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Fuzzy Translation and Fuzzy Multiplication on D-Algebras

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ABSTRACT

The concept of fuzzy set (FS) is one of the beautiful branches in Mathematics. This concept was initiated by Zadeh [1]. Since that time, many studies have been considered this concept by different ways in the field of pure and applied Mathematics. In this article, we introduced the notion of FT and FM on a D-algebras Ω, where FT is a fuzzy Translation and FM is a fuzzy Multiplication. We proved some characterizations of FT and FM of sub-algebra and d-ideal of a D-algebras Ω. Moreover, the notion of FM-αλ-T has been investigated on the D-algebras Ω. Furthermore, some results on the d-homomorphism of FT and FM which based on the fuzzy d-ideal of a D-algebras are presented.

1. Introduction

Throughout this study we mean by Ω and Γ to be two D-algebras, FT(F-α-T) is a fuzzy Translation, FM(F-λ-M) is a fuzzy multiplication and FM-αλ-T is a Fuzzy Magnified-αλ-Translation. Recently, much attention has been given to study the concept of fuzzy algebra which is one of the influential brunches in Mathematics. Zadeh in [1] provided the concept of fuzzy set. This concept has been applied on several types of algebraic concepts such as rings, modules, groups, topologies and vector spaces. The study of [2] investigated the idea of the fuzzy d-ideal of a D-algebras with some of its properties have been proved. The concept of a D-algebras which is considered as another popularization of BCK-algebras has been provided by Neggers and Kim [3]. Then some of the relations among BCK-algebras and D-algebras were discussed. A paper by Lee et. al [4] presented the FT and the FM of fuzzy sub-algebras of BCK/BCI-algebras. They discussed the relations among the FT and the FM. Furthermore, the notion of Q-ideal and fuzzy Q-ideal in Q-algebras has been investigated by Mostafa et. al [5]. The study of Hameed et. al [6] presented the definition of FT and FM of CI-algebras, and several properties of this notion were studied. While, the FT and the FM of Q-algebras were given by Hameed and Mohammed [7]. In addition, the notion of ω -FT with ω -FM on a BP-algebras are introduced by Prasanna et. al [8]. Priya and Ramachandran [9] provided the FT and FM of a PS-algebras. Then, the homomorphism and the Cartesian product of the FT and FM of a PS-algebras are also presented by Priya and Ramachandran [10]. Now days, Alshehri [11] studied the FT and FM of a BRK-algebras and some of their properties were discussed. In this paper we introduced the notion of FT and FM on D-algebras and discussed some of its properties. The contents of this paper have been structured as follows: In section two, some basic definitions and previous results that are needed in this research are presented. While, in section three, the FT and FM of d-sub-algebras are presented. Section four contains the FT and FM of d-ideals. Section five, followed by the Cartesian product of FT and FM of d-ideals. The notion of FM- $\alpha\lambda$ -T of a D-algebras has been stated in section six. In section seven, the homomorphism of the FT and FM of a D-algebras has been studied. Finally, the conclusions and further research scope of this paper are given in section eight.

2. Basic concepts

In this section, some of the previous results that are needed in this study are presented. We start with the following observations which are given as follows.

FS = Fuzzy Set(subset)

FS-Algebra = Fuzzy Sub-algebra

FdI = Fuzzy d-ideal.

Definition 2.1 [2] The system $(\Omega, *, 0)$ is said to be D-algebras, if all the following conditions are holds:

i. $u * u = 0$,

ii. $0 * u = 0$,

iii. $u * v = 0$ and $v * u = 0$ implies $u = v$ for each $u, v \in \Omega$.

Definition 2.2 [2] Let Ω be a D-algebras. Then, $\Phi \neq S \subset \Omega$ is said to be sub-algebra of Ω if $u * v \in S$ where $u, v \in S$.

Definition 2.3 [2] Let Ω be a D-algebras with $H \subset \Omega$, then H is called d-ideal of Ω if all the following are fulfilled:

i. $0 \in H$,

ii. $u * v \in H$ and $v \in H$ implies $u \in H$,

iii. $u \in H$ and $v \in \Omega$ implies $u * v \in H$.

Definition 2.4 [2] Let ψ be a FS of Ω, then ψ is called FdI of Ω if all the following are fulfilled:

i. $\psi(0) \geq \psi(u)$,

ii. $\psi(u) \geq \min\{\psi(u * v), \psi(v)\}$,

iii. $\psi(u * v) \geq \min\{\psi(u), \psi(v)\}$ for each $u, v \in \Omega$.

Definition 2.5 [2] The FS ψ of a D-algebras Ω is said to be FS-algebra of Ω if $\psi(u * v) \geq \min\{\psi(u), \psi(v)\}$ where $u, v \in \Omega$.

Definition 2.6 [2] The FS ψ of a set Ω is the function $\psi : \Omega \to [0,1]$.

Definition 2.7 [2] Let ψ_1 and ψ_2 be two FS of Ω. Then, the Cartesian product of ψ_1 and ψ_2 is the mapping $\psi_1 \times \psi_2 : \Omega \times \Omega \to [0,1]$ which is defined as $(\psi_1 \times \psi_2)(u, v) = \min\{\psi_1(u), \psi_2(v)\}, \forall u, v \in \Omega$.

Definition 2.8 [2] Let ψ be a FS of Γ and $f : \Omega \to \Gamma$ be a mapping of a D-algebras. Then, the mapping ψ^f is the inverse image of ψ under f such that $\psi^f(u) = \psi(f(u))$ for each $u \in \Omega$.

Theorem 2.1 [2] Let ψ_1 and ψ_2 are two FdI of a D-algebras Ω. Then, $\psi_1 \times \psi_2$ is FdI of $\Omega \times \Omega$.

Definition 2.9 [3] Let $(\Omega, *, 0)$ and $(\Gamma, *', 0')$ are two D-algebras. The mapping $f : \Omega \to \Gamma$ is said to be d-homomorphism if for each $u, v \in \Omega$ we have $f(u * v) = f(u) *' f(v)$.

Definition 2.10 [4] Let ψ be a FS of Ω and $\alpha \in [0, T]$. Then, FT (F-α-T) of ψ is the mapping $\psi_\alpha^T : \Omega \to [0,1]$ such that $\psi_\alpha^T(u) = \psi(u) + \alpha$ for each $u \in \Omega$.

Definition 2.11 [4] Let ψ be a FS of Ω with $\lambda \in [0,1]$. Then, FM (F-λ-M) of ψ is the mapping $\psi_\lambda^M : \Omega \to [0,1]$ such that $\psi_\lambda^M(u) = \lambda \cdot \psi(u)$ for each $u \in \Omega$.

3. FT AND FM OF D-SUB-ALGEBRAS

This section presents the notion of FT and FM of a D-algebras. In what follows, let $(\Omega, *, 0)$ be a D-algebras. Then, for any FS ψ of Ω we symbolize $T = 1 - \sup\{\psi(u) : u \in \Omega\}$ unless otherwise we mentioned.

Definition 3.1 A F-α-T set ψ_α^T of a FS ψ is called F-α-T sub-algebra if $\psi_\alpha^T(u * v) \geq \min\{\psi_\alpha^T(u), \psi_\alpha^T(v)\}$ for each $u, v \in \Omega$ and $\alpha \in [0, T]$.

Definition 3.2 A F-λ-M set ψ_λ^M of a FS ψ is called F-λ-M sub-algebra if $\psi_\lambda^M(u * v) \geq \min\{\psi_\lambda^M(u), \psi_\lambda^M(v)\}$ for each $u, v \in \Omega$ and $\lambda \in [0,1]$.

Example 3.1 Let $(\Omega = \{0,1,2\}, *, 0)$ be a D-algebras given as follows [3]:

*	0	1	2
0	0	0	0
1	2	0	2
2	1	1	0

Table 3.1

Define the FS $\psi : \Omega \to [0,1]$ by

$$\psi(u) = \begin{cases} 0.7 & , u = 0 \\ 0.02 & , u \neq 0 \end{cases}$$

Then, ψ is a FS-algebra of Ω. Here $T = 1 - \sup\{\psi(u) : u \in \Omega\} \Rightarrow T = 1 - 0.7 = 0.3$. Take $\alpha = 0.1 \in [0, T]$, and $\psi_\alpha^T : \Omega \to [0,1]$ is defined by

$$\psi_\alpha^T = \begin{cases} 0.7 + \alpha & , u = 0 \\ 0.02 + \alpha & , u \neq 0 \end{cases}$$

which satisfies $\psi_\alpha^T(u) = \psi(u) + \alpha$, $\forall u \in \Omega$. Then, it's a F-α-T. Furthermore, if we take $\lambda = 0.2 \in [0,1]$, then $\psi_\lambda^M : \Omega \to [0,1]$ is defined by

$$\psi_\lambda^M = \begin{cases} \lambda \cdot (0.7) & , u = 0 \\ \lambda \cdot (0.02) & , u \neq 0 \end{cases}$$

which satisfies $\psi_\lambda^M(u) = \lambda \cdot \psi(u)$, $\forall u \in \Omega$. Then, it's a F-λ-M.

Theorem 3.1 For any FS-algebra ψ of a D-algebras Ω, the FT ψ_α^T of ψ is a FS-algebra of Ω where $\alpha \in [0, T]$.

Proof: Suppose that ψ is a FS-algebra of a D-algebras Ω, then $\forall u, v \in \Omega$ we get

$$\psi(u * v) \geq \min\{\psi(u), \psi(v)\} \Rightarrow \psi(u * v) + \alpha \geq \min\{\psi(u), \psi(v)\} + \alpha$$

$$\geq \min\{\psi(u) + \alpha, \psi(v) + \alpha\}$$

$$= \min\{\psi_\alpha^T(u), \psi_\alpha^T(v)\}.$$

That is $\psi_\alpha^T(u*v)\ge\min\{\psi_\alpha^T(u),\psi_\alpha^T(v)\}$. Therefore, ψ_α^T is a FS-algebra of Ω. □

Theorem 3.2 Let ψ be a FS of a D-algebras Ω. Then, if a FT ψ_α^T of ψ is a FS-algebra of Ω, then so is ψ where $\alpha\in[0,T]$.

Proof: Since the FT ψ_α^T of ψ is a FS-algebra of Ω, then $\forall u,v\in\Omega$ we have

$\psi_\alpha^T(u*v)\ge\min\{\psi_\alpha^T(u),\psi_\alpha^T(v)\}\Rightarrow\psi(u*v)+\alpha$

$\ge\min\{\psi(u)+\alpha,\psi(v)+\alpha\}=\min\{\psi(u),\psi(v)\}+\alpha$

$\Rightarrow\psi(u*v)\ge\min\{\psi(u),\psi(v)\}$. Therefore, ψ is a FS-algebra of Ω. □

Theorem 3.3 For any FS-algebra ψ of a D-algebras Ω with $\lambda\in[0,1]$, the FM ψ_λ^M of ψ is also FS-algebra of Ω.

Proof: Since ψ is a FS-algebra of a D-algebras Ω, then $\forall u,v\in\Omega$ we get

$\psi(u*v)\ge\min\{\psi(u),\psi(v)\}\Rightarrow\lambda\cdot\psi(u*v)\ge\lambda\cdot\min\{\psi(u),\psi(v)\}$

$\ge\min\{\lambda\cdot\psi(u),\lambda\cdot\psi(v)\}$

$=\min\{\psi_\lambda^M(u),\psi_\lambda^M(v)\}$.

Thus, $\psi_\lambda^M(u*v)\ge\min\{\psi_\lambda^M(u),\psi_\lambda^M(v)\}$. Therefore, ψ_λ^M is a FS-algebra of Ω. □

Theorem 3.4 Let ψ be a FS of a D-algebras Ω with $\lambda\in[0,1]$. If the FM ψ_λ^M of ψ is a FS-algebra of Ω, then ψ is a FS-algebra of Ω.

Proof: Suppose ψ_λ^M is a FS-algebra of Ω, then $\forall u,v\in\Omega$ we get

$\psi_\lambda^M(u*v)\ge\min\{\psi_\lambda^M(u),\psi_\lambda^M(v)\}\Rightarrow\lambda\cdot\ \psi(u*v)$

$\ge\min\{\lambda\cdot\psi(u),\lambda\cdot\psi(v)\}=\lambda\cdot\ \min\{\psi(u),\psi(v)\}$.

That is

$\lambda\cdot\psi(u*v)\ge\lambda\cdot\min\{\psi(u),\psi(v)\}\Rightarrow\psi(u*v)\ge\min\{\psi(u),\psi(v)\}$.

Hence, ψ is a FS-algebra of Ω. □

4. FT AND FM OF D-IDEALS

In this section, we presented the notion of FT and FM of d-ideals.

Definition 4.1 A F-α-T set ψ_α^T of a FS ψ is said to be F-α-T d-ideal of Ω, if all of the following conditions are holds:

i. $\psi_\alpha^T(0)\ge\psi_\alpha^T(u)$,

ii. $\psi_\alpha^T(u)\ge\min\{\psi_\alpha^T(u*v),\psi_\alpha^T(v)\}$,

iii. $\psi_\alpha^T(u*v)\ge\min\{\psi_\alpha^T(u),\psi_\alpha^T(v)\}$, $\forall u,v\in\Omega$ and $\alpha\in[0,T]$.

Definition 4.2 A F-λ-M set ψ_λ^M of a FS ψ is said to be F-λ-M d-ideal of Ω, if all of the following conditions are holds:

i. $\psi_\lambda^M(0)\ge\psi_\lambda^M(u)$,

ii. $\psi_\lambda^M(u)\ge\min\{\psi_\lambda^M(u*v),\psi_\lambda^M(v)\}$,

iii. $\psi_\lambda^M(u*v)\ge\min\{\psi_\lambda^M(u),\psi_\lambda^M(v)\}$, $\forall u,v\in\Omega$ and $\lambda\in[0,1]$.

Theorem 4.1 Let ψ be a FS of a D-algebras Ω and ψ_α^T be a FT of ψ with $\alpha\in[0,T]$. Then, ψ is FdI of Ω iff ψ_α^T is FdI of Ω.

Proof: Suppose that ψ is FdI of Ω, then for each $u,v\in\Omega$ we have

i. $\psi(0)\ge\psi(u)\Rightarrow\psi(0)+\alpha\ge\psi(u)+\alpha\Rightarrow\psi_\alpha^T(0)\ge\psi_\alpha^T(u)$.

ii. $\psi(u)\ge\min\{\psi(u*v),\psi(v)\}\Rightarrow$

$\psi(u)+\alpha\ge\min\{\psi(u*v),\psi(v)\}+\alpha$

$\ge\min\{\psi(u*v)+\alpha,\psi(v)+\alpha\}$

$=\min\{\psi_\alpha^T(u*v),\psi_\alpha^T(v)\}$.

That is $\psi_\alpha^T(u)\ge\min\{\psi_\alpha^T(u*v),\psi_\alpha^T(v)\}$.

iii. $\psi(u*v)\ge\min\{\psi(u),\psi(v)\}\Rightarrow$

$\psi(u*v)+\alpha\ge\min\{\psi(u),\psi(v)\}+\alpha$

$\ge\min\{\psi(u)+\alpha,\psi(v)+\alpha\}$

$=\min\{\psi_\alpha^T(u),\psi_\alpha^T(v)\}$.

That is $\psi_\alpha^T(u*v)\ge\min\{\psi_\alpha^T(u),\psi_\alpha^T(v)\}$. Therefore, ψ_α^T is FdI of Ω.

Conversely, let ψ_α^T of ψ be FdI of Ω for some $\alpha\in[0,T]$. Then, for each $u,v\in\Omega$ we have:

i. $\psi_\alpha^T(0)=\psi(0)+\alpha\ge\psi(u)+\alpha\Rightarrow\psi(0)\ge\psi(u)$.

ii. $\psi_\alpha^T(u)\ge\min\{\psi_\alpha^T(u*v),\psi_\alpha^T(v)\}\Rightarrow\psi(u)+\alpha\ge\min\{\psi(u*v)$ $+\alpha,\psi(v)+\alpha\}\Rightarrow\psi(u)+\alpha\ge\min\{\psi(u*v),\psi(v)\}+\alpha$

$\Rightarrow\psi(u)\ge\min\{\psi(u*v),\psi(v)\}$.

iii. $\psi_\alpha^T(u*v)\ge\min\{\psi_\alpha^T(u),\psi_\alpha^T(v)\}\Rightarrow\psi(u*v)+\alpha\ge\min\{\psi(u)$ $+\alpha,\psi(v)+\alpha\}\Rightarrow\psi(u*v)+\alpha\ge\min\{\psi(u),\psi(v)\}+\alpha$

$\Rightarrow\psi(u*v)\ge\min\{\psi(u),\psi(v)\}$. Thus, ψ is FdI of Ω. □

Theorem 4.2 Let ψ be a FS of a D-algebras Ω and ψ_λ^M be a FM of ψ where $\lambda\in[0,1]$. Then, ψ is FdI of Ω iff ψ_λ^M is FdI of Ω.

Proof: Suppose that ψ be FdI of Ω, then for each $u,v\in\Omega$ we have

i. $\psi(0)\ge\psi(u)\Rightarrow\lambda\cdot\psi(0)\ge\lambda\cdot\psi(u)\Rightarrow\psi_\lambda^M(0)\ge\psi_\lambda^M(u)$.

ii. $\psi(u)\ge\min\{\psi(u*v),\psi(v)\}\Rightarrow$

$\lambda\cdot\psi(u)\ge\lambda\cdot\min\{\psi(u*v),\psi(v)\}$

$\ge\min\{\lambda\cdot\psi(u*v),\lambda\cdot\psi(v)\}$

$=\min\{\psi_\lambda^M(u*v),\psi_\lambda^M(v)\}$.

That is $\psi_\lambda^M(u)\ge\min\{\psi_\lambda^M(u*v),\psi_\lambda^M(v)\}$.

iii. $\psi(u*v)\ge\min\{\psi(u),\psi(v)\}\Rightarrow$

$\lambda\cdot\psi(u*v)\ge\lambda\cdot\min\{\psi(u),\psi(v)\}$

$\ge\min\{\lambda\cdot\psi(u),\lambda\cdot\psi(v)\}$

$=\min\{\psi_\lambda^M(u),\psi_\lambda^M(v)\}$.

That is $\psi_\lambda^M(u*v)\ge\min\{\psi_\lambda^M(u),\psi_\lambda^M(v)\}$. Therefore, ψ_λ^M is FdI of Ω.

Conversely, let ψ_λ^M of ψ be FdI of Ω for some $\lambda\in[0,1]$. Then, for each $u,v\in\Omega$ we have:

i. $\psi_\lambda^M(0)=\lambda\cdot\psi(0)\ge\lambda\cdot\psi(u)\Rightarrow\psi(0)\ge\psi(u)$.

ii. $\psi_{\lambda}^{M}(u) \geq \min\{\psi_{\lambda}^{M}(u*v), \psi_{\lambda}^{M}(v)\} \Rightarrow \lambda\cdot\psi(u)$
$\geq \min\{\lambda\cdot\psi(u*v), \lambda\cdot\psi(v)\} \Rightarrow \lambda\cdot\psi(u)$
$\geq \lambda\cdot\min\{\psi(u*v), \psi(v)\}$
$\Rightarrow \psi(u) \geq \min\{\psi(u*v), \psi(v)\}$.

iii. $\psi_{\lambda}^{M}(u*v) \geq \min\{\psi_{\lambda}^{M}(u), \psi_{\lambda}^{M}(v)\}$
$\Rightarrow \lambda\cdot\psi(u*v) \geq \min\{\lambda\cdot\psi(u), \lambda\cdot\psi(v)\}$
$\Rightarrow \lambda\cdot\psi(u*v) \geq \lambda\cdot\min\{\psi(u), \psi(v)\}$
$\Rightarrow \psi(u*v) \geq \min\{\psi(u), \psi(v)\}$. Thus, ψ is FdI of Ω. □

Theorem 4.3 The Intersection of two FdI translation of a D-algebras Ω is FdI translation of a D-algebras Ω.

Proof: Let $\psi_{\alpha_1}^{T}$ and $\psi_{\alpha_2}^{T}$ are two FdI translation of a FdI ψ of Ω with $\alpha_1, \alpha_2 \in [0,T]$. Then, for each $u,v \in \Omega$ we have:

i. $(\psi_{\alpha_1}^{T} \cap \psi_{\alpha_2}^{T})(0) = \min\{\psi_{\alpha_1}^{T}(0), \psi_{\alpha_2}^{T}(0)\} = \min\{\psi(0)+\alpha_1, \psi(0)+\alpha_2\}$
$\geq \min\{\psi(u)+\alpha_1, \psi(u)+\alpha_2\}$
$= \min\{\psi_{\alpha_1}^{T}(u), \psi_{\alpha_2}^{T}(u)\}$
$= (\psi_{\alpha_1}^{T} \cap \psi_{\alpha_2}^{T})(u)$.

ii. $(\psi_{\alpha_1}^{T} \cap \psi_{\alpha_2}^{T})(u) = \min\{\psi_{\alpha_1}^{T}(u), \psi_{\alpha_2}^{T}(u)\} = \min\{\psi(u)+\alpha_1, \psi(u)+\alpha_2\}$
$\geq \min\{\min\{\psi(u*v), \psi(v)\}+\alpha_1, \min\{\psi(u*v), \psi(v)\}+\alpha_2\}$
$\geq \min\{\min\{\psi(u*v)+\alpha_1, \psi(v)+\alpha_1\}, \min\{\psi(u*v)+\alpha_2, \psi(v)+\alpha_2\}\}$
$\geq \min\{\min\{\psi(u*v)+\alpha_1, \psi(u*v)+\alpha_2\}, \min\{\psi(v)+\alpha_1, \psi(v)+\alpha_2\}\}$
$\geq \min\{\min\{\psi_{\alpha_1}^{T}(u*v), \psi_{\alpha_2}^{T}(u*v)\}, \min\{\psi_{\alpha_1}^{T}(v), \psi_{\alpha_2}^{T}(v)\}\}$
$= \min\{\psi_{\alpha_1}^{T} \cap \psi_{\alpha_2}^{T}(u*v), \psi_{\alpha_1}^{T} \cap \psi_{\alpha_2}^{T}(v)\}$.

iii. $(\psi_{\alpha_1}^{T} \cap \psi_{\alpha_2}^{T})(u*v) = \min\{\psi_{\alpha_1}^{T}(u*v), \psi_{\alpha_2}^{T}(u*v)\}$
$= \min\{\psi(u*v)+\alpha_1, \psi(u*v)+\alpha_2\}$
$\geq \min\{\min\{\psi(u), \psi(v)\}+\alpha_1, \min\{\psi(u), \psi(v)\}+\alpha_2\}$
$\geq \min\{\min\{\psi(u)+\alpha_1, \psi(v)+\alpha_1\}, \min\{\psi(u)+\alpha_2, \psi(v)+\alpha_2\}\}$
$\geq \min\{\min\{\psi(u)+\alpha_1, \psi(u)+\alpha_2\}, \min\{\psi(v)+\alpha_1, \psi(v)+\alpha_2\}\}$
$\geq \min\{\min\{\psi_{\alpha_1}^{T}(u), \psi_{\alpha_2}^{T}(u)\}, \min\{\psi_{\alpha_1}^{T}(v), \psi_{\alpha_2}^{T}(v)\}\}$
$= \min\{\psi_{\alpha_1}^{T} \cap \psi_{\alpha_2}^{T}(u), \psi_{\alpha_1}^{T} \cap \psi_{\alpha_2}^{T}(v)\}$.

Therefore $\psi_{\alpha_1}^{T} \cap \psi_{\alpha_2}^{T}$ is FdI translation of Ω. □

Corollary 4.1 The intersection of a finite family of FdI translation of a D-algebras Ω is FdI translation of a D-algebras Ω.

Proof: Clear from Theorem 4.3. □

Theorem 4.4 The Intersection of two FdI multiplication of a D-algebras Ω is FdI multiplication of a D-algebras Ω.

Proof: Suppose that $\psi_{\lambda_1}^{M}$ and $\psi_{\lambda_2}^{M}$ are FdI multiplication of a FdI ψ of Ω, with $\lambda_1, \lambda_2 \in [0,1]$. Then, for each $u,v \in \Omega$ we have:

i. $(\psi_{\lambda_1}^{M} \cap \psi_{\lambda_2}^{M})(0) = \min\{\psi_{\lambda_1}^{M}(0), \psi_{\lambda_2}^{M}(0)\} = \min\{\lambda_1\cdot\psi(0), \lambda_2\cdot\psi(0)\}$
$\geq \min\{\lambda_1\cdot\psi(u), \lambda_2\cdot\psi(u)\}$
$= \min\{\psi_{\lambda_1}^{M}(u), \psi_{\lambda_2}^{M}(u)\}$
$= (\psi_{\lambda_1}^{M} \cap \psi_{\lambda_2}^{M})(u)$.

ii. $(\psi_{\lambda_1}^{M} \cap \psi_{\lambda_2}^{M})(u) = \min\{\psi_{\lambda_1}^{M}(u), \psi_{\lambda_2}^{M}(u)\} = \min\{\lambda_1\cdot\psi(u), \lambda_2\cdot\psi(u)\}$
$\geq \min\{\lambda_1\cdot\min\{\psi(u*v), \psi(v)\}, \lambda_2\cdot\min\{\psi(u*v), \psi(v)\}\}$
$\geq \min\{\min\{\lambda_1\cdot\psi(u*v), \lambda_1\cdot\psi(v)\}, \min\{\lambda_2\cdot\psi(u*v), \lambda_2\cdot\psi(v)\}\}$
$\geq \min\{\min\{\lambda_1\cdot\psi(u*v), \lambda_2\cdot\psi(u*v)\}, \min\{\lambda_1\cdot\psi(v), \lambda_2\cdot\psi(v)\}\}$
$\geq \min\{\min\{\psi_{\lambda_1}^{M}(u*v), \psi_{\lambda_2}^{M}(u*v)\}, \min\{\psi_{\lambda_1}^{M}(v), \psi_{\lambda_2}^{M}(v)\}\}$
$= \min\{\psi_{\lambda_1}^{M} \cap \psi_{\lambda_2}^{M}(u*v), \psi_{\lambda_1}^{M} \cap \psi_{\lambda_2}^{M}(v)\}$.

iii. $(\psi_{\lambda_1}^{M} \cap \psi_{\lambda_2}^{M})(u*v) = \min\{\psi_{\lambda_1}^{M}(u*v), \psi_{\lambda_2}^{M}(u*v)\}$
$= \min\{\lambda_1\cdot\psi(u*v), \lambda_2\cdot\psi(u*v)\}$
$\geq \min\{\lambda_1\cdot\min\{\psi(u), \psi(v)\}, \lambda_2\cdot\min\{\psi(u), \psi(v)\}\}$
$\geq \min\{\min\{\lambda_1\cdot\psi(u), \lambda_1\cdot\psi(v)\}, \min\{\lambda_2\cdot\psi(u), \lambda_2\cdot\psi(v)\}\}$
$\geq \min\{\min\{\lambda_1\cdot\psi(u), \lambda_2\cdot\psi(u)\}, \min\{\lambda_1\cdot\psi(v), \lambda_2\cdot\psi(v)\}\}$
$\geq \min\{\min\{\psi_{\lambda_1}^{M}(u), \psi_{\lambda_2}^{M}(u)\}, \min\{\psi_{\lambda_1}^{M}(v), \psi_{\lambda_2}^{M}(v)\}\}$
$= \min\{\psi_{\lambda_1}^{M} \cap \psi_{\lambda_2}^{M}(u), \psi_{\lambda_1}^{M} \cap \psi_{\lambda_2}^{M}(v)\}$.

Therefore $\psi_{\lambda_1}^{M} \cap \psi_{\lambda_2}^{M}$ is FdI multiplication of Ω. □

Corollary 4.2 The intersection of a finite family of FdI multiplication of a D-algebras Ω is FdI multiplication of a D-algebras Ω.

Proof: Clear from Theorem 4.4. □

5. CARTESIAN PRODUCT OF FT AND FM OF D-IDEALS

This section presents the Cartesian product of FT and FM of d-ideals.

Definition 5.1 Let $\psi_{\alpha_1}^{T}$ and $\psi_{\alpha_2}^{T}$ be two fuzzy translation of a D-algebras Ω. Then, the Cartesian product of $\psi_{\alpha_1}^{T}$ and $\psi_{\alpha_2}^{T}$ is symbolized by $\psi_{\alpha_1}^{T} \times \psi_{\alpha_2}^{T} : \Omega\times\Omega \to [0,1]$ and given as

$$(\psi_{\alpha_1}^{T} \times \psi_{\alpha_2}^{T})(u,v) = \min\{\psi_{\alpha_1}^{T}(u), \psi_{\alpha_2}^{T}(v)\},\ \forall u,v \in \Omega$$

and $\alpha_1, \alpha_2 \in [0,T]$.

Definition 5.2 Let $\psi_{\lambda_1}^{M}$ and $\psi_{\lambda_2}^{M}$ be two fuzzy multiplication of a D-algebras Ω. Then, the Cartesian product of $\psi_{\lambda_1}^{M}$ and $\psi_{\lambda_2}^{M}$ is symbolized by $\psi_{\lambda_1}^{M} \times \psi_{\lambda_2}^{M} : \Omega\times\Omega \to [0,1]$ and given as

$$(\psi_{\lambda_1}^{M} \times \psi_{\lambda_2}^{M})(u,v) = \min\{\psi_{\lambda_1}^{M}(u), \psi_{\lambda_2}^{M}(v)\},\ \forall u,v \in \Omega$$

and $\lambda_1, \lambda_2 \in [0,1]$.

Theorem 5.1 Let ψ and χ be two FdI of a D-algebras Ω. Furthermore, let $T = \min\{T_{\psi}, T_{\chi}\}$ where $T_{\psi} = 1 - \sup\{\psi(u) : u \in \Omega\}$ and $T_{\chi} = 1 - \sup\{\chi(u) : u \in \Omega\}$ where $\alpha \in [0,T]$. Then, the FT of $\psi\times\chi$ is FdI of $\Omega\times\Omega$.

Proof: Suppose that ψ and χ are FdI of a D-algebras Ω with $\alpha \in [0,T]$. By Theorem 4.1, ψ_{α}^{T} and χ_{α}^{T} are FdI of Ω. By Theorem 2.1, we have $\psi_{\alpha}^{T} \times \chi_{\alpha}^{T}$ is FdI of $\Omega\times\Omega$. Now, let $u,v \in \Omega$ then

$(\psi\times\chi)_{\alpha}^{T}(u,v) = (\psi\times\chi)(u,v) + \alpha = \min\{\psi(u), \chi(v)\} + \alpha$
$= \min\{\psi(u)+\alpha, \chi(v)+\alpha\}$
$= \min\{\psi_{\alpha}^{T}(u), \chi_{\alpha}^{T}(v)\}$
$= (\psi_{\alpha}^{T} \times \chi_{\alpha}^{T})(u,v)$ for each $(u,v) \in \Omega\times\Omega$.

Therefore, $\psi_\alpha^T \times \chi_\alpha^T$ is FdI of $\Omega\times\Omega$. □

Theorem 5.2 Let ψ and χ be two FdI of a D-algebras Ω. Furthermore, let $T=\min\{T_\psi, T_\chi\}$ where $T_\psi = 1-\sup\{\psi(u): u\in\Omega\}$ and $T_\chi = 1-\sup\{\chi(u): u\in\Omega\}$ where $\lambda\in[0,1]$. Then, the FM of $\psi\times\chi$ is FdI of $\Omega\times\Omega$.

Proof: Suppose that ψ and χ are FdI of a D-algebras Ω with $\lambda\in[0,1]$. By Theorem 4.2, ψ_λ^M and χ_λ^M are FdI of Ω. By Theorem 2.1, we have $\psi_\lambda^M\times\chi_\lambda^M$ is FdI of $\Omega\times\Omega$. Now, let $u,v\in\Omega$ then

$(\psi\times\chi)_\lambda^M(u,v) = \lambda\cdot(\psi\times\chi)(u,v) = \lambda\cdot\min\{\psi(u),\chi(v)\}$

$=\min\{\lambda\cdot\psi(u), \lambda\cdot\chi(v)\}$

$=\min\{\psi_\lambda^M(u), \chi_\lambda^M(v)\}$

$=\left(\psi_\lambda^M\times\chi_\lambda^M\right)(u,v)$ for each $(u,v)\in\Omega\times\Omega$.

Therefore, $\psi_\lambda^M\times\chi_\lambda^M$ is FdI of $\Omega\times\Omega$. □

Theorem 5.3 Let ψ and χ be two FS of a D-algebras Ω where $\psi_\alpha^T\times\chi_\alpha^T$ is FdI of $\Omega\times\Omega$ and $\alpha\in[0,T]$. Then,

i. Either $\psi_\alpha^T(0)\ge\psi_\alpha^T(u)$ or $\chi_\alpha^T(0)\ge\chi_\alpha^T(u)$ for each $u\in\Omega$.

ii. If $\psi_\alpha^T(0)\ge\psi_\alpha^T(u)$ for each $u\in\Omega$, then either $\chi_\alpha^T(0)\ge\psi_\alpha^T(u)$ or $\chi_\alpha^T(0)\ge\chi_\alpha^T(u)$.

iii. If $\chi_\alpha^T(0)\ge\chi_\alpha^T(u)$ for each $u\in\Omega$, then either $\psi_\alpha^T(0)\ge\psi_\alpha^T(u)$ or $\psi_\alpha^T(0)\ge\chi_\alpha^T(u)$.

Proof: i. Assume that $\psi_\alpha^T(0)<\psi_\alpha^T(u)$ and $\chi_\alpha^T(0)<\chi_\alpha^T(u)$ for some $u,v\in\Omega$. Then,

$\left(\psi_\alpha^T\times\chi_\alpha^T\right)(u,v)=\min\{\psi_\alpha^T(u),\chi_\alpha^T(v)\}$

$>\min\{\psi_\alpha^T(0),\chi_\alpha^T(0)\}=\left(\psi_\alpha^T\times\chi_\alpha^T\right)(0,0)\Rightarrow$

$\left(\psi_\alpha^T\times\chi_\alpha^T\right)(u,v)>\left(\psi_\alpha^T\times\chi_\alpha^T\right)(0,0)$ which is a contradiction. Hence, we got the result.

ii. Assume that $\chi_\alpha^T(0)<\psi_\alpha^T(u)$ and $\chi_\alpha^T(0)<\chi_\alpha^T(u)$ then, there exist $u,v\in\Omega$ such that

$\left(\psi_\alpha^T\times\chi_\alpha^T\right)(0,0)=\min\{\psi_\alpha^T(0),\chi_\alpha^T(0)\}=\chi_\alpha^T(0)$ and

$\left(\psi_\alpha^T\times\chi_\alpha^T\right)(u,v)=\min\{\psi_\alpha^T(u),\chi_\alpha^T(v)\}>\chi_\alpha^T(0)\Rightarrow$

$\left(\psi_\alpha^T\times\chi_\alpha^T\right)(u,v)>\left(\psi_\alpha^T\times\chi_\alpha^T\right)(0,0)$ which is a contradiction. Thus, the proof is completed.

The proof of the last point is similar to the proof of point two. □

Theorem 5.4 Let ψ and χ be two FS of a D-algebras Ω where $\psi_\lambda^M\times\chi_\lambda^M$ is FdI of $\Omega\times\Omega$ and $\lambda\in[0,1]$. Then,

i. Either $\psi_\lambda^M(0)\ge\psi_\lambda^M(u)$ or $\chi_\lambda^M(0)\ge\chi_\lambda^M(u)$ for each $u\in\Omega$.

ii. If $\psi_\lambda^M(0)\ge\psi_\lambda^M(u)$ for each $u\in\Omega$, then either $\chi_\lambda^M(0)\ge\psi_\lambda^M(u)$ or $\chi_\lambda^M(0)\ge\chi_\lambda^M(u)$.

iii. If $\chi_\lambda^M(0)\ge\chi_\lambda^M(u)$ for each $u\in\Omega$, then either $\psi_\lambda^M(0)\ge\psi_\lambda^M(u)$ or $\psi_\lambda^M(0)\ge\chi_\lambda^M(u)$.

Proof: i. Assume that $\psi_\lambda^M(0)<\psi_\lambda^M(u)$ and $\chi_\lambda^M(0)<\chi_\lambda^M(u)$ for some $u,v\in\Omega$. Then,

$\left(\psi_\lambda^M\times\chi_\lambda^M\right)(u,v)=\min\{\psi_\lambda^M(u),\chi_\lambda^M(v)\}$

$>\min\{\psi_\lambda^M(0),\chi_\lambda^M(0)\}=\left(\psi_\lambda^M\times\chi_\lambda^M\right)(0,0)\Rightarrow$

$\left(\psi_\lambda^M\times\chi_\lambda^M\right)(u,v)>\left(\psi_\lambda^M\times\chi_\lambda^M\right)(0,0)$ which is a contradiction. Hence, we achieved the result.

ii. Assume that $\chi_\lambda^M(0)<\psi_\lambda^M(u)$ and $\chi_\lambda^M(0)<\chi_\lambda^M(u)$ for some $u,v\in\Omega$. Then,

$\left(\psi_\lambda^M\times\chi_\lambda^M\right)(0,0)=\min\{\psi_\lambda^M(0),\chi_\lambda^M(0)\}=\chi_\lambda^M(0)$ and

$\left(\psi_\lambda^M\times\chi_\lambda^M\right)(u,v)=\min\{\psi_\lambda^M(u),\chi_\lambda^M(v)\}>\chi_\lambda^M(0)\Rightarrow$

$\left(\psi_\lambda^M\times\chi_\lambda^M\right)(u,v)>\left(\psi_\lambda^M\times\chi_\lambda^M\right)(0,0)$ which is a contradiction. Thus, the proof is completed.

By similar way we can prove point three. □

Theorem 5.5 Let ψ and χ be two FS of a D-algebras Ω such that $\psi_\alpha^T\times\chi_\alpha^T$ is FdI of $\Omega\times\Omega$ where $\alpha\in[0,T]$. Then, either ψ or χ is FdI of Ω.

Proof: To show that χ is FdI. From Theorem 5.3(i), we have $\psi_\alpha^T(0)\ge\psi_\alpha^T(u)$ or $\chi_\alpha^T(0)\ge\chi_\alpha^T(u)$ for each $u\in\Omega$. Thus,

i. Let $\chi_\alpha^T(0)\ge\chi_\alpha^T(u)$ then,

$\chi(0)+\alpha\ge\chi(u)+\alpha\Rightarrow\chi(0)\ge\chi(u)$.

ii. By Theorem 5.3 (iii), we get $\psi_\alpha^T(0)\ge\psi_\alpha^T(u)$ or $\psi_\alpha^T(0)\ge\chi_\alpha^T(u)$ for each $u\in\Omega$. If $\psi_\alpha^T(0)\ge\chi_\alpha^T(u)$ then, $\left(\psi_\alpha^T\times\chi_\alpha^T\right)(0,u)=\min\{\psi_\alpha^T(0),\chi_\alpha^T(u)\}=\chi_\alpha^T(u)$. ….(1)

Since $\psi_\alpha^T\times\chi_\alpha^T$ is FdI of $\Omega\times\Omega$ then for each $(u_1,u_2),(v_1,v_2)\in\Omega\times\Omega$ we have

$\left(\psi_\alpha^T\times\chi_\alpha^T\right)(u_1,u_2)=\min\{\psi_\alpha^T(u_1),\chi_\alpha^T(u_2)\}$

$\ge\min\{\min\{\psi_\alpha^T(u_1*v_1),\psi_\alpha^T(v_1)\},\min\{\chi_\alpha^T(u_2*v_2),\chi_\alpha^T(v_2)\}\}$

$\ge\min\{\min\{\psi_\alpha^T(u_1*v_1),\chi_\alpha^T(u_2*v_2)\},\min\{\psi_\alpha^T(v_1),\chi_\alpha^T(v_2)\}\}$

$\ge\min\{\left(\psi_\alpha^T\times\chi_\alpha^T\right)((u_1*v_1),(u_2*v_2)),\left(\psi_\alpha^T\times\chi_\alpha^T\right)(v_1,v_2)\}$.

That is $\left(\psi_\alpha^T\times\chi_\alpha^T\right)(u_1,u_2)\ge\min\{\left(\psi_\alpha^T\times\chi_\alpha^T\right)((u_1*v_1),(u_2*v_2)),\left(\psi_\alpha^T\times\chi_\alpha^T\right)(v_1,v_2)\}$.

Now, if $u_1=v_1=0$, then we have

$\left(\psi_\alpha^T\times\chi_\alpha^T\right)(0,u_2)\ge\min\{\left(\psi_\alpha^T\times\chi_\alpha^T\right)(0,u_2*v_2),\left(\psi_\alpha^T\times\chi_\alpha^T\right)(0,v_2)\}$ and by using (1), we get

$\chi_\alpha^T(u_2)\ge\min\{\chi_\alpha^T(u_2*v_2),\chi_\alpha^T(v_2)\}\Rightarrow$

$\chi(u_2)+\alpha\ge\min\{\chi(u_2*v_2)+\alpha,\chi(v_2)+\alpha\}\Rightarrow$

$\chi(u_2)+\alpha\ge\min\{\chi(u_2*v_2),\chi(v_2)\}+\alpha\Rightarrow$

$\chi(u_2)\ge\min\{\chi(u_2*v_2),\chi(v_2)\}$.

iii. $\left(\psi_\alpha^T\times\chi_\alpha^T\right)(u_1*v_1,u_2*v_2)\ge\min\{\left(\psi_\alpha^T\times\chi_\alpha^T\right)(u_1,u_2),\left(\psi_\alpha^T\times\chi_\alpha^T\right)(v_1,v_2)\}$, put $u_1=v_1=0$, then we have

$\left(\psi_{\alpha}^{\mathrm{T}} \times \chi_{\alpha}^{\mathrm{T}}\right)(0,u_2 * v_2) \geq \min\{\left(\psi_{\alpha}^{\mathrm{T}} \times \chi_{\alpha}^{\mathrm{T}}\right)(0,u_2), \left(\psi_{\alpha}^{\mathrm{T}} \times \chi_{\alpha}^{\mathrm{T}}\right)(0,v_2)\}$.

By using (1), we have

$\chi_{\alpha}^{\mathrm{T}}(u_2 * v_2) \geq \min\{\chi_{\alpha}^{\mathrm{T}}(u_2), \chi_{\alpha}^{\mathrm{T}}(v_2)\} \Rightarrow$

$\chi(u_2 * v_2) + \alpha \geq \min\{\chi(u_2) + \alpha, \chi(v_2) + \alpha\} \Rightarrow$

$\chi(u_2 * v_2) + \alpha \geq \min\{\chi(u_2), \chi(v_2)\} + \alpha \Rightarrow$

$\chi(u_2 * v_2) \geq \min\{\chi(u_2), \chi(v_2)\}$.

Therefore, χ is FdI of Ω. The second part can be checked by similar way. □

Theorem 5.6 Let ψ and χ be two FS of a D-algebras Ω such that $\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}$ is FdI of $\Omega \times \Omega$ where $\lambda \in [0,1]$. Then, either ψ or χ is FdI of Ω.

Proof: To show that χ is FdI of Ω. From Theorem 5.4(i), we have $\psi_{\lambda}^{\mathrm{M}}(0) \geq \psi_{\lambda}^{\mathrm{M}}(u)$ or $\chi_{\lambda}^{\mathrm{M}}(0) \geq \chi_{\lambda}^{\mathrm{M}}(u)$ for each $u \in \Omega$. Thus,

i. Let $\chi_{\lambda}^{\mathrm{M}}(0) \geq \chi_{\lambda}^{\mathrm{M}}(u)$ then,

$\lambda \cdot \chi(0) \geq \lambda \cdot \chi(u) \Rightarrow \chi(0) \geq \chi(u)$.

ii. By Theorem 5.4(iii), we have $\psi_{\lambda}^{\mathrm{M}}(0) \geq \psi_{\lambda}^{\mathrm{M}}(u)$ or $\psi_{\lambda}^{\mathrm{M}}(0) \geq \chi_{\lambda}^{\mathrm{M}}(u)$ for each $u \in \Omega$. If $\psi_{\lambda}^{\mathrm{M}}(0) \geq \chi_{\lambda}^{\mathrm{M}}(u)$ then,

$\left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(0,u) = \min\{\psi_{\lambda}^{\mathrm{M}}(0), \chi_{\lambda}^{\mathrm{M}}(u)\} = \chi_{\lambda}^{\mathrm{M}}(u)$. ....(1)

Since $\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}$ is FdI of $\Omega \times \Omega$ then for each $(u_1,u_2),(v_1,v_2) \in \Omega \times \Omega$ we have

$\left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(u_1,u_2) = \min\{\psi_{\lambda}^{\mathrm{M}}(u_1), \chi_{\lambda}^{\mathrm{M}}(u_2)\}$

$\geq \min\{\min\{\psi_{\lambda}^{\mathrm{M}}(u_1 * v_1), \psi_{\lambda}^{\mathrm{M}}(v_1)\},$

$\min\{\chi_{\lambda}^{\mathrm{M}}(u_2 * v_2), \chi_{\lambda}^{\mathrm{M}}(v_2)\}\}$

$\geq \min\{\min\{\psi_{\lambda}^{\mathrm{M}}(u_1 * v_1), \chi_{\lambda}^{\mathrm{M}}(u_2 * v_2)\}, \min\{\psi_{\lambda}^{\mathrm{M}}(v_1), \chi_{\lambda}^{\mathrm{M}}(v_2)\}\}$

$\geq \min\{\left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)((u_1 * v_1),(u_2 * v_2)), \left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(v_1,v_2)\}$.

That is $\left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(u_1,u_2) \geq \min\{\left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)((u_1 * v_1),(u_2 * v_2)), \left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(v_1,v_2)\}$.

Now, if $u_1 = v_1 = 0$, then we have

$\left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(0,u_2) \geq \min\{\left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(0,u_2 * v_2), \left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(0,v_2)\}$ and by using (1) we get $\chi_{\lambda}^{\mathrm{M}}(u_2) \geq \min\{\chi_{\lambda}^{\mathrm{M}}(u_2 * v_2), \chi_{\lambda}^{\mathrm{M}}(v_2)\} \Rightarrow$

$\lambda \cdot \chi(u_2) \geq \min\{\lambda \cdot \chi(u_2 * v_2), \lambda \cdot \chi(v_2)\} \Rightarrow$

$\lambda \cdot \chi(u_2) \geq \lambda \cdot \min\{\chi(u_2 * v_2), \chi(v_2)\} \Rightarrow$

$\chi(u_2) \geq \min\{\chi(u_2 * v_2), \chi(v_2)\}$.

iii. $\left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(u_1 * v_1, u_2 * v_2) \geq \min\{\left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(u_1,u_2), \left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(v_1,v_2)\}$, put $u_1 = v_1 = 0$, then we have

$\left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(0,u_2 * v_2) \geq \min \left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(0,u_2), \left(\psi_{\lambda}^{\mathrm{M}} \times \chi_{\lambda}^{\mathrm{M}}\right)(0,v_2)\}$. By using (1) we get

$\chi_{\lambda}^{\mathrm{M}}(u_2 * v_2) \geq \min\{\chi_{\lambda}^{\mathrm{M}}(u_2), \chi_{\lambda}^{\mathrm{M}}(v_2)\} \Rightarrow$

$\lambda \cdot \chi(u_2 * v_2) \geq \min\{\lambda \cdot \chi(u_2), \lambda \cdot \chi(v_2)\} \Rightarrow$

$\lambda \cdot \chi(u_2 * v_2) \geq \lambda \cdot \min\{\chi(u_2), \chi(v_2)\} \Rightarrow$

$\chi(u_2 * v_2) \geq \min\{\chi(u_2), \chi(v_2)\}$. Therefore, χ is FdI of Ω. The second part can be checked by similar way. □

6. FM- $\alpha\lambda$ -T OF D-ALGEBRAS

This section contains the idea of FM-$\alpha\lambda$-T of a D-algebras.

Definition 6.1 [11] Let ψ be a FS of Ω with $\alpha \in [0,\mathrm{T}]$ and $\mathrm{T} = 1 - \sup\{\psi(u) : u \in \Omega\}$ where, $\lambda \in [0,1]$. The mapping $\psi_{\alpha\lambda}^{\mathrm{TM}} : \Omega \to [0,1]$ is called FM-$\alpha\lambda$-T of ψ if it satisfies $\psi_{\alpha\lambda}^{\mathrm{TM}} = \alpha \cdot \psi(u) + \lambda$.

Example 6.1 Consider a D-algebras Ω which presented in Example 3.1. The FS ψ of Ω is given by

$$\psi(u) = \begin{cases} 0.7 & ,u = 0 \\ 0.02 & ,u \neq 0 \end{cases}$$

Then, ψ is a FS-algebra of Ω. Here $\mathrm{T} = 1 - \sup\{\psi(u) : u \in \Omega\} \Rightarrow \mathrm{T} = 1 - 0.7 = 0.3$. Take $\alpha = 0.2 \in [0,\mathrm{T}]$ and $\lambda = 0.4 \in [0,1]$. The mapping $\psi_{\alpha\lambda}^{\mathrm{TM}} : \Omega \to [0,1]$ is defined as

$$\psi_{\alpha\lambda}^{\mathrm{TM}} = \begin{cases} \alpha \cdot (0.7) + \lambda & ,u = 0 \\ \alpha \cdot (0.02) + \lambda & ,u \neq 0 \end{cases}$$

which satisfies $\psi_{\alpha\lambda}^{\mathrm{TM}} = \alpha \cdot \psi(u) + \lambda$, $\forall u \in \Omega$. Then, it's a FM-$\alpha\lambda$-T.

Theorem 6.1 Let ψ be a FS of Ω where $\alpha \in [0,\mathrm{T}], \lambda \in [0,1]$ with $\psi_{\alpha\lambda}^{\mathrm{TM}} : \Omega \to [0,1]$ is a FM-$\alpha\lambda$-T of ψ. Then, ψ is a FS-algebra of Ω iff $\psi_{\alpha\lambda}^{\mathrm{TM}}$ is a FS-algebra of Ω.

Proof: Since ψ is a FS-algebra of Ω, then, for each $u,v \in \Omega$ we have $\psi(u * v) \geq \min\{\psi(u), \psi(v)\} \Rightarrow$

$\alpha \cdot \psi(u * v) + \lambda \geq \alpha \cdot \min\{\psi(u), \psi(v)\} + \lambda$

$\geq \alpha \cdot \min\{\psi(u) + \lambda, \psi(v) + \lambda\}$

$\geq \min\{\alpha \cdot \psi(u) + \lambda, \alpha \cdot \psi(v) + \lambda\}$

$= \min\{\psi_{\alpha\lambda}^{\mathrm{TM}}(u), \psi_{\alpha\lambda}^{\mathrm{TM}}(v)\}$.

That is $\psi_{\alpha\lambda}^{\mathrm{TM}}(u * v) \geq \min\{\psi_{\alpha\lambda}^{\mathrm{TM}}(u), \psi_{\alpha\lambda}^{\mathrm{TM}}(v)\}$. Hence $\psi_{\alpha\lambda}^{\mathrm{TM}}$ is a FS-algebra of Ω.

Conversely, assume $\psi_{\alpha\lambda}^{\mathrm{TM}}$ be a FS-algebra of Ω, then, for each $u,v \in \Omega$ we have

$\psi_{\alpha\lambda}^{\mathrm{TM}}(u * v) \geq \min\{\psi_{\alpha\lambda}^{\mathrm{TM}}(u), \psi_{\alpha\lambda}^{\mathrm{TM}}(v)\}$ implies

$\alpha \cdot \psi(u * v) + \lambda \geq \min\{\alpha \cdot \psi(u) + \lambda, \alpha \cdot \psi(v) + \lambda\}$

$\geq \alpha \cdot \min\{\psi(u), \psi(v)\} + \lambda$.

That is $\alpha \cdot \psi(u * v) + \lambda \geq \alpha \cdot \min\{\psi(u), \psi(v)\} + \lambda \Rightarrow$ $\psi(u * v) \geq \min\{\psi(u), \psi(v)\}$. Therefore, ψ is a FS-algebra of Ω. □

Theorem 6.2 Let ψ be a FS of Ω where $\alpha \in [0,\mathrm{T}], \lambda \in [0,1]$ and $\psi_{\alpha\lambda}^{\mathrm{TM}} : \Omega \to [0,1]$ is a FM-$\alpha\lambda$-T of ψ. Then, ψ is FdI of Ω iff $\psi_{\alpha\lambda}^{\mathrm{TM}}$ is FdI of Ω.

Proof: Since ψ is FdI of Ω, then, for each $u,v \in \Omega$ we have $\psi(0) \geq \psi(u)$, $\psi(u) \geq \min\{\psi(u * v), \psi(v)\}$

and $\psi(u*v) \geq \min\{\psi(u), \psi(v)\}$. Thus,

i. $\psi(0) \geq \psi(u) \Rightarrow \alpha \cdot \psi(0) + \lambda \geq \alpha \cdot \psi(u) + \lambda \Rightarrow \psi^{\mathrm{TM}}_{\alpha\lambda}(0) \geq \psi^{\mathrm{TM}}_{\alpha\lambda}(u)$.

ii. $\psi(u) \geq \min\{\psi(u*v), \psi(v)\} \Rightarrow$

$$\alpha \cdot \psi(u) + \lambda \geq \alpha \cdot \min\{\psi(u*v), \psi(v)\} + \lambda$$
$$\geq \min\{\alpha \cdot \psi(u*v) + \lambda, \alpha \cdot \psi(v) + \lambda\}$$
$$= \min\{\psi^{\mathrm{TM}}_{\alpha\lambda}(u*v), \psi^{\mathrm{TM}}_{\alpha\lambda}(v)\}.$$

That is $\psi^{\mathrm{TM}}_{\alpha\lambda}(u) \geq \min\{\psi^{\mathrm{TM}}_{\alpha\lambda}(u*v), \psi^{\mathrm{TM}}_{\alpha\lambda}(v)\}$.

iii. $\psi(u*v) \geq \min\{\psi(u), \psi(v)\} \Rightarrow$

$$\alpha \cdot \psi(u*v) + \lambda \geq \alpha \cdot \min\{\psi(u), \psi(v)\} + \lambda$$
$$\geq \min\{\alpha \cdot \psi(u) + \lambda, \alpha \cdot \psi(v) + \lambda\}$$
$$= \min\{\psi^{\mathrm{TM}}_{\alpha\lambda}(u), \psi^{\mathrm{TM}}_{\alpha\lambda}(v)\}.$$

That is $\psi^{\mathrm{TM}}_{\alpha\lambda}(u*v) \geq \min\{\psi^{\mathrm{TM}}_{\alpha\lambda}(u), \psi^{\mathrm{TM}}_{\alpha\lambda}(v)\}$. Hence $\psi^{\mathrm{TM}}_{\alpha\lambda}$ is FdI of Ω.

Conversely, assume $\psi^{\mathrm{TM}}_{\alpha\lambda}$ be FdI of Ω, then for each $u, v \in \Omega$ we have

i. $\psi^{\mathrm{TM}}_{\alpha\lambda}(0) \geq \psi^{\mathrm{TM}}_{\alpha\lambda}(u) \Rightarrow \alpha \cdot \psi(0) + \lambda \geq \alpha \cdot \psi(u) + \lambda \Rightarrow \psi(0) \geq \psi(u)$.

ii. $\psi^{\mathrm{TM}}_{\alpha\lambda}(u) \geq \min\{\psi^{\mathrm{TM}}_{\alpha\lambda}(u*v), \psi^{\mathrm{TM}}_{\alpha\lambda}(v)\} \Rightarrow$

$$\alpha \cdot \psi(u) + \lambda \geq \min\{\alpha \cdot \psi(u*v) + \lambda, \alpha \cdot \psi(v) + \lambda\}$$
$$\geq \alpha \cdot \min\{\psi(u*v), \psi(v)\} + \lambda.$$

That is $\alpha \cdot \psi(u) + \lambda \geq \alpha \cdot \min\{\psi(u*v), \psi(v)\} + \lambda$ which implies $\psi(u) \geq \min\{\psi(u*v), \psi(v)\}$.

iii. $\psi^{\mathrm{TM}}_{\alpha\lambda}(u*v) \geq \min\{\psi^{\mathrm{TM}}_{\alpha\lambda}(u), \psi^{\mathrm{TM}}_{\alpha\lambda}(v)\} \Rightarrow$

$$\alpha \cdot \psi(u*v) + \lambda \geq \min\{\alpha \cdot \psi(u) + \lambda, \alpha \cdot \psi(v) + \lambda\}$$
$$\geq \alpha \cdot \min\{\psi(u), \psi(v)\} + \lambda.$$

That is $\alpha \cdot \psi(u*v) + \lambda \geq \alpha \cdot \min\{\psi(u), \psi(v)\} + \lambda \Rightarrow \psi(u*v) \geq \min\{\psi(u), \psi(v)\}$. Therefore ψ is FdI of Ω. □

7. HOMOMORPHISM OF FT AND FM OF D-ALGEBRAS

In this section, we provided the homomorphism of FT and FM of a D-algebras and proved some results which are based on the FS-algebra and FdI of a D-algebras Ω.

Theorem 7.1 If $f : (\Omega, *, 0) \to (\Gamma, *', 0')$ is a d-homomorphism with $\psi^{\mathrm{T}}_{\alpha}$ is a FT of a FS ψ. Then, the pre-image of $\psi^{\mathrm{T}}_{\alpha}$ is defined as $f^{-1}(\psi^{\mathrm{T}}_{\alpha}) = \psi^{\mathrm{T}}_{\alpha}(f(u))$ for each $u \in \Omega$. If ψ is FdI of a D-algebras Γ, then $f^{-1}(\psi^{\mathrm{T}}_{\alpha})$ is FdI of a D-algebras Ω.

Proof: Since ψ is FdI of a D-algebras Γ, then for each $v_1, v_2 \in \Gamma$ there exist $u_1, u_2 \in \Omega$ such that $f(u_1) = v_1$ and $f(u_2) = v_2$. Thus,

i. $\psi(0') \geq \psi(v_1) \Rightarrow \psi(0') + \alpha \geq \psi(v_1) + \alpha \Rightarrow$
$\psi(f(0)) + \alpha \geq \psi(f(u_1)) + \alpha \Rightarrow \psi^{\mathrm{T}}_{\alpha}(f(0)) \geq \psi^{\mathrm{T}}_{\alpha}(f(u_1)) \Rightarrow$
$f^{-1}(\psi^{\mathrm{T}}_{\alpha})(0) \geq f^{-1}(\psi^{\mathrm{T}}_{\alpha})(u)$.

ii. $\psi(v_1) \geq \min\{\psi(v_1 *' v_2), \psi(v_2)\}$ which implies

$$\psi(v_1) + \alpha \geq \min\{\psi(v_1 *' v_2), \psi(v_2)\} + \alpha$$
$$\geq \min\{\psi(v_1 *' v_2) + \alpha, \psi(v_2) + \alpha\}$$
$$\geq \min\{\psi^{\mathrm{T}}_{\alpha}(f(u_1) *' f(u_2)), \psi^{\mathrm{T}}_{\alpha}(f(u_2))\}$$
$$\geq \min\{\psi^{\mathrm{T}}_{\alpha}(f(u_1 * u_2)), \psi^{\mathrm{T}}_{\alpha}(f(u_2))\}$$
$$= \min\{f^{-1}(\psi^{\mathrm{T}}_{\alpha})(u_1 * u_2), f^{-1}(\psi^{\mathrm{T}}_{\alpha})(u_2)\}.$$

That is $f^{-1}(\psi^{\mathrm{T}}_{\alpha})(u_1) \geq \min\{f^{-1}(\psi^{\mathrm{T}}_{\alpha})(u_1 * u_2), f^{-1}(\psi^{\mathrm{T}}_{\alpha})(u_2)\}$.

iii. $\psi(v_1 *' v_2) \geq \min\{\psi(v_1), \psi(v_2)\} \Rightarrow$

$\cdot\, \psi(v_1 *' v_2) + \alpha \geq \min\{\psi(v_1), \psi(v_2)\} + \alpha$

$\Rightarrow \psi(v_1 *' v_2) + \alpha \geq \min\{\psi(v_1) + \alpha, \psi(v_2) + \alpha\}$

$\Rightarrow \psi(f(u_1) *' f(u_2)) + \alpha \geq \min\{\psi(f(u_1)) + \alpha, \psi(f(u_2)) + \alpha\}$

$\Rightarrow \psi^{\mathrm{T}}_{\alpha}(f(u_1 * u_2)) \geq \min\{\psi^{\mathrm{T}}_{\alpha}(f(u_1)), \psi^{\mathrm{T}}_{\alpha}(f(u_2))\}$

$\Rightarrow f^{-1}(\psi^{\mathrm{T}}_{\alpha})(u_1 * u_2) \geq \min\{f^{-1}(\psi^{\mathrm{T}}_{\alpha})(u_1), f^{-1}(\psi^{\mathrm{T}}_{\alpha})(u_2)\}$.

Hence, $f^{-1}(\psi^{\mathrm{T}}_{\alpha})$ is FdI of a D-algebras Ω. □

Theorem 7.2 If $f : (\Omega, *, 0) \to (\Gamma, *', 0')$ is a d-homomorphism with $\psi^{\mathrm{M}}_{\lambda}$ is a FM of the FS ψ. Then, the pre-image of $\psi^{\mathrm{M}}_{\lambda}$ is defined as $f^{-1}(\psi^{\mathrm{M}}_{\lambda}) = \psi^{\mathrm{M}}_{\lambda}(f(u))$ for each $u \in \Omega$. If ψ is FdI of a D-algebras Γ, then $f^{-1}(\psi^{\mathrm{M}}_{\lambda})$ is FdI of a D-algebras Ω.

Proof: Since ψ is FdI of a D-algebras Γ, then for each $v_1, v_2 \in \Gamma$ there exist $u_1, u_2 \in \Omega$ such that $f(u_1) = v_1$ and $f(u_2) = v_2$. Thus,

i. $\psi(0') \geq \psi(v_1) \Rightarrow \lambda \cdot \psi(0') \geq \lambda \cdot \psi(v_1) \Rightarrow$
$\lambda \cdot \psi(f(0)) \geq \lambda \cdot \psi(f(u_1)) \Rightarrow \psi^{\mathrm{M}}_{\lambda}(f(0)) \geq \psi^{\mathrm{M}}_{\lambda}(f(u_1)) \Rightarrow$
$f^{-1}(\psi^{\mathrm{M}}_{\lambda})(0) \geq f^{-1}(\psi^{\mathrm{M}}_{\lambda})(u)$.

ii. $\psi(v_1) \geq \min\{\psi(v_1 *' v_2), \psi(v_2)\} \Rightarrow$

$$\lambda \cdot \psi(v_1) \geq \lambda \cdot \min\{\psi(v_1 *' v_2), \psi(v_2)\}$$
$$\geq \min\{\lambda \cdot \psi(v_1 *' v_2), \lambda \cdot \psi(v_2)\}$$
$$\geq \min\{\psi^{\mathrm{M}}_{\lambda}(f(u_1) *' f(u_2)), \psi^{\mathrm{M}}_{\lambda}(f(u_2))\}$$
$$\geq \min\{\psi^{\mathrm{M}}_{\lambda}(f(u_1 * u_2)), \psi^{\mathrm{M}}_{\lambda}(f(u_2))\}$$
$$= \min\{f^{-1}(\psi^{\mathrm{M}}_{\lambda})(u_1 * u_2), f^{-1}(\psi^{\mathrm{M}}_{\lambda})(u_2)\}.$$

That is
$f^{-1}(\psi^{\mathrm{M}}_{\lambda})(u_1) \geq \min\{f^{-1}(\psi^{\mathrm{M}}_{\lambda})(u_1 * u_2), f^{-1}(\psi^{\mathrm{M}}_{\lambda})(u_2)\}$.

iii. $\psi(v_1 *' v_2) \geq \min\{\psi(v_1), \psi(v_2)\}$ which implies

$\lambda \cdot \psi(v_1 *' v_2) \geq \lambda \cdot \min\{\psi(v_1), \psi(v_2)\}$

$\Rightarrow \lambda \cdot \psi(v_1 *' v_2) \geq \min\{\lambda \cdot \psi(v_1), \lambda \cdot \psi(v_2)\}$

$\Rightarrow \lambda \cdot \psi(f(u_1) *' f(u_2)) \geq \min\{\lambda \cdot \psi(f(u_1)), \lambda \cdot \psi(f(u_2))\}$

$\Rightarrow \psi^{\mathrm{M}}_{\lambda}(f(u_1 * u_2)) \geq \min\{\psi^{\mathrm{M}}_{\lambda}(f(u_1)), \psi^{\mathrm{M}}_{\lambda}(f(u_2))\}$

$\Rightarrow f^{-1}(\psi^{\mathrm{M}}_{\lambda})(u_1 * u_2) \geq \min\{f^{-1}(\psi^{\mathrm{M}}_{\lambda})(u_1), f^{-1}(\psi^{\mathrm{M}}_{\lambda})(u_2)\}$.

Therefore, $f^{-1}(\psi^{\mathrm{M}}_{\lambda})$ is FdI of Ω. □

Corollary 7.1 Let $f : (\Omega, *, 0) \to (\Gamma, *', 0')$ be a d-homomorphism with $\psi^{\mathrm{T}}_{\alpha}$ is a FT of the FS ψ. Then, the pre-image of $\psi^{\mathrm{T}}_{\alpha}$ is defined as $f^{-1}(\psi^{\mathrm{T}}_{\alpha}) = \psi^{\mathrm{T}}_{\alpha}(f(u))$ for each $u \in \Omega$. If ψ is a FS-algebra of a D-algebras Γ, then $f^{-1}(\psi^{\mathrm{T}}_{\alpha})$ is a FS-algebra of a D-algebras Ω.

Proof: Since ψ is a FS-algebra of a D-algebras Γ, then for each $v_1, v_2 \in \Gamma$ there exist $u_1, u_2 \in \Omega$ such that $f(u_1) = v_1$, $f(u_2) = v_2$. Thus,

$\psi(v_1 *' v_2) \geq \min\{\psi(v_1), \psi(v_2)\}$

$\Rightarrow \psi(v_1 *' v_2) + \alpha \geq \min\{\psi(v_1), \psi(v_2)\} + \alpha$

$\Rightarrow \psi(v_1 *' v_2) + \alpha \geq \min\{\psi(v_1) + \alpha, \psi(v_2) + \alpha\}$

$\Rightarrow \psi(f(u_1) *' f(u_2)) + \alpha \geq \min\{\psi(f(u_1)) + \alpha, \psi(f(u_2)) + \alpha\}$

$\Rightarrow \psi_\alpha^{T}(f(u_1 * u_2)) \geq \min\{\psi_\alpha^{T}(f(u_1)), \psi_\alpha^{T}(f(u_2))\}$

$\Rightarrow f^{-1}(\psi_\alpha^{T})(u_1 * u_2) \geq \min\{f^{-1}(\psi_\alpha^{T})(u_1), f^{-1}(\psi_\alpha^{T})(u_2)\}$.

Therefore $f^{-1}(\psi_\alpha^{T})$ is a FS-algebra of a D-algebras Ω. □

Corollary 7.2 Let $f : (\Omega, *, 0) \to (\Gamma, *', 0')$ be a d-homomorphism with ψ_λ^{M} is a FM of the FS ψ. Then, the pre-image of ψ_λ^{M} is defined as $f^{-1}(\psi_\lambda^{M}) = \psi_\lambda^{M}(f(u))$ for each $u \in \Omega$. If ψ is a FS-algebra of a D-algebras Γ, then $f^{-1}(\psi_\lambda^{M})$ is a FS-algebra of a D-algebras Ω.

Proof: Since ψ is a FS-algebra of a D-algebras Γ, then for each $v_1, v_2 \in \Gamma$ there exist $u_1, u_2 \in \Omega$ such that $f(u_1) = v_1$ and $f(u_2) = v_2$. Thus, we have

$\psi(v_1 *' v_2) \geq \min\{\psi(v_1), \psi(v_2)\}$

$\Rightarrow \lambda \cdot \psi(v_1 *' v_2) \geq \lambda \cdot \min\{\psi(v_1), \psi(v_2)\}$

$\Rightarrow \lambda \cdot \psi(v_1 *' v_2) \geq \min\{\lambda \cdot \psi(v_1), \lambda \cdot \psi(v_2)\}$

$\Rightarrow \lambda \cdot \psi(f(u_1) *' f(u_2)) \geq \min\{\lambda \cdot \psi(f(u_1)), \lambda \cdot \psi(f(u_2))\}$

$\Rightarrow \psi_\lambda^{M}(f(u_1 * u_2)) \geq \min\{\psi_\lambda^{M}(f(u_1)), \psi_\lambda^{M}(f(u_2))\}$

$\Rightarrow f^{-1}(\psi_\lambda^{M})(u_1 * u_2) \geq \min\{f^{-1}(\psi_\lambda^{M})(u_1), f^{-1}(\psi_\lambda^{M})(u_2)\}$.

Therefore $f^{-1}(\psi_\lambda^{M})$ is a FS-algebra of Ω. □

Definition 7.1 Let $f : \Omega \to \Omega$ be an endomorphism and ψ_α^{T} be a FT of a FS ψ of a D-algebras Ω. Then, $(\psi_\alpha^{T})_f$ is a new FS of Ω defined by $(\psi_\alpha^{T})_f(u) = (\psi_\alpha^{T})(f(u)) = \psi(f(u)) + \alpha$ for each $u \in \Omega$ and $\alpha \in [0, T]$.

Definition 7.2 Let $f : \Omega \to \Omega$ be an endomorphism and ψ_λ^{M} be a FM of a FS ψ of a D-algebras Ω. Then, $(\psi_\lambda^{M})_f$ is a new FS of Ω defined by $(\psi_\lambda^{M})_f(u) = (\psi_\lambda^{M})(f(u)) = \lambda \cdot \psi(f(u))$ for each $u \in \Omega$ and $\lambda \in [0,1]$.

Theorem 7.3 Let $f : \Omega \to \Omega$ be an endomorphism of a D-algebras Ω. If ψ is FdI of Ω, then $(\psi_\alpha^{T})_f$ is FdI of a D-algebras Ω.

Proof: Let $u, v \in \Omega$, then

i. $(\psi_\alpha^{T})_f(0) = \psi_\alpha^{T}(f(0)) = \psi(f(0)) + \alpha \geq \psi(f(u)) + \alpha = \psi_\alpha^{T}(f(u)) = (\psi_\alpha^{T})_f(u) \Rightarrow (\psi_\alpha^{T})_f(0) \geq (\psi_\alpha^{T})_f(u)$.

ii $(\psi_\alpha^{T})_f(u) = \psi_\alpha^{T}(f(u)) = \psi(f(u)) + \alpha$

$\geq \min\{\psi(f(u * v)), \psi(f(v))\} + \alpha$

$\geq \min\{\psi(f(u * v)) + \alpha, \psi(f(v)) + \alpha\}$

$\geq \min\{\psi_\alpha^{T}(f(u * v)), \psi_\alpha^{T}(f(v))\}$

$= \min\{(\psi_\alpha^{T})_f(u * v), (\psi_\alpha^{T})_f(v)\}$.

That is $(\psi_\alpha^{T})_f(u) \geq \min\{(\psi_\alpha^{T})_f(u * v), (\psi_\alpha^{T})_f(v)\}$.

iii. $(\psi_\alpha^{T})_f(u * v) = \psi_\alpha^{T}(f(u * v)) = \psi(f(u * v)) + \alpha$

$\geq \min\{\psi(f(u)), \psi(f(v))\} + \alpha$

$\geq \min\{\psi(f(u)) + \alpha, \psi(f(v)) + \alpha\}$

$\geq \min\{\psi_\alpha^{T}(f(u)), \psi_\alpha^{T}(f(v))\}$

$= \min\{(\psi_\alpha^{T})_f(u), (\psi_\alpha^{T})_f(v)\}$.

That is $(\psi_\alpha^{T})_f(u * v) \geq \min\{(\psi_\alpha^{T})_f(u), (\psi_\alpha^{T})_f(v)\}$.

Therefore, $(\psi_\alpha^{T})_f$ is FdI of a D-algebras Ω. □

Theorem 7.4 Let $f : \Omega \to \Omega$ be an endomorphism of a D-algebras Ω. If ψ is FdI of Ω, then $(\psi_\lambda^{M})_f$ is FdI of a D-algebras Ω.

Proof: Let $u, v \in \Omega$, then

i. $(\psi_\lambda^{M})_f(0) = \psi_\lambda^{M}(f(0)) = \lambda \cdot \psi(f(0)) \geq \lambda \cdot \psi(f(u)) = \psi_\lambda^{M}(f(u)) = (\psi_\lambda^{M})_f(u) \Rightarrow (\psi_\lambda^{M})_f(0) \geq (\psi_\lambda^{M})_f(u)$.

ii. $(\psi_\lambda^{M})_f(u) = \psi_\lambda^{M}(f(u)) = \lambda \cdot \psi(f(u))$

$\geq \lambda \cdot \min\{\psi(f(u * v)), \psi(f(v))\}$

$\geq \min\{\lambda \cdot \psi(f(u * v)), \lambda \cdot \psi(f(v))\}$

$\geq \min\{\psi_\lambda^{M}(f(u * v)), \psi_\lambda^{M}(f(v))\}$

$= \min\{(\psi_\lambda^{M})_f(u * v), (\psi_\lambda^{M})_f(v)\}$.

That is $(\psi_\lambda^{M})_f(u) \geq \min\{(\psi_\lambda^{M})_f(u * v), (\psi_\lambda^{M})_f(v)\}$.

iii. $(\psi_\lambda^{M})_f(u * v) = \psi_\lambda^{M}(f(u * v))$

$= \lambda \cdot \psi(f(u * v)) \geq \lambda \cdot \min\{\psi(f(u)), \psi(f(v))\}$

$\geq \min\{\lambda \cdot \psi(f(u)), \lambda \cdot \psi(f(v))\}$

$\geq \min\{\psi_\lambda^{M}(f(u)), \psi_\lambda^{M}(f(v))\}$

$= \min\{(\psi_\lambda^{M})_f(u), (\psi_\lambda^{M})_f(v)\}$.

That is $(\psi_\lambda^{M})_f(u * v) \geq \min\{(\psi_\lambda^{M})_f(u), (\psi_\lambda^{M})_f(v)\}$.

Therefore, $(\psi_\lambda^{M})_f$ is FdI of a D-algebras Ω. □

Theorem 7.5 Let $f : (\Omega, *, 0) \to (\Gamma, *', 0')$ be an epimorphism. If $(\psi_\alpha^{T})_f$ is FdI of a D-algebras Ω, then ψ is FdI of a D-algebras Γ.

Proof: Since $(\psi_\alpha^{T})_f$ is FdI of D-algebras Ω, then for each $u_1, u_2 \in \Omega$ there exist $v_1, v_2 \in \Gamma$ such that $f(u_1) = v_1$ and $f(u_2) = v_2$. Thus,

i. $(\psi_\alpha^{T})_f(0) \geq (\psi_\alpha^{T})_f(u_1) \Rightarrow \psi_\alpha^{T}(f(0)) \geq \psi_\alpha^{T}(f(u_1)) \Rightarrow \psi(f(0)) + \alpha \geq \psi(v_1) + \alpha \Rightarrow \psi(0') \geq \psi(v_1)$.

ii. $(\psi_\alpha^{T})_f(u_1) \geq \min\{(\psi_\alpha^{T})_f(u_1 * u_2), (\psi_\alpha^{T})_f(u_2)\} \Rightarrow \psi_\alpha^{T}(f(u_1)) \geq \min\{\psi_\alpha^{T}(f(u_1 * u_2)), \psi_\alpha^{T}(f(u_2))\} \Rightarrow \psi(f(u_1)) + \alpha \geq \min\{\psi(f(u_1) *' f(u_2)) + \alpha, \psi(f(u_2)) + \alpha\} \Rightarrow \psi(v_1) + \alpha \geq \min\{\psi(v_1 *' v_2), \psi(v_2)\} + \alpha \Rightarrow \psi(v_1) \geq \min\{\psi(v_1 *' v_2), \psi(v_2)\}$.

iii. $(\psi_\alpha^{T})_f(u_1 * u_2) \geq \min\{(\psi_\alpha^{T})_f(u_1), (\psi_\alpha^{T})_f(u_2)\} \Rightarrow \psi_\alpha^{T}(f(u_1 * u_2)) \geq \min\{\psi_\alpha^{T}(f(u_1)), \psi_\alpha^{T}(f(u_2))\} \Rightarrow \psi(f(u_1) *' f(u_2)) + \alpha \geq \min\{\psi(f(u_1) + \alpha, \psi(f(u_2)) + \alpha\} \Rightarrow \psi(v_1 *' v_2) + \alpha \geq \min\{\psi(v_1), \psi(v_2)\} + \alpha \Rightarrow \psi(v_1 *' v_2) \geq \min\{\psi(v_1), \psi(v_2)\}$. Therefore, ψ is FdI of a D-algebras Γ. □

Theorem 7.6 Let $f:(\Omega,*,0)\to(\Gamma,*',0')$ be an epimorphism. If $(\psi_\lambda^M)_f$ is FdI of a D-algebras Ω, then ψ is FdI of a D-algebras Γ.

Proof: Since $(\psi_\lambda^M)_f$ is FdI of a D-algebras Ω, then for each $u_1,u_2\in\Omega$ there exist $v_1,v_2\in\Gamma$ such that $f(u_1)=v_1$ and $f(u_2)=v_2$. Thus,

i. $(\psi_\lambda^M)_f(0)\ge(\psi_\lambda^M)_f(u_1)\Rightarrow\psi_\lambda^M(f(0))\ge\psi_\lambda^M(f(u_1))\Rightarrow$
$\lambda\cdot\psi(f(0))\ge\lambda\cdot\psi(v_1)\Rightarrow\psi(0')\ge\psi(v_1)$.

ii. $(\psi_\lambda^M)_f(u_1)\ge\min\{(\psi_\lambda^M)_f(u_1*u_2),(\psi_\lambda^M)_f(u_2)\}\Rightarrow$
$\psi_\lambda^M(f(u_1))\ge\min\{\psi_\lambda^M(f(u_1*u_2)),\psi_\lambda^M(f(u_2))\}\Rightarrow$
$\lambda\cdot\psi(f(u_1))\ge\min\{\lambda\cdot\psi(f(u_1)*'f(u_2)),\lambda\cdot\psi(f(u_2))\}\Rightarrow$
$\lambda\cdot\psi(v_1)\ge\lambda\cdot\min\{\psi(v_1*'v_2),\psi(v_2)\}\Rightarrow$
$\psi(v_1)\ge\min\{\psi(v_1*'v_2),\psi(v_2)\}$.

iii. $(\psi_\lambda^M)_f(u_1*u_2)\ge\min\{(\psi_\lambda^M)_f(u_1),(\psi_\lambda^M)_f(u_2)\}\Rightarrow$
$\psi_\lambda^M(f(u_1*u_2))\ge\min\{\psi_\lambda^M(f(u_1)),\psi_\lambda^M(f(u_2))\}\Rightarrow$
$\lambda\cdot\psi(f(u_1)*'f(u_2))\ge\min\{\lambda\cdot\psi(f(u_1),\lambda\cdot\psi(f(u_2))\}\Rightarrow$
$\lambda\cdot\psi(v_1*'v_2)\ge\lambda\cdot\min\{\psi(v_1),\psi(v_2)\}\Rightarrow$
$\psi(v_1*'v_2)\ge\min\{\psi(v_1),\psi(v_2)\}$. Therefore, ψ is FdI of a D-algebras Γ. □

Theorem 7.7 Let $f:(\Omega,*,0)\to(\Gamma,*',0')$ be a d-homomorphism. If ψ is FdI of a D-algebras Γ, then $(\psi_\alpha^T)_f$ is FdI of a D-algebras Ω.

Proof: Since ψ is FdI of a D-algebras Γ, then for each $v_1,v_2\in\Gamma$ there exist $u_1,u_2\in\Omega$ such that $f(u_1)=v_1$ and $f(u_2)=v_2$. Thus,

i. $\psi(0')\ge\psi(v_1)\Rightarrow\psi(0')+\alpha\ge\psi(v_1)+\alpha\Rightarrow$
$\psi(f(0))+\alpha\ge\psi(f(u_1))+\alpha\Rightarrow\psi_\alpha^T(f(0))\ge\psi_\alpha^T(f(u_1))=$
$(\psi_\alpha^T)_f(0)\ge(\psi_\alpha^T)_f(u_1)$.

ii. $\psi(v_1)\ge\min\{\psi(v_1*'v_2),\psi(v_2)\}\Rightarrow$
$\psi(v_1)+\alpha\ge\min\{\psi(v_1*'v_2),\psi(v_2)\}+\alpha\Rightarrow$
$\psi(v_1)+\alpha\ge\min\{\psi(v_1*'v_2)+\alpha,\psi(v_2)+\alpha\}\Rightarrow$
$\psi_\alpha^T(f(u_1))\ge\min\{\psi_\alpha^T(f(u_1)*'f(u_2)),\psi_\alpha^T(f(u_2))\}=$
$(\psi_\alpha^T)_f(u_1)\ge\min\{(\psi_\alpha^T)_f(u_1*u_2),(\psi_\alpha^T)_f(u_2)\}$.

iii. $\psi(v_1*'v_2)\ge\min\{\psi(v_1),\psi(v_2)\}$ implies
$\psi(v_1*'v_2)+\alpha\ge\min\{\psi(v_1),\psi(v_2)\}+\alpha\Rightarrow$
$\psi(v_1*'v_2)+\alpha\ge\min\{\psi(v_1)+\alpha,\psi(v_2)+\alpha\}\Rightarrow$
$\psi_\alpha^T(f(u_1)*'f(u_2))\ge\min\{\psi_\alpha^T(f(u_1)),\psi_\alpha^T(f(u_2))\}=$
$(\psi_\alpha^T)_f(u_1*u_2)\ge\min\{(\psi_\alpha^T)_f(u_1),(\psi_\alpha^T)_f(u_2)\}$. Thus, $(\psi_\alpha^T)_f$ is FdI of a D-algebras Ω. □

Theorem 7.8 Let $f:(\Omega,*,0)\to(\Gamma,*',0')$ be a d-homomorphism. If ψ is FdI of a D-algebras Γ, then $(\psi_\lambda^M)_f$ is FdI of a D-algebras Ω.

Proof: Since ψ is FdI of a D-algebras Γ, then for each $v_1,v_2\in\Gamma$ there exist $u_1,u_2\in\Omega$ such that $f(u_1)=v_1$ and $f(u_2)=v_2$. Thus,

i. $\psi(0')\ge\psi(v_1)\Rightarrow\lambda\cdot\psi(0')\ge\lambda\cdot\psi(v_1)\Rightarrow\lambda\cdot\psi(f(0))$
$\ge\lambda\cdot\psi(f(u_1))\Rightarrow\psi_\lambda^M(f(0))\ge\psi_\lambda^M(f(u_1))=(\psi_\lambda^M)_f(0)\ge(\psi_\lambda^M)_f(u_1)$.

ii. $\psi(v_1)\ge\min\{\psi(v_1*'v_2),\psi(v_2)\}\Rightarrow$
$\lambda\cdot\psi(v_1)\ge\lambda\cdot\min\{\psi(v_1*'v_2),\psi(v_2)\}\Rightarrow$
$\lambda\cdot\psi(v_1)\ge\min\{\lambda\cdot\psi(v_1*'v_2),\lambda\cdot\psi(v_2)\}\Rightarrow$
$\psi_\lambda^M(f(u_1))\ge\min\{\psi_\lambda^M(f(u_1)*'f(u_2)),\psi_\lambda^M(f(u_2))\}=$
$(\psi_\lambda^M)_f(u_1)\ge\min\{(\psi_\lambda^M)_f(u_1*u_2),(\psi_\lambda^M)_f(u_2)\}$.

iii. $\psi(v_1*'v_2)\ge\min\{\psi(v_1),\psi(v_2)\}\Rightarrow\lambda\cdot\psi(v_1*'v_2)$
$\ge\lambda\cdot\min\{\psi(v_1),\psi(v_2)\}\Rightarrow\lambda\cdot\psi(v_1*'v_2)$
$\ge\min\{\lambda\cdot\psi(v_1),\lambda\cdot\psi(v_2)\}$
$\Rightarrow\psi_\lambda^M(f(u_1)*'f(u_2))\ge\min\{\psi_\lambda^M(f(u_1)),\psi_\lambda^M(f(u_2))\}$
$=(\psi_\lambda^M)_f(u_1*u_2)\ge\min\{(\psi_\lambda^M)_f(u_1),(\psi_\lambda^M)_f(u_2)\}$.

Thus, $(\psi_\lambda^M)_f$ is FdI of a D-algebras Ω. □

Conclusion

As a conclusion, the notion of FT and FM on a D-algebras has been introduced. Certain results that concern FS-algebra and FdI were proved. Moreover, we proved that the FS ψ of the D-algebra Ω become FdI iff the FT ψ_α^T of ψ is a FdI (resp. FM ψ_λ^M of ψ). Furthermore, we also showed that the FS ψ of a D-algebras Ω is a FS-algebra of Ω iff the $\psi_{\alpha\lambda}^{TM}$ is a FS-algebra of Ω (resp. FdI). In addition, some results on the homomorphism of a FT with FM of a D-algebras Ω were studied. As an extension, this article may include the study of anti-FT and anti-FM on a D-algebras Ω.

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الترجمات الضبابية والضرب الضبابي على الجبر-D

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[2] قسم الرياضيات ، كلية التربية للعلوم الصرفة ، جامعة الانبار ، الرمادي ، العراق

الملخص

يعتبر مفهوم الرياضيات الضبابية احد الفروع الجميلة في الرياضيات. هذا المفهوم تم تقديمه من قبل لطفي زاده [1]. منذ ذلك الوقت, تم النظر الى هذا المفهوم بطرق مختلفة في مجال الرياضيات البحتة والتطبيقية. في هذا البحث قدمنا مفهوم الترجمة الضبابية والضرب الضبابي على الجبر D . كذلك اثبتنا بعض المكافئات التي تعتمد على الجبر الجزئي والمثالي الضبابي من النوع D .اضف الى ذلك قدمنا بعض نتائج التشاكل للترجمة الضبابية والضرب الضبابي على الجبر من النوع D .