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Generalized N* ideal closed sets in Nano N* ideal topological Spaces With Some Properties

Nabila I. Aziz , Taha H. Jasim

Department of Mathematics , College of Computer Science and Mathematics, Tikrit University, Iraq

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Corresponding Author:

Name: Nabila I. Aziz

E-mail: :

nabila.be@tu.edu.iq

tahahameed@tu.edu iq

Tel:

ABSTRACT

In this paper, we will study a new class of sets and said to be generalized N* ideal -closed sets in nano N* ideal topological spaces and its properties. Furthermore the relationships were introduced and notation.

1. Introduction

In 1913 [1] studied the notation of Nano topology. [2]study Nano generalized alpha closed sets in Nano topological Spaces , [3] introduced Nano g*αClosed sets in nano topology. [4] introduced new class of open sets in nano topological spaces. [5] studied a new types of nano topology via nano ideals. [6] studied alpha generalized closed sets in ideal topology . (2018-2019) [7-8] studied simple forms of nano open sets in ideal nano topological spaces, unified approach of several sets in ideal nano topological spaces.[9] studied the nano ideal generalized closed set in nano ideal topological space. [10] presented continuous maps and hausdorff spaces in nano ideal spaces. In (2019) [11] studied the generalized classes of ideal nano topological space , [12] presented new sets in ideal topological space. [13] studied generalized *I β-closed sets in ideal topological spaces. [14] studied On Some New Notions in Nano Ideal Topological Spaces .In (2020) [15] presented nano Iαg-closed sets and normality via nIαg-closed sets in nano ideal topology In(2019) presented New generalized classes of ideal nano topological space[17] studied ideal with micro topological space[18] introduced nano ideal α regular- closed sets in nano ideal topological spaces. [19] presented topological approach for rough sets by using j-nearly concepts via ideals.In this paper, we introduce and investigate a new class of closed sets called generalized n* ideal -closed sets in nano n* ideal toplogical spaces and also discuss the relationship with some new existing closed sets in nano n* ideal topological Spaces.

2 Preliminaries

Definition 2.1 [1] : Let w be a non-empty set and R be Relation Equivalence on w, then w is called the indistinguishable relationship. and (W, R) area of approximation, $X \subseteq W$.

A) $L_{R(X).} = \cup_{x\in U}\{R(X) : R(X) \subseteq X \}$.

B) $U_{R(X).}$ That is $U_{R(X)} = \cup_{x\in U}\{R(X) : R(X) \cap X \neq \emptyset\}$.

C) $B_{R(X)} = \{U_{R(X)} - L_{R(X)}\}$.

Definition 2.2 [1]: W is universe, R be an equivalence relation on W , $X \subseteq W$. $\tau_{R(X)} = \{W, \phi, L_{R(X)}, U_{R(X)}, B_{R(X)}\}$, $(W, \tau_{R(X)})$ is said to be Nano top. space. The sets of $\tau_{R(X)}$ are named as nano open sets .

Definition 2.3 [1] : $(\varphi, \tau_{R(X)})$ nano topological Space, $X \subseteq \varphi$. ,$P \subseteq \varphi$:

i. n int(P)= ∪ {G: G⊆ P, G is nano open sets}

ii. n cl(P)= ∩ { G: P⊆ G, G is nano closed sets}.

Definition 2.4[20]: (ω, $\tau^*_{R(X)}$) is nano N*topological Space, A⊆ ω then A is said to be $N^*\beta$ regular -open set ,if ∃ N regular open set D ∋ D ⊆ A ⊆ $N^*\beta cl(D)$. symbolized by $N^*R\beta o(x)$.

Definition 2.5[20] : The nano N*topological Space (δ, $\tau^*_{R(X)}$) , A⊆ δ , A named as N*α- regular open set ,if ∃ N* Regular open set F , F ⊆ A ⊆ N * α cl(F).

Definition 2.6[21]: (W, $\tau_{R(X)}$) is nano topological space, β ⊆ W is called nanoN*open sets if β satisfy .

1- For each Q∈ β ,∃Nα-Open Set G ∋ Q∈G⊆ NCl(β).

2- If β ∩ [$U_{R(X)}$]^c ≠ ∅ then $U_{R(X)}$ ⊆ β , symbolized by $\tau^*_{R(X)}$.

Definition 2.7[21]: (γ, $\tau^*_{R(X)}$) N*Space , Q ⊆ γ called

a) N*-alpha-Open set if Q⊆ N*int (N*cl (N*int (Q))) , symbolized by N*α -open set.

b) N*-beta-Open set if Q⊆ N*cl (N*int (N*cl(Q))) , symbolized by N*β-open set.

Definition 2.8[22]: let I be an Ideal on a topological space (X, τ) which satisfies the following conditions. (1) H ∈ I , F ⊂ H → F ∈ I (2) H ∈ I and F ∈ I → H ∪ F ∈ I.

Definition 2.9[5]: (W, N) is nano topological space with ideal I on W called Ideal nano top. space symbolized by (W, N , I). $M_{n(x)} = \{M_n \mid x \in M_n, M_n \in N\}$,is the collection of nano open set(x) ⊇ x.

Definition 2.10 [19]: S ⊆ (U, N , I) is called n*-closed if $S^*_n \subseteq S$. supplement of N*-c(x) is called N*-O(x)

Definition 2.11[5]: Let (μ, N) is nano top. S. And ideal I on μ. The Ncl* called nano∗-closure , ncl* (H) = H ∪ $H^*{}_n$, H ⊆ X.

Theorem 2.12 [5-9] :The Nano top. s. (μ, N) with an Ideal (j, k) on μ ,H, J ⊆μ. Then

1. $H \subseteq J \Rightarrow H_n^* \subseteq J_n^*$.
2. $j \subseteq k \Rightarrow H_n^*(k) \subseteq H_n^*(j)$.
3. $H_n^* = N\text{-}cl(H_n^*) \subseteq N\text{-}cl(H)$ (H_n^* be nano closed ⊆ ncl(H)).
4. $(H_n^*)_n^* \subseteq H_n^*$

Lemma 2.13[5-9] : (μ, N , I) be nano topological space with ideal I , $M \subseteq M^*_n$, then $M^*{}_n = ncl(M^*{}_n) = ncl(M)$

Theorem 2.14 [5-9] ncl* satisfies the conditions:

1. M ⊆ ncl* (M). 2. ncl* (∅) = ∅ , ncl* (μ) = μ. 3. If H ⊆F, thus ncl* (H) ⊆ ncl* (F). 4. ncl* (H) ∪ ncl* (F) = ncl* (H ∪ F) 5. ncl* (ncl* (H)) = ncl* (H)

Definition 2.15[7,11]: H ⊆ (γ, N , I) is called

1. .Nano α-I-Open set (briefly α-NI-open) if H ⊂ nint(ncl*(nint(H))

2 .Nano β-I-Open set (briefly β-NI-open) if H ⊂ ncl*(nint(ncl*(H))).

3 NI Regular Open set if H =nint (ncl* (H)) and NI regular closed if ncl* (nint (H)) = H.

Definition 2.16[18] :(γ, $\tau^*_{R(X)}$) nano N ∗ space, H⊆ γ , H is said to be N* generalized α-closed set in nano N* topological space ↔ L ⊆ N*αCl(H), H⊆L , L is N* α − open set.

3 Generalized N* ideal closed set in Nano N* ideal τopological Spaces With Some Properties.

Definition 3.1: (μ, $\tau^*_{R(X)}$,j) be nano N* ideal topological space.

Let $(\ )_{n^*}^{**}$ a set operator from F(μ) to F(μ) , [F(μ) ⊆μ].

For A ⊆ μ , $A_{n^*}^{**}$ (j, N*)={ x ∈ μ: G_{n^*} ∩ A ∉ j , ∀ $G_{n^*} \in G_{n^*}(x)$} said Nano n* locally Function of A, j and n* (briefly $A_{n^*}^{**}$).

Theorem 3.2 . (μ, $\tau^*_{R(x)}$, J) is nano N* ideal topological space and ideals j, k on μ and A, B ⊆μ. Then

(i) $A \subseteq B \Rightarrow A_{n^*}^{**} \subseteq B_{n^*}^{**}$

(ii) $j \subseteq k \Rightarrow A_{n^*}^{**}(k) \subseteq A_{n^*}^{**}(j)$.

(iii) $A_{n^*}^{**} = N^*\text{-}Cl(A_{n^*}^{**}) \subseteq N^*\text{-}Cl(A)$ [$A_{n^*}^{**}$ is Nano N* closed subset of N*-Cl(A)].

(iv) $(A_{n^*}^{**})_{n^*}^{**} \subseteq A_{n^*}^{**}$.

Proof. (i) Let A ⊂ B and x ∈ $A_{n^*}^{**}$. let x ∉ $B_{n^*}^{**}$. → G_{n^*} ∩B ∈ j ,G_{n^*} ∈ G_{n^*} (x). Since G_{n^*} ∩ A ⊆ G_{n^*} ∩ B and G_{n^*} ∩ B ∈ j, we obtain G_{n^*} ∩ A ∈ j from def. of ideal. Hence, x ∉ $A_{n^*}^{**}$. This is a contradiction. Its Clear that , $A_{n^*}^{**} \subseteq B_{n^*}^{**}$..

(ii) j ⊆ k and x ∈ $A_{n^*}^{**}$.(k) , G_{n^*} ∩ A ∉ k ∀ G_{n^*} ∈ G_{n^*} (x). since G_{n^*} ∩ A ∉ j. then x ∈ $A_{n^*}^{**}$ (j).

(iii) x ∈ $A_{n^*}^{**}$. ∀ G_{n^*} ∈ G_{n^*} (x), G_{n^*} ∩ A ∉ I. → G_{n^*} ∩ A ≠ ∅. Hence x ∈ N*-cl(A).

(iv) From (iii), $(A_{n^*}^{**})_{n^*}^{**} \subseteq N^*\text{-}Cl(A_{n^*}^{**}) = A_{n^*}^{**}$., since $A_{n^*}^{**}$ Is Nano N* closed set. Hence $(A_{n^*}^{**})_{n^*}^{**} \subseteq A_{n^*}^{**}$.

The opposite of [i, ii and iii] in Theorem 3.3. not satisfy.

Example3.3: (i) Let μ = {a, b, c, d}, X = {a, b} ⊂ μ, μ /R={{a}, {c}, {b, d}} and N = { μ, ∅, {a }, {b,d}, {a,b,d}}, N*O-(x)={ μ, ∅, {a}, {b},{d},{a , b},{a, d},{b, d}, { a, b, d }} and the ideal I ={ ∅, {a}} .A = {a,c} , B = {a,d}, $A_{n^*}^{**}$ = {c} ⊂ $B_{n^*}^{**}$ = { b, c, d } but A ⊄ B.

(ii) Let μ = { a, b, c, d } be the universe, X = {a, b} ⊂ μ, μ /R = {{a}, {c}, {b, d}}

N = {U, ∅, {a},{b,d}, { a, b, d }}, N*O-(x)={ U, ∅, {a},{b},{d},{a,b},{a,d},{b,d}, { a, b, d }} and the ideals be I ={ ∅, {a}} , J = {∅, {b}}, I⊄ J, A = {a,c,d}. $A_{n^*}^{**}$(I)={c,d} ⊂ $A_{n^*}^{**}$(j)={a,c ,d}.

(iii) Let μ = {a,b,c,d} be the universe, X = {a, d} ⊂ μ, μ /R={{a}, {c}, {b, d}} and N = { μ, ∅, {a},{b,d},{a,b,d}}, N*O-(x)={ μ, ∅, {a},{b},{d},{a,b},{a ,d},{b,d},{a,b,d}} with ideal I ={ ∅, {a}}. A = {b,c,d}, we have N*-cl(A) = { b, c, d }, $A_{n^*}^{**}$ = {c, d} , N*-cl $A_{n^*}^{**}$ = {c, d}, N*-cl(A) ⊄ $A_{n^*}^{**}$ = N*-cl($A_{n^*}^{**}$).

Theorem 3.4:. If (μ, $\tau^*_{R(X)}$,j) nano N* ideal topological space, and B ⊆ $B_{n^*}^{**}$, then $B_{n^*}^{**}$ = N*-cl($B_{n^*}^{**}$) = N*-cl(B).

Proof: B ⊆μ, $B_{n^*}^{**}$= N*-cl($B_{n^*}^{**}$) ⊆ N*-cl(B), by using Theorem 3.6 [iii] B ⊆ $B_{n^*}^{**}$→ N*-cl(B) ⊆ N*-cl($B_{n^*}^{**}$) and So $B_{n^*}^{**}$= N*-cl($B_{n^*}^{**}$) = N*-cl(B).

Theorem 3.5:. $(\mu, \tau^*_{R(X)}, j)$ be nano N* ideal topological space and ideal j on μ , A ⊆ X. If A ⊆ $A_{n^*}^{**}$, thus $A_{n^*}^{**}$ = N*-cl($A_{n^*}^{**}$) = N*-cl(A) = N*-cl** (A).

Definition 3.6: $(\mu, \tau^*_{R(X)}, I)$ is nano N* ideal topological space . The set operator N*cl** is said to be n*- closure ** [n*-cl**(A)=A∪ $A_{n^*}^{**}$] , A ⊆ X .

Definition 3.7: $(\mu, \tau^*_{R(X)}, I)$ be nano N^* ideal topological Space .The set operator N*int** is said to be n*- interior* * [n*-int**(A) = A∩ $A_{n^*}^{**}$] for A ⊆ X .

Theorem 3. 8:. $(\mu, \tau^*_{R(X)}, j)$ is nano N* ideal topological space and ideal j on μ , A ⊆ μ. If A ⊆ $A_{n^*}^{**}$, then

(i) n*-Cl(A) = n*-Cl** (A)

(ii) n*-int(μ − A) = n*-int** (μ − A).

Proof. (i) by using Th. 2.5.

Proof. (ii) let A ⊆ $A_{n^*}^{**}$, then N*-cl(A) = N*-cl** (A) by (i) and so X –N*-cl(A) = X –N*-cl** (A). Therefore, N*-int(X − A) = N*-int** (X − A).

Definition 3.9: Let$(\mu, \tau^*_{R(X)}, j)$ be nano N* ideal topological space .The subset A of μ is said to be nano N^* I- open set (briefly N*I-o(x)) if A ⊆ N* int ($A_{n^*}^{**}$).

Example 3.10: μ = { 1, 2 , 3, 4, 5}, μ /R={{1},{2,3 } ,{4, 5 }}, X={ 1,2 }

I={∅ ,{2}}, $T_{R(x)}$ ={∅, μ,{ 1 },{2,3},{1,2,3} }, $\tau^*_{R(X)}$ ={∅, U, { 1 } ,{ 2},{3 },{1,2},{1,3},{2,3},{1,2,3},{1,2,3,4},{1,2,3,5} }.

N*IO(x)={ ∅, μ,{1},{3},{1,3}}

Remark 3.11:. It is clear that N*I-open and N*-open are independent

Example 3.12. μ = {a, b, c, d} be the universe, X = {a, b} ⊂ μ

μ /R = {{a},{c},{ b, d }} and N = {∅, μ, { a}, { b, d }, { a, b, d }}

N*O(x) = {∅, μ, {a},{b},{d},{b, d},{ a ,d },{a , b},{ a, b, d }}

and the ideal J ={ ∅, {a}}. (i) For A = {a, b, d}, we have $A_{n^*}^{**}$ = {b, c, d} , N*-int($A_{n^*}^{**}$) = {b, d} ⇒ A ⊈ N*-int($A_{n^*}^{**}$)., A ln N* but A is not an element an N*IO(x).

(ii) For A = {b}, $A_{n^*}^{**}$={b, c, d} , N*int($A_{n^*}^{**}$) = {b, d} ⇒ A ⊆ N*int($A_{n^*}^{**}$). A ∈ N*I-open set , A ∉ N*-o(x).

Definition 3.13: $(\mu, \tau^*_{R(X)}, j)$ be nano N* ideal topological space , A ⊆ μ. , A is said to be

a) n* RI- o(x)[A = n* int(ncl** ((A))].

b) n* αI-o(x)[A ⊆ n*int(n*cl**(n*int(A)))].

c) n* βI-o(x)[A ⊆ n*cl**[n*int(n*cl**(A))]. denoted by N*RIO(μ, x) (Respectively, N*αIo(μ, x) ,N*βIo-(μ, x)) .

Example 3.14: Let μ ={ 1, 2 , 3, 4, 5}, μ /R={{1},{2,3 } ,{4, 5 }}, X={ 1,2 } ,I={∅ ,{2}}

$\tau_{R(x)}$ ={∅, μ,{ 1 },{2,3},{1,2,3} }, $\tau^*_{R(X)}$ ={∅, μ, ,{ 1 } ,{ 2},{3 },{1,2},{1,3},{2,3},{1,2,3},{1,2,3,4},{1,2,3,5} }.

N*IO(x)={ ∅, μ,{1},{3},{1,3}}

N* RI -o(x)= {∅, μ,{1},{2},{3},{1,2},{1,3},{2,3}}..

N* αI -o(x)= {∅, μ,{ 1},{2},{3},{1,2},{1,3},{2,3},{1,2,3},{1,2,3,4},{1,2,3,5 }}.

N*βIo(x)={∅, μ,{1},{2},{3}},{1,2},{1,3},{2,3},{1,4 },{1,5},{3,4},{3,5},{1,2,4},{1,2,5},{2,3,4},{2,3,5},{ 3,4,5},{1,3,4},{1 ,3,5},{1,2,4,5},{2,3,4, },{1,3,4,5},{1,4,5},{1,2,3} ,{1,2,3,4},{ 1,2,3,5}}.

Definition 3.15 : $(\sigma, T^*_{R(X)}, j)$ be nano N* ideal topological space , A⊆ σ , A is said to be N* regular βI -open set . If there is a N* regular − I open set D. ∋ D ⊆ A ⊆ N *βcl(D).

Recall example 3.13**:** N* R β I -o(x)= {∅, μ,{1},{2},{3},{1,2},{1,3},{2,3}}.

Definition 3.16: $(\mu, \tau^*_{R(X)}, j)$ is nano N* ideal topological space , A⊆ μ , A called N^* regular αI −o(x) if there is N* regular I open set D , ∋ D⊆A⊆ N^*αcl(D).

Recall example 3.13:

N^*rαIo(x)={∅, μ,{1},{ 2},{3},{1,2},{1,3},{2,3},{1,4},{1,5},{1,4,5},{2,4,5},{2,4},{2,5}{3,4},{3,5},{3,4,5}{1,2,4},{1,2,5},{1,2,4 ,5},{1,3,4},{1,3,5},{1,3,4,5},{2,3,4},{2,3,5},{2,3,4,5 }}.

Proposition 3.17: N *I –o(x) is N *α I-o(x) , the opposite is not satisfies

Proof: Assume B is n*I-open set ⇒ B⊆ n*int($A_{n^*}^{*}$) ⊆ n*-int(n*-cl* (B)) .

⊆ n*int ((n*cl* (n*int (B))). B is n * αI-open set.

Recall example 3.13: A={1,2,3,4} is N* αI -o(x) , not N*I-o(x).

Proposition 3.18: N * αI -open set is N* βI-open set , the opposite is not satisfy**.**

Proof: Since n*int(A) ⊆ A, n* cl**(n int(A)) ⊆ n* cl**(A).

n*int(N*cl**(n*int(A))) ⊆ n* int(n*cl**(A)))⊆n*cl**(n*int(n* cl**(A))). A is N * βI-o(x).

Recall Example 3.13: A= {1,2,4} is N*βI-o(x) , not N* αI -o(x).

Proposition 3.19: N * regular β I -open set is N * regular α I-open set , the opposite doesn't satisfies

Proof: Assume A is n*βαI –o(x) ⇒ ∃ D⊆A⊆ n*β cl(D) And D is n*RI -open set Since n*β cl(D) ⊆ n*αcl (D) ⇒ D⊆A⊆ n*αcl (D) . A is n*RαI –o(x).

Recall example 3.13: μ ={ 1, 2 , 3, 4, 5}, μ /R={{1},{2,3 } ,{4, 5 }}, X={ 1,2 } ,I={∅ ,{2}}

A= {3,4,5} is N^*RαI −o(x) , not n* R β I -o(x)

Proposition 3.20: N *R αI -open set is N*αI -open set , the opposite doesn't satisfies

Proof: Assume A is N*RαI –o(x) ⇒ ∃ D⊆A⊆ N*αcl (D)

And D is N *RI -open set ⇒ D= N* int(N*cl** ((D)) ⊆ N*int (D) ⊆ (N*cl**(N*int(D)) ⊆ N*int(N*cl**(N*int(D)) ⇒ D is N*αI –o(x).

Recall example 3.13: A={1,2,3,5} N* αI -o(x) , not N^*RαI − o(x).

Proposition 3.21: N *R βI -open set is N*βI -open set ,the opposite doesn't satisfied
Proof: Assume A is n*RβI –o(x) ⇒ ∃ T⊆A⊆ n*βcl (T)
And T is n *RI –o(x) take A=T ⇒ A= n* int(ncl** ((A)) ⊆ ncl**(n* int(ncl** ((A))⇒ A is n*βI –o(x).
Recall Example 3.13:A= {1,2,4} is N*βI-o(x) , not N* R βI –o(x).

Definition 3.22 : (μ, $\tau^*_{R(X)}$,I) is nano N* ideal topological space , E ⊆ μ , E is called n* generalized α I-closed set in nano N* ideal topological space ↔ Y ⊆ n*αcl(E)., E⊆Y , Y is a n*αI-o(x).
Recall Example 3.13: E={1,2,3} ,G= {1,2,3,4}⇒ G ⊆ n *αcl(E) = μ ⇒ E is N *g αI − c(x).

Definition 3.23: (μ, $T^*_{R(x)}$,I) be nano N* ideal topological space , F ⊆ μ , F is said to be N*regular generalized αI −closed set in nano N* ideal topological space ↔ D ⊆N*αcl(F), whenever F⊆D , D is a n*R αI- open set.
Recall example 3.13: U= {2,3}, D={2,3,4,5}, D ⊆ N*αcl(U) ⇒U is n *rg αI -c(x).

Definition 3.24: (μ, $\tau^*_{R(X)}$,I) be nano N* ideal topological space, Q ⊆ μ, Q is said n* generalized β I– c(x) in nano N* ideal topological space ↔ D ⊆ N*βcl(Q) , Q ⊆D , D is N*βI-o(x)
Recall example 3.13: Q={1,2,3} , D= {1, 2,3,4}
⇒ N*βcl(Q)= μ ⇒ D ⊆ μ ⇒Q is N *g βI -c(x).

Definition 3.25: (μ, $\tau^*_{R(X)}$,I) is nano N* ideal topological space., Q ⊆ μ, Q called n* regular generalized β I–c(x) in nano N* ideal topological space ↔ Z⊆ n*βcl(Q) , Q⊆Z , Z is N*RβI-o(x) .
Recall example 3.13: A= {2,3} , D= {2,3} , D ⊆N*βcl(A) ⇒ A is N*RgβI-c(x) .

Proposition 3.26: n *gβ I-c(x) and n *g αI-c(x) are independent.

Example 3.27: φ= { i , j, k , l}, φ /R={{i}, { j, k,l } }, X={ j ,k }, I= {∅,{j ,l}}
$\tau_{R(x)}$ ={∅, φ,{ j , k , l } }, N *o(x)={∅, φ, ,{ j } , { k}, {l }, {j , k}, {j , l}, {k, l}, {j, k ,l}}= N * α o(x)
N *gβI o(x)={∅,φ,{j},{k }, {l}, {j, k}, {j, l}, {k, i}, {l, k}, {i, l}, {i, j}, {i, k, j}, {i, k, l}, {k, j, l }}.
N *g αI-o(x)={∅,φ,{j},{k},{l},{j ,k},{j, l},{I ,j, k},{k ,j ,l}}.
A={I ,j, k} is N *g αI-c(x). but A is not N *gβ I-c(x) .
A={k ,i} is n *gβ I-c(x) ,not n *g αI-c(x).

Proposition 3.28: n *r g αI -c(x) With n *g αI-c(x). are independent.
Recall example 3.32: N *Rg αI -o(x)={∅, μ,{ j}, {k}, {l}, {j , k}, {j, l}, {i , j, k}, {k , j , l}, {k, l},{i ,k ,l}}
N *g αI-o(x)={∅, μ,{j},{k}, {l}, {j , k}, {j, l},{i ,j, k},{k , j , l}}.
A={k ,l} is N*rg* αI -c(x) but A is not N*g* αI-c(x).

Proposition 3. 29: N *Rg βI -c(x) is N * Rg αI -c(x) , the converse doesn't satisfy.
Proof: Assume K is n *r g βI -c(x) ⇒ D ⊆N*βcl(k) , k⊆D , D is a n *R βI- o(x)
n *βcl(K) ⊆ n *αcl(K), and every N *R βI -o(x) is N *R αI -O(x) ⇒ D ⊆ N *αcl(K) ,D is N *R αI -o(x) ⇒ K is N * Rg αI –closed set ∎
Recall example 3.32: N *Rg αI -o(x)={∅, μ,{j},{k}, {l}, {j , k}, {j, l}, {i , j, k}, {k , j , l},{k, l},{i ,k ,l}}
N *R gβI-o(x)={ ∅, μ,{ j } ,{ k}, {l },{j, k},{j, l},{k ,l}}.
M={k ,j }, D={ k ,j ,l}⇒ M is N*R g αI -c(x) , not N *RgβI −c(x) Since {k , j}, N*β-o(x) , but D ⊈N*βcl(M) ={k ,j} . M is not N *Rg βI −c(x) .

Proposition 3. 30: n *g βI -c(x) with n *R g αI -c(x) are independent..
Recall example 3.32: F={i, k}, D={ k ,i}⇒ F is N* gβI -c(x) , not N *gRαI −c(x) , D ⊆n*αcl(F) but {k ,i} is not N*RαI-o(x), not N *R g αI − c(x) .
Recall example 3.14:
N*Rg*αIc(x)={∅,U,{1},{2},{3},{1,2},{1,3},{2,3},{1,4},{1,5},{1,4,5},{2,4,5},{2,4},{2,5},{3,4},{3,5},{3,4,5}{1,2,4},{1,2,5},{1,2,4,5},{1,3,4},{1,3,5},{1,3,4,5},{2,3,4},{2,3,5},{2,3,4,5}}.
N*g*αI-c(x)={∅,U,{1},{2},{3},{1,2},{1,3},{2,3},{1,2,3},{1,2,3,4},{1,2,3,5}}.
A={1,2,3,4} is N*g*αI-c(x) but A is not N*Rg*αI-c(x).

Proposition 3.31: n *rg βI -c(x) is n * g βI − c (x) . converse doesn't satisfy
Proof: Assume F is n *rg βI -c(x) ⇒ D ⊆ n *βCl(F) , F⊆D , D Is n *R βI –o(x).∵ n *R βI –o(x) is n *βI –o(x) , D ⊆ n *βcl(F) ⇒ F is n * g βI −c (x).∎
Recall example 3.32: M={i, j}, D={ i ,j}⇒ M is N* gβI -c(x) , not N *RgβI −c(x) , D ⊆N*βcl(M) ,{i ,j} not N*RβI-o(x), not N *R g βIc(x) .

Conclusions
from our study in the light of the theoretical part and examples illustrated, we can
conclude the generalized N* ideal -closed set can be study in(micro, soft) topological spaces

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المجموعات المغلقةالمعممة N * المثاليه في الفضاءات التبوبولوجية النانويه N* المثالية وخصائصها

نبيلة ابراهيم عزيز ، طه حميد جاسم
قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، العراق

الملخص

سوف ندرس فئة جديدة من المجموعات اسميناها المجموعات المغلقةالمعممة * N المثاليه في الفضاءات التبوبولوجية النانويه * N المثالية وخصائصها. علاوة على ذلك ، تم تقديم العلاقات بينها.