(X) or rtx ∈ H + J(X) or stx ∈ H + J(X). Since x ∈ F, implies that either rsx ∈(H + J(X)) ∩ F or rtx ∈ (H + J(X)) ∩ F or stx ∈ (H + J(X)) ∩ F. Thus since J(X) = J (F), we get either rsx ∈ (H + J(F)) ∩ F or rtx ∈ (H + J(F)) ∩ F or stx ∈ (H + J(F)) ∩ F. Hence by lemma (2 . 10), we have either rsx ∈ (H ∩ F) + J(F) or rtx ∈ (H ∩ F) + J(F) or stx ∈ (H ∩ F) + J(F). Thus H ∩ F is nearly quasi 2-absorbing in F.

Proposition (2 . 15)

Let H be a submodule of an R-module X, and H+J(X) is nearly quasi-2-absorbing submodule of X, then H is nearly quasi-2-absorbing submodule in X.

Proof

To prove that H is nearly quasi 2-absorbing submodule let rstx ∈ H, r , s , t ∈ R , x ∈ X ,since H ⊆ H+J(X), then rstx ∈ H + J(X). But H+J(X) is nearly quasi-2-absorbing in X, so either rsx ∈ H + J(X) + J(X) = H + J(X) or rtx ∈ H + J(X) + J(X) = H + J(X) or stx ∈ H + J(X) + J(X) = H + J(X). Thus H is nearly quasi-2- absorbing in X.

Proposition (2.16)

Let H be proper submodule of an R-module X such that H is nearly quasi 2-absorbing submodule of X, then S^{-1}H is nearly quasi 2-absorbing submodule of S^{-1}R – module S^{-1}X .

Proof

Assume that $\frac{a}{s_1}\frac{b}{s_2}\frac{c}{s_3}\frac{x}{s_4} \in S^{-1}H$, where $\frac{a}{s_1},\frac{b}{s_2},\frac{c}{s_3} \in S^{-1}R$ and $\frac{x}{s_4} \in S^{-1}X$, $a , b , c \in R$, $x \in X$, $s_1 , s_2, s_3 , s_4 \in S$, *it follows that* $\frac{abcx}{t} \in S^{-1}H$ where $s_1 s_2 s_3 s_4 = t \in S$. then there exists $t_1 \in S$ such that $t_1abcx \in$ H , since H is nearly quasi 2-absorbing in X then either $abxt_1 \in$ H + J(X) or $acxt_1 \in H + J(X)$ or $bcxt_1 \in$ H + J(X), which implies that either $\frac{a}{s_1}\frac{b}{s_2}\frac{x}{s_4}\frac{t_1}{t_1} \in S^{-1}[H+ J(X)] = S^{-1}H + J(S^{-1}X)$ or $\frac{a}{s_1}\frac{c}{s_3}\frac{x}{s_4}\frac{t_1}{t_1} \in S^{-1}[H+ J(X)] = S^{-1}H + J(S^{-1}X)$ or $\frac{b}{s_2}\frac{c}{s_3}\frac{x}{s_4}\frac{t_1}{t_1} \in S^{-1}[H+ J(X)] = S^{-1}H + J(S^{-1}X)$. *Thus* $S^{-1}H$ *is nearly quasi 2-absorbing submodule of* $S^{-1}X$.

" Recall that a submodule E of an R-module X is small in X , if for any submodule F of X such that X = E + F then F = X [5]".

Also , "recall that an R-epimorphism φ: X → X` is called small epimorphism if Kerφ small submodule of X [5]."

"Lemma (2 . 17) [5, corollary 9 . 1 . 5]

If an R-epimorphism *φ : X → X` is small*, then φ( $J$(X))= $J$(X`) and φ^{-1}(J(X`)) = J(X)".

Proposition (2 . 18)

Let *φ : X → X`* be a small R-epimorphism , and E is a nearly quasi-2-absorbing submodule of *X`*, then φ^{-1}(E) is a nearly quasi-2-absorbing of *X*.

Proof

Assume that rstx ∈ φ^{-1}(E) , r , s , t ∈ R , x ∈ X , so rstφ(x) ∈ E. But E is a nearly quasi 2-absorbing submodule of X` , then either rsφ(x) ∈ E + $J$(X`) or rtφ(x) ∈ E + J(X`) or stφ(x) ∈ E + $J$(X`). Thus either rsx ∈ φ^{-1}(E) + φ^{-1}((X`)) or rtx ∈ $\varphi^{-1}$(E) + $\varphi^{-1}$($J$(X`)) or stx ∈ φ^{-1}(E) + φ^{-1}(J(X`)). But φ is small epimorphism, implies that $\varphi^{-1}$( (X`)) = J(X) by lemma (2 . 17). Hence we have either rsx ∈ φ^{-1}(E) + *J(X)* or rtx ∈ φ^{-1}(E) + *J(X)* or stx ∈ φ^{-1}(E) + *J(X).*

Proposition (2 . 19)

Let *φ : X → X`* be a small R-epimorphism . and E be a proper nearly quasi-2-absorbing submodule of *X* such that Ker *φ* ⊆ E , then *φ*(E) is a nearly quasi-2-absorbing submodule of *X`*.

Proof

It is clear that *φ*(E) is proper submodule of *X`* , if not suppose that *φ*(E)= *X`*, let x ∈ *X* such that *φ*(x) ∈ *X`* = *φ*(E), implies that there exist e ∈ E such that φ(e) = φ(x) . implies that φ(e – x ) = 0 , it follows that e – x ∈ Ker *φ* ⊆ E , implies that x ∈ E , thus E = *X* contradiction , Now assume that rst*x`* ∈ φ (E) , r , s , t ∈ R, *x`* ∈ *X`*, since φ is an epimorphism , then there exist *x* ∈ *X* such that φ (x) = *x`*. Hence rst φ (x) ∈ *φ*(E) , implies that there exist e ∈ E such that rst φ (x) = *φ*(e), hence φ (rst x - e) = 0 , so rstx – e ∈ Ker *φ* ⊆ E, implies that rstx ∈ E . But E is a nearly quasi 2-

absorbing in X, so either rsx ∈ E + $\jmath$ (X) or rtx ∈ E + $\jmath$ (X) or stx ∈ E + $\jmath$ (X), it follows that either rs$x^{`}$∈ φ (E) + φ ( $\jmath$(X)) or rt$x^{`}$∈ φ (E) + φ ($\jmath$(X)) or st$x^{`}$∈ φ (E) + φ ( $\jmath$(X)). Thus by lemma ( 2 . 17) , we have either rs$x^{`}$∈ φ (E) + $\jmath(X^{`})$ or rt$x^{`}$∈ φ (E) + $\jmath(X^{`})$ or st$x^{`}$∈ φ (E) + $\jmath(X^{`})$.

Corollary (2 . 20)

Let E be a submodule of an R-module X, and F be a small submodule of X contained in E. Then $\frac{E}{F}$ is a nearly quasi 2-absorbing submodule of $\frac{X}{F}$ if and only if E is a nearly quasi 2-absorbing in X.

Proof

Let π: $X \rightarrow \frac{X}{F}$ be a natural R- epimophism , then the result follow by proposition (2 . 18) and proposition (2 . 19).

Proposition (2.21)

Let $X^{`}$ and $X^{``}$be R-modules and and $X = X^{`} \oplus X^{``}$ if $E = E^{`} \oplus E^{``}$ is a nearly quasi 2-absorbing submodule in $X$, with $E \subseteq \jmath(X)$ , where $E^{`}$ , $E^{``}$ are submodules of $X^{`}$, $X^{``}$ respectively. Then $E^{`}$ and $E^{``}$ are nearly quasi 2-absorbing submodules in $X^{`}$ and $X^{``}$ respectively.

Proof

Assume that rst$x^{`}$ ∈ $E^{`}$, r , s , t ∈ R , $x^{`}$ ∈ $X^{`}$. Then rst($x^{`}$, 0) ∈ $E = E^{`} \oplus E^{``}$. *But* $E$ is a nearly quasi 2-absorbing in $X$, then either rs($x^{`}$, 0) ∈ $E + \jmath(X) = \jmath(X)$ or rt($x^{`}$, 0) ∈ $E + \jmath(X) = \jmath(X)$ or st($x^{`}$, 0) ∈ $E + \jmath(X) = \jmath(X)$ . But $\jmath(X) = (X^{`}) \oplus (X^{``})$. It follows that either rs($x^{`}$, 0) ∈ $\jmath(X^{`}) \oplus \jmath(X^{``})$ or rt($x^{`}$, 0) ∈ $\jmath(X^{`}) \oplus \jmath(X^{``})$ or st($x^{`}$, 0) ∈ $\jmath(X^{`}) \oplus \jmath(X^{``})$. Thus either rs$x^{`}$ ∈ $\jmath(X^{`}) \subseteq E^{`} + \jmath(X^{`})$ or rt$x^{`}$ ∈ $\jmath(X^{`}) \subseteq E^{`} + \jmath(X^{`})$ or st$x^{`}$ ∈ $\jmath(X^{`}) \subseteq E^{`} + \jmath(X^{`})$. Hence $E^{`}$ is a nearly quasi 2-absorbing submodule in $X^{`}$.

By similar way $E^{``}$ is a nearly quasi 2-absorbing submodule of $X^{``}$.

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قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

الملخص

كل الحلقات المسخدمة في هذا البحث هي حلقات ابدالية بمحايد و كل المقاسات المعرفة عليها هي مقاسات يسارية أحادية. المقاس الجزئي الفعلي E من المقاس X يدعى مقاس جزئي اولي ظاهري قريب اذا كان abx ∈ E حيث x ∈ X , a , b ∈ R يؤدي الى اما ax ∈ E +ȷ(X) او bx ∈ E +ȷ(X) حيث ان ȷ(X) هو جذر جاكوبسن، هذا التعريف قادنا لكي نقدم مفهوم المقاس الجزئي المستحوذ من النمط – 2 الظاهري كأعمام للمقاس الجزئي الاولي الظاهري القريب و المقاس الجزئي المستحوذ من النمط – 2، حيث انه يدعى المقاس الجزئي الفعلي E بانه مقاسا جزئيا مستحوذا من النمط – 2 ظاهريا قريبا اذا كان abcm ∈ E حيث a , b , c ∈ R , m ∈ X يؤدي الى انه اما abm∈ E + ȷ(X) او acm∈ E + ȷ(X) او bcm ∈ E + ȷ(X). درسنا معظم المفاهيم الأساسية لهذا المفهوم ، بالإضافة لذلك درسنا علاقة هذا المفهوم مع أصناف أخرى من المقاسات. كذلك اعطينا مكافئات و امثلة عليه.