Generalized N* ideal closed sets in Nano N* ideal topological Spaces With Some Properties
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**ABSTRACT**

In this paper, we will study a new class of sets and said to be generalized N* ideal -closed sets in nano N* ideal topological spaces and its properties. Furthermore the relationships were introduced and notation.

1. Introduction


2 Preliminaries

**Definition 2.1** [1] : Let w be a non-empty set and R be Relation Equivalence on w, then w is called the indistinguishable relationship. and (W, R) area of approximation, X ⊆ W.
A) L_{R(X)} = \bigcup_{x \in X} R(x) : R(x) \subseteq X .
B) \bar{U}_{R(X)}. That is \bar{U}_{R(X)} = \bigcup_{x \in X} \{R(x) : R(x) \cap X \neq \emptyset \}.
C) B_{R(X)} = \{U \setminus L_{R(X)}\}.

**Definition 2.2** [1]: W is universe, R be an equivalence relation on W , X ⊆ W, \tau_{R(X)} = \{W, \phi, L_{R(X)}, U_{R(X)}, B_{R(X)}\}, (W, \tau_{R(X)}) is said to be Nano top. space. The sets of \tau_{R(X)} are named as nano open sets.

**Definition 2.3** [1] : (\varphi, \tau_{R(X)}) nano topological Space, X ⊆ \varphi . P ⊆ \varphi:
\begin{enumerate}
  \item i. n \text{int}(P) = \bigcup \{G : G \subseteq P, G \text{ is nano open sets}\}
  \item ii. n \text{cl}(P) = \bigcap \{ G : P \subseteq G, G \text{ is nano closed sets}\}.
\end{enumerate}
Definition 2.4[20]: \((\omega, \tau^{*}_{R(X)})\) is nano \(N^{*}\)topological Space, \(A \subseteq \omega\) then A is said to be \(N^{*}\) regular open set, if \(\exists \ \text{\textit{3 regular open set}} \ D \ 
subseteq A \subseteq N^{*} (\text{rel}(D)),\) symbolized by \(N^{*} \text{\textit{R}}(\omega)\).

Definition 2.5[20]: The nano \(N^{*}\)topological Space \((\delta, \tau^{*}_{R(X)})\), \(\subseteq \delta\), A named as \(N^{*}\alpha\)-regular open set, if \(\exists \ N^{*}\alpha\)-regular open set \(F, \ F \subseteq A \subseteq N^{*} \alpha(C(F))\).

Definition 2.6[21]: \((W, \tau_{R(X)})\) is nano topological space, \(\beta \subseteq W\) is called nano open sets if \(\beta\) satisfy:
1. For each \(Q \subseteq \beta, \exists\alpha\)-open Set \(G \subseteq Q \subseteq NCl(\beta)\).
2. If \(\beta \cap [U_{R(X)}]^{*} \neq \emptyset\) then \(U_{R(X)} \subseteq \beta\), symbolized by \(\tau^{*}_{R(X)}\).

Definition 2.7[21]: \((\gamma, \tau^{*}_{R(X)})\) N\(^{*}\)Space, \(Q \subseteq \gamma\) called
a) \(N^{*}\)-alpha-open set if \(\emptyset \subseteq N^{*}\text{int}(N^{*}\text{cl}(Q))\), symbolized by \(N^{*}\alpha\)-open set.
b) \(N^{*}\)beta-open set if \(\emptyset \subseteq N^{*}\text{cl}(N^{*}\text{int}(Q))\), symbolized by \(N^{*}\beta\)-open set.

Definition 2.8[22]: Let \(I\) be an Ideal on a topological space \((X, \tau)\) which satisfies the following conditions.
1. \(H \subseteq I, F \cap H \subseteq F \subseteq I\).
2. \(H \subseteq I \Rightarrow H_{\alpha} \subseteq I\).
3. \(H_{\beta} = N^{*}\text{cl}(H_{\alpha}) \subseteq N^{*}\text{cl}(H) (H_{\alpha} \neq N^{*}\text{cl}(H))\).
4. \(H_{\alpha} \subseteq H_{\beta}\).

Lemma 2.13[5-9]: \((\mu, N, I)\) is nano topological space with ideal \(I, MS M_{\alpha}, \text{then} M_{\alpha}^{*} = N^{*}\text{cl}(M_{\alpha}) = ncl(M)\).

Theorem 2.14[5-9]: ncl satisfies the conditions:
1. \(M \subseteq ncl(M)\).
2. \(ncl(\emptyset) = \emptyset\).
3. If \(H \subseteq F\), then \(ncl(H) \subseteq ncl(F)\).
4. \(ncl(H \cup ncl(F) = ncl(H) \cup ncl(F)\).
5. \(ncl(H) = ncl(H)\).

Definition 2.15[7,11]: \(H \subseteq (\gamma, N, I)\) is called
1. Nano \(\alpha\)-Open set (briefly nano-\(\alpha\)-open set) if \(H \subseteq nint(ncl(nint(H)))\).
2. Nano \(\beta\)-Open set (briefly nano-\(\beta\)-open set) if \(H \subseteq ncl(nint(ncl(H)))\).
3. NI Regular Open set if \(H = nint(ncl(H))\) and NI regular closed if \(ncl(nint(H)) = H\).

Definition 2.16[18]: \((\gamma, \tau^{*}_{R(X)})\) nano \(N^{*}\) space, \(H \subseteq \gamma\), H is said to be \(N^{*}\) generalized closed set in nano \(N^{*}\) topological space \(L \subseteq N^{*}\text{cl}(H), H \subseteq L\), L is \(N^{*}\) \(\alpha\)-open set.

3 Generalized \(N^{*}\)ideal closed set in Nano \(N^{*}\) ideal topological Spaces With Some Properties.

Definition 3.1: \((\mu, \tau^{*}_{R(X)})\) be nano \(N^{*}\) ideal topological space, 

Let \(J_{\alpha}\) is a set operator from \(F(\mu)\) to \(F(\mu)\), \[F(\mu) \subseteq \mu\].

For \(A \subseteq \mu, A_{\alpha} \subseteq \{x \in \mu | G_{\alpha} \cap A \subseteq J_{\alpha} \subseteq G_{\alpha}(x)\}\), said Nano \(N^{*}\) locally Function of \(A, j\) and \(n^{*}\) (briefly \(A_{\alpha}\)).

Theorem 3.2: \((\mu, \tau^{*}_{R(X)})\) is nano \(N^{*}\) ideal topological space and ideals \(j, k \subseteq A, B \subseteq \mu\).

Then
(i) \(A \subseteq B \Rightarrow A_{\alpha} \subseteq B_{\alpha}\).
(ii) \(j \subseteq k \Rightarrow A_{\alpha}(j) \subseteq A_{\alpha}(k)\).
(iii) \(A_{\alpha} = N^{*}\text{Cl}(A_{\alpha}) \subseteq N^{*}\text{Cl}(A)\) [\(A_{\alpha}\) is Nano closed set of \(N^{*}\text{Cl}(A)\)].
A"**" = N*-cl(A"**") = N*-cl(A) = N*-cl** (A).

**Definition 3.6:** (μ, τ**R**_{X,j}) is nano N* ideal topological space. The set operator N*cl** is said to be n*-closure ** [n*-cl**(A)=A \cup A"**", A \subseteq X].

**Definition 3.7:** (μ, τ**R**_{X,j}) be nano N* ideal topological Space. The set operator N*int** is said to be n*-interior* *[n*-int**(A) = An A"**", for A \subseteq X].

**Theorem 3.8:** (μ, τ**R**_{X,j}) is nano N* ideal topological space and ideal j on μ, \( \alpha \subseteq \mu \). If A \subseteq A"**", then

(i) n*-Cl(A) = n*-Cl** (A)

(ii) n*-int(A) = n*-int** (μ - A).

Proof (i) by using Th. 2.5.

Proof (ii) let A \subseteq X. Then A"**" = N*-cl** (A) by (i) and so X - N*-cl(A) = X - N*-cl** (A). Therefore, N*int(X - A) = N*int** (X - A).

**Definition 3.9:** Let(μ, τ**R**_{X,j}) be nano N* ideal topological space. The subset A of μ is said to be nano N* I-open set (briefly N*I-o(x)) if A \subseteq N* int (A"**").

**Example 3.10:** μ = \{1, 2, 3, 4, 5\}, μ /
R = \{1\}, \{2, 3\}, \{4, 5\} \}, X = \{1, 2, 3\}

I = \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 2, 5\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}, \{1, 2, 4, 5\}, \{1, 3, 4, 5\}, \{1, 2, 3, 4, 5\}.

**N*I-o(x) = \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 2, 5\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}, \{1, 2, 4, 5\}, \{1, 3, 4, 5\}, \{1, 2, 3, 4, 5\}.

**Remark 3.11:** It is clear that N*I and N*open are independent.

**Example 3.12:** μ = \{a, b, c\} be the universe, X = \{a, b, c\} \subseteq μ

μ/R = \{\{a\}, \{a, c\}, \{b, d\} \} and N = \{\{a\}, \{a, d\}, \{a, b, d\}\}.

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N*I-o(x) = \{\{a\}, \{a, d\}, \{a, b, d\}\}.

and the ideal I = \{\{a\}\}.

(i) For A = \{a, b, d\}, we have A"**" = \{a, b, c, d\}, \n*N*int(A"**") = \{a, d\} \Rightarrow A \subseteq N*int(A"**")

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(ii) For A = \{b\}, A"**" = \{b, c\} \Rightarrow N*int(A"**") = \{b\} \Rightarrow A \subseteq N*int(A"**")

A \subseteq N*I-open set, A \subseteq N*I-o(x).

**Definition 3.13:** (μ, τ**R**_{X,j}) be nano N* ideal topological space. A \subseteq μ, \( \alpha \subseteq \mu \). Is said to be

a) n* RI- o(x) \Rightarrow A \subseteq n* int(cl**(A)).

b) n* αI-o(x) \Rightarrow A \subseteq n* int(\{n*\int(cl**(A))\}).

c) n* βI-o(x) \Rightarrow A \subseteq n* cl**(n*int(cl**(A))).

denoted by N*RI(μ, x), N*αI(μ, x), N*β(μ, x).

**Example 3.14:** Let μ = \{1, 2, 3, 4, 5\}, μ /
R = \{\{1\}, \{2, 3\}, \{4, 5\}\}, X = \{1, 2\}

\( \tau_{R(X)} = \{\{a\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}.

N*I-o(x) = \{\{a\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}.

N* αI-o(x) = \{\{a\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}.

N* βI-o(x) = \{\{a\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}.
Proof: Assume A is n RβI - o(x) ⇒ ∃ T ≤ A ≤ n αβcl (T) 
And T is n αRI - o(x) take A= T ⇒ A = n int(ncl*(A)) ⊆ ncl*(n int(ncl*(A))) ⇒ is n αβI - o(x).
Recall Example 3.13: A= [1, 2, 4] is N αβI - o(x), but N RβI - o(x).

**Definition 3.22**: (μ, τ*Rx(I), l) is nano N α ideal topological space, E ⊆ μ , E is called n α generalized α-1 closed set in nano N α ideal topological space ↔ Y ⊆ n α inv(E), E Y, Y is a n ααl - o(x).
Recall Example 3.13: E= [1, 2, 3], G= [1, 2, 3, 4] ⇒ G ⊆ n α inv(E) = μ ⇒ E is N g αl - c(x).

**Definition 3.23**: (μ, T*Rx(I), l) be nano N α ideal topological space , F ⊆ μ , F is said to be N α regular generalized αl - closed set in nano N α ideal topological space ↔ D ⊆ N α cl(F), whenever D ≤ D , D is a n Rβαl - open set. 
Recall Example 3.13: U= (2, 3), D= [2, 3, 4, 5], D ⊆ N α cl(U) ⇒ U is N g αl - c(x).

**Definition 3.24**: (μ, T*Rx(I), l) be nano N α ideal topological space, Q ⊆ μ, Q is said n α generalized βI - c(x) in nano N α ideal topological space ↔ D ⊆ N βI cl(Q), Q ≤ D , D is N βI - o(x).
Recall Example 3.13: Q= [1, 2, 3], D= [1, 2, 3, 4] ⇒ N βI cl(Q) = μ ⇒ D ⊆ μ = Q is N g βI - c(x).

**Definition 3.25**: (μ, T*Rx(I), l) be nano N α ideal topological space, Q ⊆ μ, Q called n α regular generalized βI - c(x) in nano N α ideal topological space ↔ Z ⊆ N βI cl(Q), Q ≤ Z, Z is N RβI - o(x).
Recall Example 3.13: A= [2, 3], D= [2, 3, 4], D ⊆ N βI cl(A) ⇒ A is N RβI - c(x).

**Proposition 3.26**: n g βI - c(x) and n g αl - c(x) are independent.

**Example 3.26**: φ= { i, j, k, l}, φ /R= { j, i, k, l } , X= { j, k, l }, l= { φ, [j, l] } τ R(I)= {φ, φ, [ j, l ] } , N *αo(x)= {φ, φ , { j }, { k, l }, [ j, k ], [ k, l ] } , N *αo(x)={φ, φ, [ j }, { k, l ] } , N *βI o(x)= {φ, [ j ], [ k, l ] } , N *βI o(x)= {φ, φ, [ j ], [ k, l ] } , N *αβI o(x)= {φ, φ, [ j ], [ k, l ] } , N *αβI o(x)= {φ, φ, [ j ], [ k, l ] } , N *αβI o(x)= {φ, φ, [ j ], [ k, l ] } , N *αβI o(x)= {φ, φ, [ j ], [ k, l ] } , N *αβI o(x)= {φ, φ, [ j ], [ k, l ] } , A= {j, k } , N *g βI - c(x). but A is not N *g βI - c(x) .
A= {i, j} is n g βI - c(x), but not n g αl - c(x).

**Proposition 3.28**: n r αl - c(x) With n g αl - c(x) and n Rg βI - c(x) - x are independent.

References


مجموعات المغلقة المعممة N* المثالية في الفضاءات التبولوجية النانويه N* المثالية

وخصائصها

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الملخص

سوف ندرس فئة جديدة من المجموعات اسمها المجموعات المغلقة المعممة N* المثالية في الفضاءات التبولوجية النانويه N* المثالية وخصائصها. علاوة على ذلك، تم تقديم العلاقات بينها.