Fuzzy Translation and Fuzzy Multiplication on D-Algebras
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ABSTRACT
The concept of fuzzy set (FS) is one of the beautiful branches in Mathematics. This concept was initiated by Zadeh [1]. Since that time, many studies have been considered this concept by different ways in the field of pure and applied Mathematics. In this article, we introduced the notion of FT and FM on a D-algebras \( \Omega \), where FT is a fuzzy Translation and FM is a fuzzy Multiplication. We proved some characterizations of FT and FM of sub-algebra and d-ideal of a D-algebras \( \Omega \). Moreover, the notion of FM-\( \alpha \lambda \)-T has been investigated on the D-algebras \( \Omega \). Furthermore, some results on the d-homomorphism of FT and FM which based on the fuzzy d-ideal of a D-algebras are presented.

Keywords: D-algebras, Fuzzy d-ideal, Fuzzy multiplication, Fuzzy magnified-\( \alpha \lambda \)-translation, Fuzzy set, d-homomorphism.

1. Introduction
Throughout this study we mean by \( \Omega \) and \( \Gamma \) to be two D-algebras, FT\( (\alpha \lambda -T) \) is a fuzzy Translation, FM\( (\alpha \lambda -M) \) is a fuzzy multiplication and FM-\( \alpha \lambda -T \) is a Fuzzy Magnified-\( \alpha \lambda \)-Translation. Recently, much attention has been given to study the concept of fuzzy algebra which is one of the influential branches in Mathematics. Zadeh in [1] provided the concept of fuzzy set. This concept has been applied on several types of algebraic concepts such as rings, modules, groups, topologies and vector spaces. The study of [2] investigated the idea of the fuzzy d-ideal of a D-algebras with some of its properties have been proved. The concept of a D-algebras which is considered as another popularization of BCK-algebras has been provided by Neggers and Kim [3]. Then some of the relations among BCK-algebras and D-algebras were discussed. A paper by Lee et. al [4] presented the FT and the FM of fuzzy sub-algebras of BCK/BCI-algebras. They discussed the relations among the FT and the FM. Furthermore, the notion of Q-ideal and fuzzy Q-ideal in Q-algebras has been investigated by Mostafa et. al [5]. The study of Hameed et. al [6] presented the definition of FT and FM of CI-algebras, and several properties of this notion were studied. While, the FT and the FM of Q-algebras were given by Hameed and Mohammed [7]. In addition, the notion of \( \omega \)-FT with \( \omega \)-FM on a BP-algebras are introduced by Prasanna et. al [8]. Priya and Ramachandran [9] provided the FT and FM of a PS-algebras. Then, the homomorphism and the Cartesian product of the FT and FM of a PS-algebras are also presented by Priya and Ramachandran [10]. Now days, Alshehri [11] studied the FT and FM of a BRK-algebras and some of their properties were discussed. In this paper we introduced the notion of FT and FM on D-algebras and discussed some of its properties. The contents of this paper have been structured as follows: In section two, some basic definitions and previous results that are needed in this research are presented. While, in section three, the FT and FM of d-sub-algebras are presented. Section four contains the FT and FM of d-ideals. Section five, followed by the Cartesian product of FT and FM of d-ideals. The notion of FM-\( \alpha \lambda \)-T of a D-algebras has been stated in section six. In section seven, the homomorphism of the FT and FM of a D-algebras has been studied. Finally, the conclusions and further research scope of this paper are given in section eight.
2. Basic concepts

In this section, some of the previous results that are needed in this study are presented. We start with the following observations which are given as follows.

\[ \Omega \] is said to be \( \bigstar \)-D-algebras, if all the following conditions are holds:

i. \( u \bigstar u = 0 \),

ii. \( 0 \bigstar u = 0 \),

iii. \( u \bigstar v = 0 \) and \( v \bigstar u = 0 \) implies \( u = v \) for each \( u, v \in \Omega \).

**Definition 2.2** [2] Let \( \Omega \) be a \( \bigstar \)-D-algebra. Then, \( \Phi \neq S \subset \Omega \) is said to be sub-algebra of \( \Omega \) if \( u \bigstar v \in S \) where \( u, v \in S \).

**Definition 2.3** [2] Let \( \Omega \) be a \( \bigstar \)-D-algebras with \( \bigstar H \subset \Omega \), then \( \bigstar H \) is called d-ideal of \( \Omega \) if all the following are fulfilled:

i. \( 0 \in \bigstar H \),

ii. \( u \bigstar v \in \bigstar H \) and \( v \in \bigstar H \) implies \( u \in \bigstar H \),

iii. \( u \in \bigstar H \) and \( v \in \bigstar H \) implies \( u \bigstar v \in \bigstar H \).

**Definition 2.4** [2] Let \( \psi \) be a FS of \( \Omega \), then \( \psi \) is called \( \bigstar \)-ideal of \( \Omega \) if all the following are fulfilled:

i. \( \psi(0) \leq \psi(u) \),

ii. \( \psi(u) \geq \min\{\psi(u \bigstar v), \psi(v)\} \),

iii. \( \psi(u \bigstar v) \geq \min\{\psi(u), \psi(v)\} \) for each \( u, v \in \Omega \).

**Definition 2.5** [2] The FS \( \psi \) of a \( \bigstar \)-D-algebras \( \Omega \) is said to be FS-algebra of \( \Omega \) if \( \psi(u \bigstar v) \geq \min\{\psi(u), \psi(v)\} \) where \( u, v \in \Omega \).

**Definition 2.6** [2] The FS \( \psi \) of a set \( \Omega \) is the function \( \psi : \Omega \rightarrow [0,1] \).

**Definition 2.7** [2] Let \( \psi_1 \) and \( \psi_2 \) be two FS of \( \Omega \). Then, the Cartesian product of \( \psi_1 \) and \( \psi_2 \) is the mapping \( \psi_1 \times \psi_2 : \Omega \times \Omega \rightarrow [0,1] \) which is defined as \( (\psi_1 \times \psi_2)(u,v) = \min\{\psi_1(u),\psi_2(v)\}, \forall u,v \in \Omega \).

**Definition 2.8** [2] Let \( \psi \) be a FS of \( \bigstar \Gamma \) and \( f : \Omega \rightarrow \Gamma \) be a mapping of a \( \bigstar \)-D-algebras. Then, the mapping \( \psi' \) is the inverse image of \( \psi \) under \( f \) such that \( \psi'(u) = \psi(f(u)) \) for each \( u \in \Omega \).

**Theorem 2.1** [2] Let \( \psi_1 \) and \( \psi_2 \) are two \( \bigstar \)-ideal of \( \bigstar \)-D-algebras \( \Omega \). Then, \( \psi_1 \times \psi_2 \) is \( \bigstar \)-ideal of \( \bigstar \Omega \).

**Definition 2.9** [3] Let \( \Omega \) and \( (\bigstar, \cdot, ') \) and \( \Gamma \) be two \( \bigstar \)-D-algebras. The mapping \( f : \Omega \rightarrow \Gamma \) is said to be \( \bigstar \)-homomorphism if for each \( u,v \in \Omega \) we have \( f(\bigstar uv) = f(u) \cdot f(v) \).

**Definition 2.10** [4] Let \( \psi \) be a FS of \( \Omega \) and \( \bigstar \alpha \in [0,1] \). Then, \( \bigstar \)-FM of \( \bigstar \)-ideal of \( \bigstar \)-D-algebras \( \Omega \), the FT \( \psi'_{\bigstar \alpha} \) of \( \psi \) is the mapping \( \psi'_{\bigstar \alpha} : \Omega \rightarrow [0,1] \) such that \( \psi'_{\bigstar \alpha}(u) = \psi(u) + \alpha \) for each \( u \in \Omega \).

**Definition 2.11** [4] Let \( \psi \) be a FS of \( \Omega \) with \( \lambda \in [0,1] \). Then, \( \bigstar \)-FM (F-\( \bigstar \)-M) of \( \psi \) is the mapping \( \psi'_{\bigstar \alpha} : \Omega \rightarrow [0,1] \) such that \( \psi'_{\bigstar \alpha}(u) = \lambda \cdot \psi(u) \) for each \( u \in \Omega \).

3. FT AND FM OF D-SUB-ALGEBRAS

This section presents the notion of FT and FM of a \( \bigstar \)-D-algebras. In what follows, let \( (\Omega, \bigstar, 0) \) be a \( \bigstar \)-D-algebras. Then, for any FS \( \psi \) of \( \Omega \) we symbolize \( T = 1 - \sup\{\psi(u) : u \in \Omega \} \) unless otherwise we mentioned.

**Definition 3.1** A F-\( \bigstar \)-T set \( \psi^T \) of a FS \( \psi \) is called F-\( \bigstar \)-T sub-algebra if \( \psi^T(u \bigstar v) \geq \min\{\psi^T(u),\psi^T(v)\} \) for each \( u,v \in \Omega \) and \( \alpha \in [0,1] \).

**Definition 3.2** A F-\( \lambda \)-M sub-algebra if \( \psi^M(u \bigstar v) \geq \min\{\psi^M(u),\psi^M(v)\} \) for each \( u,v \in \Omega \) and \( \lambda \in [0,1] \).

**Example 3.1** Let \( (\Omega = [0,1,2], \bigstar, 0) \) be a \( \bigstar \)-D-algebras given as follows [3]:

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Table 3.1

Define the FS \( \psi : \Omega \rightarrow [0,1] \) by

\[
\psi(u) = \begin{cases} 
0.7 & \text{if } u = 0 \\
0.02 & \text{if } u \neq 0 
\end{cases}
\]

Then, \( \psi \) is a FS-algebra of \( \Omega \). Here \( T = 1 - \sup\{\psi(u) : u \in \Omega \} \rightarrow T=1-0.7=0.3 \). Take \( \alpha = 0.1 \in [0,1] \), and \( \psi^T_{\bigstar \alpha} : \Omega \rightarrow [0,1] \) is defined by

\[
\psi^T_{\bigstar \alpha} = \begin{cases} 
0.7 + \alpha & \text{if } u = 0 \\
0.02 + \alpha & \text{if } u \neq 0 
\end{cases}
\]

which satisfies \( \psi^T_{\bigstar \alpha}(u) = \psi(u) + \alpha \), \( \forall u \in \Omega \). Then, it’s a F-\( \bigstar \)-T. Furthermore, if we take \( \lambda = 0.2 \in [0,1] \), then \( \psi^M : \Omega \rightarrow [0,1] \) is defined by

\[
\psi^M = \begin{cases} 
\lambda \cdot (0.7) & \text{if } u = 0 \\
\lambda \cdot (0.02) & \text{if } u \neq 0 
\end{cases}
\]

which satisfies \( \psi^M(u) = \lambda \cdot \psi(u) \), \( \forall u \in \Omega \). Then, it’s a F-\( \lambda \)-M.

**Theorem 3.1** For any FS-algebra \( \psi \) of a \( \bigstar \)-D-algebras \( \Omega \), the FT \( \psi_{\bigstar \alpha}^T \) of \( \psi \) is a FS-algebra of \( \Omega \) where \( \alpha \in [0,1] \).

**Proof:** Suppose that \( \psi \) is a FS-algebra of a \( \bigstar \)-D-algebras \( \Omega \), then \( \forall u,v \in \Omega \) we get \( \psi(\bigstar uv) \geq \min\{\psi(u),\psi(v)\} \geq \psi(\bigstar uv) + \alpha \geq \min\{\psi(u) + \alpha, \psi(v) + \alpha\} \geq \min\{\psi_{\bigstar \alpha}(u),\psi_{\bigstar \alpha}(v)\} \).

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That is \( \psi_T(\alpha + v) \geq \min\{\psi_T(\alpha), \psi_T(v)\} \). Therefore, \( \psi_T \) is a FS-algebra of \( \Omega \). □

**Theorem 3.2** Let \( \psi \) be a FS of a D-algebra \( \Omega \). Then, if \( \psi \) is a FS-algebra of \( \Omega \), then \( \psi \) is a FS-algebra of \( \Omega \) if all of the following conditions are holds:

i. \( \psi(0) \geq \psi(u) \Rightarrow \psi(0) + \alpha \geq \psi(u) + \alpha \Rightarrow \psi(0) \geq \psi(u) \)

ii. \( \psi(0) + \alpha \geq \psi(u) + \alpha \Rightarrow \psi(0) \geq \psi(u) \)

That is \( \psi_T(\alpha + v) \geq \min\{\psi_T(u), \psi_T(v)\} \). Therefore, \( \psi_T \) is a FS-algebra of \( \Omega \). □

**Theorem 3.3** For any FS-algebra \( \psi \) of a D-algebra \( \Omega \) with \( \lambda \in [0,1] \), the FM \( \psi^M \) of \( \psi \) is also FS-algebra of \( \Omega \). 

**Proof:** Since \( \psi \) is a FS-algebra of a D-algebra \( \Omega \), then for each \( u, v \in \Omega \) we get

\[
\psi(u + v) \geq \min\{\psi(u), \psi(v)\} \Rightarrow \psi(u + v) \geq \min\{\psi(u), \psi(v)\} \geq \min\{\lambda \cdot \psi(u), \lambda \cdot \psi(v)\} = \lambda \cdot \min\{\psi(u), \psi(v)\}
\]

Thus, \( \psi^M(u + v) \geq \min\{\psi^M(u), \psi^M(v)\} \). Therefore, \( \psi^M \) is a FS-algebra of \( \Omega \). □

**Theorem 3.4** Let \( \psi \) be a FS of a D-algebra \( \Omega \) with \( \lambda \in [0,1] \). If the FM \( \psi^M \) of \( \psi \) is a FS-algebra of \( \Omega \), then \( \psi^M \) is a FS-algebra of \( \Omega \). 

**Proof:** Suppose \( \psi^M \) is a FS-algebra of \( \Omega \), then for each \( u, v \in \Omega \) we get

\[
\psi^M(u + v) \geq \min\{\psi^M(u), \psi^M(v)\} \Rightarrow \psi(u + v) \geq \min\{\lambda \cdot \psi(u), \lambda \cdot \psi(v)\} = \lambda \cdot \min\{\psi(u), \psi(v)\}.
\]

That is \( \lambda \cdot \psi(u + v) \geq \lambda \cdot \min\{\psi(u), \psi(v)\} \Rightarrow \psi(u + v) \geq \min\{\psi(u), \psi(v)\} \). Hence, \( \psi^M \) is a FS-algebra of \( \Omega \). □

**4. FT AND FM OF D-IDEALS**

In this section, we presented the notion of FT and FM of d-ideals.

**Definition 4.1** A \( F-\alpha -T \) set \( \psi_T \) of a FS \( \psi \) is said to be \( F-\alpha -T \) d-ideal of \( \Omega \) if all of the following conditions are holds:

i. \( \psi_T(0) \geq \psi_T(u) \),

ii. \( \psi_T(u) \leq \min\{\psi_T(u + v), \psi_T(v)\} \),

iii. \( \psi_T(u + v) \geq \min\{\psi_T(u), \psi_T(v)\} \), \( \forall u, v \in \Omega \) and \( \alpha \in (0,T] \).

**Definition 4.2** A \( F-\lambda -M \) set \( \psi_M \) of a FS \( \psi \) is said to be \( F-\lambda -M \) d-ideal of \( \Omega \) if all of the following conditions are holds:

i. \( \psi_M(0) \geq \psi_M(u) \),

ii. \( \psi_M(u) \leq \min\{\psi_M(u + v), \psi_M(v)\} \),

iii. \( \psi_M(u + v) \geq \min\{\psi_M(u), \psi_M(v)\} \), \( \forall u, v \in \Omega \) and \( \lambda \in [0,1] \).

**Theorem 4.1** Let \( \psi \) be a FS of a D-algebras \( \Omega \) and \( \psi^M \) be a FT of \( \psi \) with \( \alpha \in [0,T] \). Then, \( \psi \) is FdI of \( \Omega \) iff \( \psi^M \) is FdI of \( \Omega \).

**Proof:** Suppose that \( \psi \) is FdI of \( \Omega \), then for each \( u, v \in \Omega \) we have

i. \( \psi(0) \geq \psi(u) \Rightarrow \psi(0) + \alpha \geq \psi(u) + \alpha \Rightarrow \psi(0) \geq \psi(u) \)

ii. \( \psi(u) + \alpha \geq \psi(0) + \alpha \Rightarrow \psi(u) \)

That is \( \psi_T(u) \geq \min\{\psi_T(u + v), \psi_T(v)\} \). Therefore, \( \psi_T \) is a FS-algebra of \( \Omega \). □

**Theorem 4.2** Let \( \psi \) be a FS of a D-algebras \( \Omega \) and \( \psi^M \) be a FM of \( \psi \) where \( \lambda \in [0,1] \). Then, \( \psi \) is FdI of \( \Omega \) iff \( \psi^M \) is FdI of \( \Omega \).

**Proof:** Suppose that \( \psi \) is FdI of \( \Omega \), then for each \( u, v \in \Omega \) we have

i. \( \psi(0) \geq \psi(u) \Rightarrow \lambda \cdot \psi(0) \geq \lambda \cdot \psi(u) \Rightarrow \psi_M(0) \geq \psi_M(u) \)

ii. \( \lambda \cdot \psi(u) \geq \lambda \cdot \min\{\psi(u + v), \psi(v)\} \)

That is \( \psi_M(u + v) \geq \min\{\psi_M(u + v), \psi_M(v)\} \). Therefore, \( \psi_M \) is a FS-algebra of \( \Omega \). □

Conversely, let \( \psi^M \) of \( \psi \) be FdI of \( \Omega \) for some \( \lambda \in [0,1] \). Then, for each \( u, v \in \Omega \) we have

i. \( \psi_M(0) = \lambda \cdot \psi(0) \Rightarrow \lambda \cdot \psi(u) \Rightarrow \psi_M(0) \geq \psi_M(u) \).
ii. \( \psi^M_{\alpha}(u) \geq \min \{ \psi^M_{\alpha}(u \ast v), \psi^M_{\beta}(v) \} \Rightarrow \lambda \cdot \psi(u) \)
\[ \geq \min \{ \lambda \cdot \psi(u \ast v), \lambda \cdot \psi(v) \} \Rightarrow \lambda \cdot \psi(u) \]
\[ \Rightarrow \psi(u) \geq \lambda \cdot \min[\psi(u \ast v), \psi(v)] \].

iii. \( \psi^M_{\alpha}(u \ast v) \geq \min \{ \psi^M_{\alpha}(u), \psi^M_{\beta}(v) \} \)
\[ \Rightarrow \lambda \cdot \psi(u \ast v) \geq \min \{ \lambda \cdot \psi(u), \lambda \cdot \psi(v) \} \]
\[ \Rightarrow \lambda \cdot \psi(u \ast v) \geq \lambda \cdot \min[\psi(u), \psi(v)] \]
\[ \Rightarrow \psi(u \ast v) \geq \min[\psi(u), \psi(v)] \]. Thus, \( \psi \) is FdI of \( \Omega \).

Theorem 4.3 The Intersection of two FdI translation of a D-algebras \( \Omega \) is FdI translation of a D-algebras \( \Omega \).

Proof: Let \( \psi^T_\alpha \) and \( \psi^T_\beta \) are two FdI translation of a FdI \( \psi \) of \( \Omega \) with \( \alpha, \beta \in [0, T] \). Then, for each \( u, v \in \Omega \) we have:

i. \( \{ \psi^T_\alpha \cap \psi^T_\beta \}(0) = \min \{ \psi^T_\alpha(0), \psi^T_\beta (0) \} = \min \{ \psi(0) + \alpha, \psi(0) + \beta \} \)
\[ \geq \min \{ \psi(u \ast v) + \alpha, \psi(u \ast v) + \beta \} \]
\[ \Rightarrow \psi(u \ast v) \geq \lambda \cdot \min[\psi(u \ast v), \psi(v)] \] where \( \alpha, \beta, \lambda \in [0, T] \).

ii. \( \{ \psi^T_\alpha \cap \psi^T_\beta \}(u \ast v) = \min \{ \psi^T_\alpha(u \ast v), \psi^T_\beta(u \ast v) \} \)
\[ \geq \min \{ \psi(u \ast v) + \alpha, \psi(u \ast v) + \beta \} \]
\[ \Rightarrow \psi(u \ast v) \geq \lambda \cdot \min[\psi(u \ast v), \psi(v)] \] where \( \alpha, \beta, \lambda \in [0, T] \).

iii. \( \{ \psi^T_\alpha \cap \psi^T_\beta \}(u \ast v) = \min \{ \psi^T_\alpha(u \ast v), \psi^T_\beta(u \ast v) \} \)
\[ \geq \min \{ \psi(u \ast v) + \alpha, \psi(u \ast v) + \beta \} \]
\[ \Rightarrow \psi(u \ast v) \geq \lambda \cdot \min[\psi(u \ast v), \psi(v)] \] where \( \alpha, \beta, \lambda \in [0, T] \).

Therefore \( \psi^T_\alpha \cap \psi^T_\beta \) is FdI translation of \( \Omega \).

5. CARTESIAN PRODUCT OF FT AND FM OF D-IDEALS

This section presents the Cartesian product of FT and FM of d-ideals.

Definition 5.1 Let \( \psi^T_\alpha \) and \( \psi^T_\beta \) be two fuzzy algebras of a D-algebras \( \Omega \). Then, the Cartesian product of \( \psi^T_\alpha \) and \( \psi^T_\beta \) is symbolized by \( \psi^T_\alpha \times \psi^T_\beta : \Omega \times \Omega \rightarrow [0, 1] \) and given as
\[ \{ \psi^T_\alpha \times \psi^T_\beta \}(u, v) = \min \{ \psi^T_\alpha(u), \psi^T_\beta(v) \}, \forall u, v \in \Omega \] and \( \alpha, \beta, \lambda \in [0, T] \).

Definition 5.2 Let \( \psi^M_\alpha \) and \( \psi^M_\beta \) be two fuzzy multiplication of a D-algebra \( \Omega \). Then, the Cartesian product of \( \psi^M_\alpha \) and \( \psi^M_\beta \) is symbolized by \( \psi^M_\alpha \times \psi^M_\beta : \Omega \times \Omega \rightarrow [0, 1] \) and given as
\[ \{ \psi^M_\alpha \times \psi^M_\beta \}(u, v) = \min \{ \psi^M \alpha(u), \psi^M_\alpha(v) \}, \forall u, v \in \Omega \] and \( \alpha, \beta, \lambda \in [0, 1] \).

Theorem 4.5 The Intersection of two FdI translation of a D-algebras \( \Omega \) is FdI translation of a D-algebras \( \Omega \).

Proof: Suppose that \( \psi^M_\alpha \) and \( \psi^M_\beta \) are FdI multiplication of a FdI \( \psi \) of \( \Omega \) with \( \lambda, \mu \in [0, 1] \). Then, for each \( u, v \in \Omega \) we have:

i. \( \{ \psi^M_\alpha \cap \psi^M_\beta \}(0) = \min \{ \psi^M_\alpha(0), \psi^M_\beta (0) \} = \min \{ \lambda \cdot \psi(0), \mu \cdot \psi(0) \} \)
\[ \geq \min \{ \lambda \cdot \psi(u \ast v), \mu \cdot \psi(v) \} \]
\[ \Rightarrow \lambda \cdot \psi(u \ast v) \geq \min \{ \lambda \cdot \psi(u), \mu \cdot \psi(v) \} \]
\[ \Rightarrow \psi(u \ast v) \geq \min \{ \lambda \cdot \psi(u), \mu \cdot \psi(v) \} \] where \( \lambda, \mu, \alpha \in [0, T] \).

ii. \( \{ \psi^M_\alpha \cap \psi^M_\beta \}(u \ast v) = \min \{ \psi^M_\alpha(u \ast v), \psi^M_\beta(u \ast v) \} \)
\[ \geq \min \{ \lambda \cdot \psi(u \ast v), \mu \cdot \psi(u \ast v) \} \]
\[ \Rightarrow \lambda \cdot \psi(u \ast v) \geq \min \{ \lambda \cdot \psi(u), \mu \cdot \psi(v) \} \]
\[ \Rightarrow \psi(u \ast v) \geq \min \{ \lambda \cdot \psi(u), \mu \cdot \psi(v) \} \] where \( \lambda, \mu, \alpha \in [0, T] \).

Then, for each \( u, v \in \Omega \) we have:

\[ \{ \psi^M_\alpha \cap \psi^M_\beta \}(u \ast v) = \min \{ \psi^M_\alpha(u \ast v), \psi^M_\beta(u \ast v) \} \]
\[ \geq \min \{ \lambda \cdot \psi(u \ast v), \mu \cdot \psi(u \ast v) \} \]
\[ \Rightarrow \lambda \cdot \psi(u \ast v) \geq \min \{ \lambda \cdot \psi(u), \mu \cdot \psi(v) \} \]
\[ \Rightarrow \psi(u \ast v) \geq \min \{ \lambda \cdot \psi(u), \mu \cdot \psi(v) \} \] where \( \lambda, \mu, \alpha \in [0, T] \).

Therefore \( \psi^M_\alpha \cap \psi^M_\beta \) is FdI translation of \( \Omega \).

Theorem 4.6 The Intersection of two FdI translation of a D-algebras \( \Omega \) is FdI translation of a D-algebras \( \Omega \).

Proof: Clear from Theorem 4.5. □

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Therefore, $\psi^T \times \chi^T \in \text{FDI of } \Omega \times \Omega$.

**Theorem 5.2** Let $\psi$ and $\chi$ be two FDI of a D-algebras $\Omega$. Furthermore, let $T = \min\{T_\psi, T_\chi\}$ where $T_\psi = 1 - \sup\{\psi(u) : u \in \Omega\}$ and $T_\chi = 1 - \sup\{\chi(u) : u \in \Omega\}$ where $\Omega \in [0,1]$. Then, the FM of $\psi \times \chi$ is FDI of $\Omega \times \Omega$.

**Proof:** Suppose that $\psi$ and $\chi$ are FDI of a D-algebras $\Omega$ with $\lambda \in [0,1]$. By Theorem 4.2, $\psi^T$ and $\chi^T$ are FDI of $\Omega$. By Theorem 2.1, we have $\psi^T \times \chi^T \in \text{FDI of } \Omega \times \Omega$. Now, let $u,v \in \Omega$ then

$$
(\psi \times \chi)^T(u,v) = \lambda \cdot (\psi \times \chi)(u,v) = \lambda \cdot \min\{\psi(u), \chi(v)\} = 
\min\{\lambda \cdot \psi(u), \lambda \cdot \chi(v)\} = 
\min\{\psi^T(u), \chi^T(v)\} = (\psi^T \times \chi^T)(u,v).
$$

Therefore, $\psi^T \times \chi^T$ is FDI of $\Omega \times \Omega$.

**Theorem 5.3** Let $\psi$ and $\chi$ be two FS of a D-algebras $\Omega$ where $\psi^T \times \chi^T$ is FDI of $\Omega \times \Omega$ and $\alpha \in [0, T]$. Then, i. Either $\psi^T(0) \geq \psi^T(u)$ or $\chi^T(0) \geq \chi^T(u)$ for each $u \in \Omega$.

ii. If $\psi^T(0) \geq \psi^T(u)$, then either $\chi^T(0) \geq \chi^T(u)$ or $\chi^T(0) \geq \chi^T(u)$ for each $u \in \Omega$.

iii. If $\chi^T(0) \geq \chi^T(u)$, then either $\psi^T(0) \geq \psi^T(u)$ or $\psi^T(0) \geq \psi^T(u)$ for each $u \in \Omega$.

**Proof:** i. Assume that $\psi^T(0) < \psi^T(u)$ and $\chi^T(0) < \chi^T(u)$ for some $u,v \in \Omega$. Then,

$$
(\psi^T \times \chi^T)(u,v) = \min\{\psi^T(u), \chi^T(v)\} > \min\{\psi^T(0), \chi^T(0)\} = (\psi^T \times \chi^T)(0,0).
$$

Thus, the proof is completed.

**Theorem 5.4** Let $\psi$ and $\chi$ be two FS of a D-algebras $\Omega$ where $\psi^T \times \chi^T$ is FDI of $\Omega \times \Omega$ and $\lambda \in [0,1]$. Then, i. Either $\psi^T(0) \geq \psi^T(u)$ or $\chi^T(0) \geq \chi^T(u)$ for each $u \in \Omega$.

ii. If $\psi^T(0) \geq \psi^T(u)$, then either $\chi^T(0) \geq \chi^T(u)$ or $\chi^T(0) \geq \chi^T(u)$ for each $u \in \Omega$.

iii. If $\chi^T(0) \geq \chi^T(u)$, then either $\psi^T(0) \geq \psi^T(u)$ or $\psi^T(0) \geq \psi^T(u)$ for each $u \in \Omega$.

**Proof:** i. Assume that $\psi^T(0) < \psi^T(u)$ and $\chi^T(0) < \chi^T(u)$ for some $u,v \in \Omega$. Then,

$$
(\psi^T \times \chi^T)(u,v) = \min\{\psi^T(u), \chi^T(v)\} > \min\{\psi^T(0), \chi^T(0)\} = (\psi^T \times \chi^T)(0,0).
$$

Thus, the proof is completed.

**Theorem 5.5** Let $\psi$ and $\chi$ be two FS of a D-algebras $\Omega$ such that $\psi^T \times \chi^T$ is FDI of $\Omega \times \Omega$ where $\alpha \in [0, T]$. Then, either $\psi$ or $\chi$ is FDI of $\Omega$.

**Proof:** To show that $\chi$ is FDI. From Theorem 5.3(i), we have $\psi^T(0) \geq \psi^T(u)$ or $\chi^T(0) \geq \chi^T(u)$ for each $u \in \Omega$. Thus, i. Let $\chi^T(0) \geq \chi^T(u)$, then $\chi(0) + \alpha \geq \chi(u)$. ii. By Theorem 5.3 (iii), we get $\psi^T(0) \geq \psi^T(u)$ or $\psi^T(0) \geq \psi^T(u)$ for each $u \in \Omega$. Thus, the proof is completed.

Since $\psi^T \times \chi^T$ is FDI of $\Omega \times \Omega$ then for each $(u_1, u_2)$ and $(v_1, v_2) \in \Omega \times \Omega$, we have

$$
(\psi^T \times \chi^T)(u_1, u_2) = \min\{\psi^T(u_1), \chi^T(u_2)\} \geq \min\{\psi^T(u_1, v_1), \chi^T(u_2, v_2)\}, \min\{\psi^T(u_1, v_1), \chi^T(u_2, v_2)\} = \min\{\psi^T(u_1, v_1), \chi^T(u_2, v_2)\}.
$$

Therefore, the proof is completed.

The proof of the last point is similar to the proof of point two.
Let $\psi$ and $\chi$ be two FS of a D-algebras $\Omega$ such that $\psi^M \times \chi^M$ is FdI of $\Omega \times \Omega$ where $\lambda \in [0,1]$. Then, either $\psi$ or $\chi$ is FdI of $\Omega$.

Proof: To show that $\chi$ is FdI of $\Omega$. From Theorem 5.4(i), we have $\psi^M(0) \geq \psi^M(u)$ for each $u \in \Omega$. Thus, $\chi(0) \geq \chi(u)$ then, $\lambda \cdot \chi(0) \geq \lambda \cdot \chi(u)$.

ii. By Theorem 5.4(iii), we have $\psi^M(0) \geq \psi^M(u)$ or $\psi^M(0) \geq \psi^M(u)$ for each $u \in \Omega$. If $\psi^M(0) \geq \chi^M(u)$ then,

$$\psi^M \times \chi^M(0,u) \geq \psi^M(u) \chi^M(u) . \quad \ldots (1)$$

Since $\psi^M \times \chi^M$ is FdI of $\Omega \times \Omega$ then for each $(u_1,v_1),(u_2,v_2) \in \Omega \times \Omega$ we have

$$\psi^M \times \chi^M(u_1,u_2) = \min\{\psi^M(u_1),\chi^M(u_2)\} \geq \min\{\psi^M(u_1, v_1),\chi^M(u_2, v_2)\} \geq \min\{\psi^M(u_1, v_1),\chi^M(u_2, v_2)\} .$$

That is $\psi^M \times \chi^M(u_1,u_2) \geq \min\{\psi^M \times \chi^M(u_1, v_1),\chi^M(u_2, v_2)\}$.

Therefore, $\chi$ is FdI of $\Omega$. The second part can be checked by similar way. \[\square\]

**6. FM-$\alpha\lambda$-T OF D-ALGEBRAS**

This section contains the idea of FM-$\alpha\lambda$-T of a D-algebras.

**Definition 6.1** [11] Let $\psi$ be a FS of $\Omega$ with $\alpha \in [0,T]$ and $T = 1 - \sup \{\psi(u) : u \in \Omega\}$, where, $\lambda \in [0,1]$. The mapping $\psi^T_{\alpha\lambda} : \Omega \to [0,1]$ is called FM-$\alpha\lambda$-T of $\psi$ if it satisfies $\psi^T_{\alpha\lambda} = \alpha \cdot \psi(u) + \lambda$.

**Example 6.1** Consider a D-algebra $\Omega$ which presented in Example 3.1. The FS $\psi$ of $\Omega$ is given by

$$\psi(u) = \begin{cases} 0.7, & u = 0 \\ 0.02, & u \neq 0 \end{cases}$$

Then, $\psi$ is a FS-algebra of $\Omega$. Here $T = 1 - \sup \{\psi(u) : u \in \Omega\}$ $\Rightarrow T = 1 - 0.7 = 0.3$. Take $\alpha = 0.2 \in [0,T]$ and $\lambda = 0.4 \in [0,1]$. The mapping $\psi^T_{\alpha\lambda} : \Omega \to [0,1]$ is defined as

$$\psi^T_{\alpha\lambda} = \alpha \cdot \psi(u) + \lambda , \quad \forall u \in \Omega.$$
and $\psi(u * v) \geq \min\{\psi(u), \psi(v)\}$. Thus, 

i. $\psi(0) = \psi(u) \Rightarrow \alpha \cdot \psi(0) + \lambda \geq \alpha \cdot \psi(u) + \lambda \Rightarrow \psi(0) = \psi(u)$. 

ii. $\psi(u) \geq \min\{\psi(u * v), \psi(v)\} \Rightarrow \alpha \cdot \psi(u) + \lambda \geq \alpha \cdot \psi(u * v) + \lambda \geq \min\{\alpha \cdot \psi(u + \lambda), \alpha \cdot \psi(v) + \lambda\} = \psi(u + \lambda)$. 

That is $\psi_u^T(0) = \min\{\psi_u^T(u * v), \psi_u^T(v)\}$. 

iii. $\psi(u * v) \geq \min\{\psi(u), \psi(v)\} \Rightarrow \alpha \cdot \psi(u * v) + \lambda \geq \alpha \cdot \min\{\psi(u), \psi(v)\} + \lambda \geq \min\{\alpha \cdot \psi(u) + \lambda, \alpha \cdot \psi(v) + \lambda\} = \psi(u + \lambda)$. 

That is $\psi_u^T(u * v) \geq \min\{\psi_u^T(u), \psi_u^T(v)\}$. Hence $\psi_u^T$ is FdI of D-$\Omega$. 

Conversely, assume $\psi_u^T$ is FdI of D-$\Omega$, then for each $u, v \in \Omega$ we have 

i. $\psi_u^T(0) = \psi_u^T(u) \Rightarrow \alpha \cdot \psi(0) + \lambda \geq \alpha \cdot \psi(u) + \lambda \Rightarrow \psi(0) = \psi(u)$. 

ii. $\psi_u^T(u) \geq \min\{\psi_u^T(u * v), \psi_u^T(v)\} \Rightarrow \alpha \cdot \psi(u) + \lambda \geq \min\{\alpha \cdot \psi(u * v) + \lambda, \alpha \cdot \psi(v) + \lambda\} \geq \alpha \cdot \min\{\psi(u), \psi(v)\} + \lambda$. 

That is $\alpha \cdot \psi(u) + \lambda \geq \alpha \cdot \min\{\psi(u), \psi(v)\} + \lambda$ which implies $\psi(u) \geq \min\{\psi(u * v), \psi(v)\}$. 

iii. $\psi_u^T(u * v) \geq \min\{\psi_u^T(u), \psi_u^T(v)\} \Rightarrow \alpha \cdot \psi(u * v) \geq \min\{\alpha \cdot \psi(u) + \lambda, \alpha \cdot \psi(v) + \lambda\} \geq \alpha \cdot \min\{\psi(u), \psi(v)\} + \lambda$. 

That is $\alpha \cdot \psi(u * v) + \lambda \geq \alpha \cdot \min\{\psi(u), \psi(v)\} + \lambda$.

7. HOMOMORPHISM OF FT AND FM OF D-ALGEBRAS

In this section, we provided the homomorphism of FT and FM of D-algebras and proved some results which are based on the FS-algebra and FdI of a D-algebras $\Omega$. 

Theorem 7.1 If $f : (\Omega, * , 0) \rightarrow (\Gamma, \cdot', 0')$ is a d-homomorphism with $\psi_u^T$ is a FT of a FS $\psi$. Then, the pre-image of $\psi_u^T$ is defined as $f^{-1}(\psi_u^T) = \psi_u^T(f(0))$ for each $u \in \Omega$. If $\psi$ is FdI of a D-algebras $\Gamma$, then $f^{-1}(\psi_u^T)$ is FdI of a D-algebras $\Omega$. 

Proof: Since $\psi$ is FdI of a D-algebras $\Gamma$, then for each $v_1, v_2 \in \Gamma$ there exist $u_1, u_2 \in \Omega$ such that $f(u_1) = v_1$ and $f(u_2) = v_2$. Thus, 

i. $\psi(0') \geq \psi(v) \Rightarrow \psi(0') + \alpha \geq \psi(v) + \alpha \Rightarrow \psi(f(0)) + \alpha \geq \psi(f(u_1)) + \alpha \Rightarrow \psi_u^T(f(0)) \geq \psi_u^T(f(u_1)) \Rightarrow f^{-1}(\psi_u^T)(0) \geq f^{-1}(\psi_u^T)(u)$. 

ii. $\psi'(v) \geq \min\{\psi(1 * v), \psi(v)\}$ which implies $\psi(1 * v) + \alpha \geq \psi(0) + \alpha \Rightarrow \psi(0') + \alpha \geq \psi(v) + \alpha$. 

That is $f^{-1}(\psi_u^T)(u) \geq \min\{f^{-1}(\psi_u^T)(u_1 * u_2), f^{-1}(\psi_u^T)(u_2)\}$. 

Hence, $f^{-1}(\psi_u^T)$ is FdI of a D-algebras $\Omega$. 

Theorem 7.2 If $f : (\Omega, * , 0) \rightarrow (\Gamma, \cdot', 0')$ is a d-homomorphism with $\psi_u^M$ is a FM of the FS $\psi$. Then, the pre-image of $\psi_u^M$ is defined as $f^{-1}(\psi_u^M) = \psi_u^M(f(0))$ for each $u \in \Omega$. If $\psi$ is FdI of a D-algebras $\Gamma$, then $f^{-1}(\psi_u^M)$ is FdI of a D-algebras $\Omega$. 

Proof: Since $\psi$ is FdI of a D-algebras $\Gamma$, then for each $v_1, v_2 \in \Gamma$ there exist $u_1, u_2 \in \Omega$ such that $f(u_1) = v_1$ and $f(u_2) = v_2$. Thus, 

i. $\psi(0') \geq \psi(v) \Rightarrow \psi(0') + \alpha \geq \psi(v) + \alpha \Rightarrow \psi(f(0)) + \alpha \geq \psi(f(u_1)) + \alpha \Rightarrow f^{-1}(\psi_u^M)(0) \geq f^{-1}(\psi_u^M)(u)$. 

ii. $\psi'(v) \geq \min\{\psi(1 * v), \psi(v)\}$ which implies $\psi(1 * v) + \alpha \geq \psi(v) + \alpha$. 

That is $f^{-1}(\psi_u^M)(u) \geq \min\{f^{-1}(\psi_u^M)(u_1 * u_2), f^{-1}(\psi_u^M)(u_2)\}$. 

Hence, $f^{-1}(\psi_u^M)$ is FdI of a D-algebras $\Omega$. 

Corollary 7.1 Let $f : (\Omega, * , 0) \rightarrow (\Gamma, \cdot', 0')$ be a d-homomorphism with $\psi_u^T$ is a FT of the FS $\psi$. Then, the pre-image of $\psi_u^T$ is defined as $f^{-1}(\psi_u^T) = \psi_u^T(f(0))$ for each $u \in \Omega$. If $\psi$ is a FS-algebra of a D-algebras $\Gamma$, then $f^{-1}(\psi_u^T)$ is a FS-algebra of a D-algebras $\Omega$. 

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Proof: Since $\psi$ is a FS-algebra of a D-algebras $\Gamma$, then for each $v_1,v_2 \in \Gamma$ there exist $u, u_2 \in \Omega$ such that $f(u_1) = v_1$, $f(u_2) = v_2$. Thus, $\psi(v_1, *v_2) \geq \min\{\psi(v_1, v_2), \psi(v_2)\}$

$\implies \psi(v_1, *v_2) + \alpha \geq \min\{\psi(v_1), \psi(v_2)\}$

$\implies \psi(v_1, *v_2) + \alpha \geq \min\{\psi(v_1) + \alpha, \psi(v_2) + \alpha\}$

$\implies \psi(f(u_1), *f(u_2)) + \alpha \geq \min\{\psi(f(u_1) + \alpha, \psi(f(u_2)) + \alpha\}$

$\implies \psi(v_1, *v_2) \geq \min\{\psi(v_1) + \alpha, \psi(v_2) + \alpha\}$

$\implies f^{-1}(\psi(M^\alpha)) \geq \min\{\psi^{-1}(v_1), \psi^{-1}(v_2)\}$

Therefore $f^{-1}(\psi(M^\alpha))$ is a FS-algebra of a D-algebras $\Omega$.

Corollary 7.2 Let $f : (\Omega, *, 0) \rightarrow (\Gamma, *, 0')$ be a d-homomorphism with $\psi(M^\alpha)$ is a FM of the FS $\psi$. Then, the pre-image of $\psi(M^\alpha)$ is defined as $f^{-1}(\psi(M^\alpha)) = \psi(M^\alpha)(f(u))$ for each $u \in \Omega$. If $\psi$ is a FS-algebra of a D-algebras $\Gamma$, then $f^{-1}(\psi(M^\alpha))$ is a FS-algebra of a D-algebras $\Omega$.

Proof: Since $\psi$ is a FS-algebra of a D-algebras $\Gamma$, then for each $v_1, v_2 \in \Gamma$ there exist $u, u_2 \in \Omega$ such that $f(u_1) = v_1$, and $f(u_2) = v_2$. Thus, we have $\psi(v_1, *v_2) \geq \min\{\psi(v_1), \psi(v_2)\}$

$\implies \lambda \cdot \psi(v_1, *v_2) \geq \lambda \cdot \min\{\psi(v_1), \psi(v_2)\}$

$\implies \lambda \cdot \psi(v_1, *v_2) \geq \min\{\lambda \cdot \psi(v_1), \lambda \cdot \psi(v_2)\}$

$\implies \psi(f(u_1), *f(u_2)) \geq \min\{\lambda \cdot \psi(f(u_1)), \lambda \cdot \psi(f(u_2))\}$

$\implies \psi(f(u_1, *u_2)) \geq \min\{\psi(f(u_1)), \psi(f(u_2))\}$

$\implies f^{-1}(\psi(M^\alpha)) \geq \min\{f^{-1}(\psi(v_1)), f^{-1}(\psi(v_2))\}$

Therefore $f^{-1}(\psi(M^\alpha))$ is a FS-algebra of $\Omega$.

Definition 7.1 Let $f : \Omega \rightarrow \Omega$ be an endomorphism and $\psi(M^\alpha)$ be a FT of a FS $\psi$ of a D-algebras $\Omega$. Then, $(\psi(M^\alpha))^\psi(f(u))$ is a new FS of $\Omega$ defined by $(\psi(M^\alpha))^\psi(f(u)) = (\psi(M^\alpha)(f(u))) = \psi(f(u)) + \alpha$ for each $u \in \Omega$ and $\alpha \in [0, T]$.

Definition 7.2 Let $f : \Omega \rightarrow \Omega$ be an endomorphism and $\psi(M^\alpha)$ be a FM of a FS $\psi$ of a D-algebras $\Omega$. Then, $(\psi(M^\alpha))^\psi(f(u))$ is a new FS of $\Omega$ defined by $(\psi(M^\alpha))^\psi(f(u)) = \lambda \cdot \psi(f(u))$ for each $u \in \Omega$ and $\lambda \in [0, 1]$.

Theorem 7.3 Let $f : \Omega \rightarrow \Omega$ be an endomorphism of a D-algebras $\Omega$. If $\psi$ is FdI of $\Omega$, then $(\psi(M^\alpha))^\psi(f(u))$ is FdI of a D-algebras $\Omega$.

Proof: Let $u, v \in \Omega$, then

i. $(\psi(M^\alpha))^\psi(f(u)) = \psi(f(u)) + \alpha \geq \psi(f(v)) + \alpha$

$\implies \psi(f(u)) + \alpha \geq \psi(f(v)) + \alpha$

$\implies \psi(f(u)) \geq \psi(f(v)) + \alpha$

$\implies \psi(f(u)) \geq \psi(f(v)) + \alpha$.

Therefore, $(\psi(M^\alpha))^\psi(f(u))$ is FdI of a D-algebras $\Omega$. □

Theorem 7.4 Let $f : \Omega \rightarrow \Omega$ be an endomorphism of a D-algebras $\Omega$. If $\psi$ is FdI of $\Omega$, then $(\psi(M^\alpha))^\psi(f(u))$ is FdI of a D-algebras $\Omega$.

Proof: Let $u, v \in \Omega$, then i. $(\psi(M^\alpha))^\psi(f(0)) = \lambda \cdot \psi(f(0)) \geq \lambda \cdot \psi(f(v))$

$\implies \psi(f(0)) = \psi(f(0)) \geq \psi(f(v))$

$\implies \psi(f(u)) = \psi(f(u)) \geq \psi(f(v))$

$\implies \psi(f(u)) \geq \psi(f(v))$

$\implies \psi(f(u)) \geq \psi(f(v))$

$\implies \psi(f(u)) \geq \psi(f(v))$

$\implies \psi(f(u)) \geq \psi(f(v))$

$\implies \psi(f(u)) \geq \psi(f(v))$

Therefore, $(\psi(M^\alpha))^\psi(f(u))$ is FdI of a D-algebras $\Omega$. □

Theorem 7.5 Let $f : \Omega \rightarrow \Omega$ be an endomorphism of a D-algebras $\Omega$. If $(\psi(M^\alpha))^\psi(f(u))$ is FdI of a D-algebras $\Omega$, then $\psi$ is FdI of a D-algebras $\Gamma$.

Proof: Since $(\psi(M^\alpha))^\psi(f(u))$ is FdI of a D-algebras $\Omega$, then for each $u, u_2 \in \Omega$ there exist $v_1, v_2 \in \Gamma$ such that $f(u_1) = v_1$, and $f(u_2) = v_2$. Thus, $\lambda \cdot \psi(f(u)) \geq \psi(f(v))$

$\implies \lambda \cdot \psi(f(u)) \geq \psi(f(v))$

$\implies \lambda \cdot \psi(f(u)) \geq \psi(f(v))$

$\implies \lambda \cdot \psi(f(u)) \geq \psi(f(v))$

$\implies \lambda \cdot \psi(f(u)) \geq \psi(f(v))$

$\implies \lambda \cdot \psi(f(u)) \geq \psi(f(v))$

$\implies \lambda \cdot \psi(f(u)) \geq \psi(f(v))$

$\implies \lambda \cdot \psi(f(u)) \geq \psi(f(v))$

$\implies \lambda \cdot \psi(f(u)) \geq \psi(f(v))$

$\implies \lambda \cdot \psi(f(u)) \geq \psi(f(v))$

Therefore, $(\psi(M^\alpha))^\psi(f(u))$ is FdI of a D-algebras $\Gamma$. □
Theorem 7.6 Let \( f : (\Omega \times 0, 0) \rightarrow (\Gamma \times 0, 0) \) be an epimorphism. If \((\psi^\lambda)_M \) is Fddl of a D-algebras \( \Omega \), then \( \psi \) is Fddl of a D-algebras \( \Gamma \).

Proof: Since \((\psi^\lambda)_M \) is Fddl of a D-algebras \( \Omega \), then for each \( a_1, a_2 \in \Omega \) there exist \( v_1, v_2 \in \Gamma \) such that \( f(a_1) = v_1 \) and \( f(a_2) = v_2 \). Thus, i. \((\psi^\lambda)_M (0) \geq (\psi^\lambda)_M (0) \geq (\psi^\lambda)_M (0) \Rightarrow \lambda \cdot (\psi^\lambda)(0) \geq \lambda \cdot (\psi^\lambda)(0) \geq (\psi^\lambda)(0) \). ii. \((\psi^\lambda)_M (a_1 \cdot a_2) \geq (\psi^\lambda)_M (a_1 \cdot a_2), (\psi^\lambda)_M (a_1 \cdot a_2) \Rightarrow (\psi^\lambda)(v_1) \geq (\psi^\lambda)(f(a_1)), (\psi^\lambda)(v_2) \Rightarrow \lambda \cdot (\psi^\lambda)(v_1 \cdot v_2) \geq \lambda \cdot (\psi^\lambda)(f(a_1 \cdot a_2)) \Rightarrow (\psi^\lambda)(v_1 \cdot v_2) \geq \lambda \cdot (\psi^\lambda)(f(a_1 \cdot a_2)) \geq (\psi^\lambda)(v_1 \cdot v_2) \). Therefore, \( \psi \) is Fddl of a D-algebras \( \Gamma \). \( \square \)

Theorem 7.7 Let \( f : (\Omega \times 0, 0) \rightarrow (\Gamma \times 0, 0) \) be a d-homomorphism. If \( \psi \) is Fddl of a D-algebras \( \Gamma \), then \((\psi^\lambda)_M \) is Fddl of a D-algebras \( \Omega \).

Proof: Since \( \psi \) is Fddl of a D-algebras \( \Gamma \), then for each \( v_1, v_2 \in \Gamma \) there exist \( a_1, a_2 \in \Omega \) such that \( f(a_1) = v_1 \) and \( f(a_2) = v_2 \). Thus, i. \((\psi^\lambda)(0) \geq (\psi^\lambda)(0) \geq (\psi^\lambda)(0) \Rightarrow \lambda \cdot (\psi^\lambda)(0) \geq \lambda \cdot (\psi^\lambda)(0) \geq (\psi^\lambda)(0) \). ii. \((\psi^\lambda)(a_1 \cdot a_2) \geq (\psi^\lambda)(a_1 \cdot a_2), (\psi^\lambda)(a_1 \cdot a_2) \Rightarrow (\psi^\lambda)(v_1) \geq (\psi^\lambda)(f(a_1)), (\psi^\lambda)(v_2) \Rightarrow \lambda \cdot (\psi^\lambda)(v_1 \cdot v_2) \geq \lambda \cdot (\psi^\lambda)(f(a_1 \cdot a_2)) \Rightarrow (\psi^\lambda)(v_1 \cdot v_2) \geq \lambda \cdot (\psi^\lambda)(f(a_1 \cdot a_2)) \geq (\psi^\lambda)(v_1 \cdot v_2) \). Therefore, \( \psi \) is Fddl of a D-algebras \( \Omega \). \( \square \)

Conclusion
As a conclusion, the notion of FT and FM on a D-algebras has been introduced. Certain results that concern FS-algebra and Fddl were proved. Moreover, we proved that the FS \( \psi \) of the D-algebras \( \Omega \) become Fddl if the FT \( \psi^\lambda \) of \( \psi \) is a Fddl (resp. FM \( \psi^\lambda \) of \( \psi \)). Furthermore, we also showed that the FS \( \psi \) of a D-algebras \( \Omega \) is a FS-algebra of \( \psi \) iff the \( \psi^\lambda \) of \( \psi \) is a FS-algebra of \( \Omega \) (resp. Fddl). In addition, some results on the homomorphism of a FT with a FM on a D-algebras \( \Omega \) were studied. As an extension, this article may include the study of anti-FT and anti-FM on a D-algebras \( \Omega \).

References
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الملخص

يعتبر مفهوم الرياضيات الضبابية أحد الفروع الجميلة في الرياضيات. هذا المفهوم تم تقديمه من قبل طفلي زاده [1] منذ ذلك الوقت، تم النظر إلى هذا المفهوم بطرق مختلفة في مجال الرياضيات النظرية والتطبيقية. في هذا البحث قمنا بمفهوم الترجمة الضبابية والضرب الضبابي على الجبر D. كذلك البتنا بعض المكافآت التي تعمد على الجبر الجزئي والمتالي الضبابي من النوع D. لضيف إلى ذلك قمنا بعض النتائج التباين للكمية D الضبابية والضرب الضبابي على الجبر من النوع D.