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On stability Conditions of Burr X Autoregressive model

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ABSTRACT

This article deals with proposed nonlinear autoregressive model based on Burr X cumulative distribution function known as Burr X AR (p), we demonstrate stability conditions of the proposed model in terms of its parameters by using dynamical approach known as local linearization method to find stability conditions of a nonzero fixed point of the proposed model, in addition the study demonstrate stability condition of a limit cycle if Burr X AR (1) model have a limit cycle of period greater than one.

Introduction

Topic of the non- linear time series is the most important and detailed topics in the last three decades. For the simple reason that most of the phenomena studied are non-linear to be closer to reality and to give more accurate explanations than linear models, Moreover, these models show the characteristics of this nonlinear nature and the most important nonlinear properties are

1. Jump phenomena which appear clearly in .Duffingėquations .

$$\ddot{x}(t) + c\dot{x}(t) + \alpha x(t) + \beta x^3(t) = F\cos wt$$

where c,α,β are real constant, $c\,\dot{x}(t)$ is the damping force , $F\cos(wt)$ is the external force and $\alpha x(t)+\beta x^3(t)$ is the restoring force

2. The amplitude frequency dependent which cause a jump phenomena

3. The existence of a limit cycles This behavior is characterized by the following van der pol equation

$$\ddot{x}(t) - \beta[1 - x^2(t)]\dot{x}(t) + \alpha x(t) = 0$$

A local linearization method is used to approximate a non- linear dynamical system near its fixed point to a linear dynamical system

Consider the van der pol equation:

$$\ddot{x} + (x^2 - 1)\dot{x} + x = 0 \quad \ldots (1.1)$$

put $\dot{x}$=y we may rewrite (1.1) in the standard state space form with $(x, y)^T$ as the state vector, then we get the following system

$$\dot{x} = y$$

$$\dot{y} = (1 - x^2)y - x \quad \ldots (1.2)$$

The only fixed point of the system (1.2) is the origin and the solution at this point is $\dot{x} = 0, \dot{y} = 0$

The first two term of Taylor expansion about this point gives local linearization as:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = L(0,0)\begin{bmatrix}x\\ y\end{bmatrix} \quad \ldots (1.3)$$

where $L(x,y)=\begin{bmatrix}0 & 1\\ -1-2xy & 1-x^2\end{bmatrix}$

Therefore around the origin the local approximation of van der pol equation is

$$\ddot{x} - \dot{x} + x = 0$$

The article proposes a new time non-linear time series model named Burr X AR(P)and study the stability conditions of a non-zero singular point of this model . Also we study the orbital stability of a limit cycle when the model possess a limit cycle of period greater than one.

2-Preliminaries

Most of proposed nonlinear autoregressive time series models based on one smooth function that jump smoothly from 0 to 1, then all distribution function are useful to represent the nonlinearity of these models . In this article we proposed a new nonlinear autoregressive time series model based on Burr X distribution function.

Let us consider the Burr X distribution with two parameters, the cumulative distribution function of this distribution has the form

$$F(x;\alpha,\lambda) = (1 - e^{-(\lambda x)^2})^{\alpha}, \; x > 0, \; \alpha > 0, \; \lambda > 0 \quad \ldots (2.1)$$

Where α is the shape parameter and λ the scale parameters. [10], [11].

The cumulative distribution function of Burr X possess the following properties

1- $\lim_{\lambda\to 0}(1-e^{-(\lambda x)^2})^{\alpha}=0$

2- $\lim_{\lambda\to \infty}(1-e^{-(\lambda x)^2})^{\alpha}=1$

3- $\lim_{\alpha\to 0}(1-e^{-(\lambda x)^2})^{\alpha}=1$

4- $\lim_{x\to \infty}(1-e^{-(\lambda x)^2})^{\alpha}=1$

5- $\lim_{x\to 0}(1-e^{-(\lambda x)^2})^{\alpha}=0$

The last model two properties are very important in our proposed and fig (2.1)
illustrate the roles of parameters α and λ

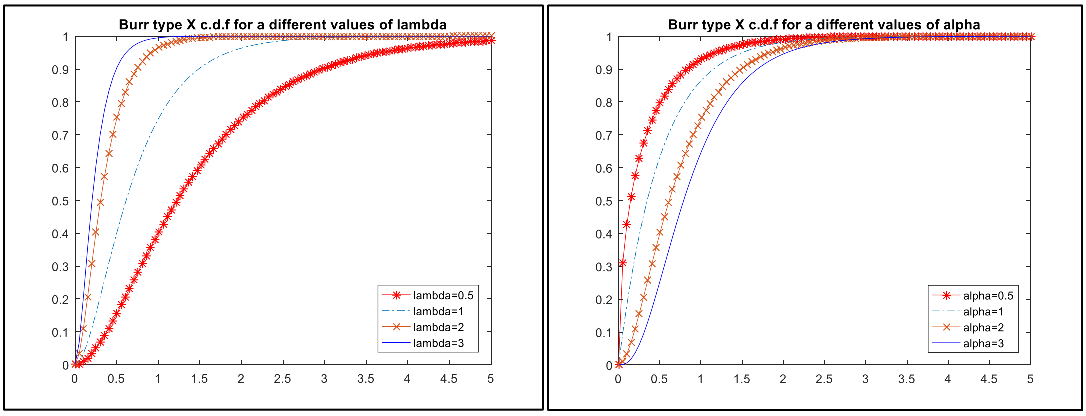

Fig (2.1): Burr Type X cumulative distribution function with different values of α and λ

Definition 2.1

Let $\{x_t\}$ be a discrete time series and t=±1,±2,±3,… the Burr X AR(P) model is defined as follows:

$x_t=\sum_{i=1}^{p}(\varphi_i+\pi_i\,(1-e^{-(\lambda x_{t-1})^2})^{\alpha})\,x_{t-i}+z_t$ … (2.2)

Where $\{z_t\}$ is white noise process, α and λ are shape and scale parameter, $\{\varphi_i\} and \{\pi_i\}, i=1,2,3,\ldots$ p are the model parameters.

In fact all nonlinear autoregressive discrete time series models can be represents as:

$x_t=f(x_{t-1},x_{t-2},\ldots,x_{t-p},Z_t)$ … (2.3)

Where $Z_t\sim iidN(0,\sigma_Z^2)$ and f be a nonlinear function

Definition 2.2

A fixed point y of $x_t=f(x_{t-1},x_{t-2},\ldots,x_{t-p})$ is defind as a point for which every trajectory of $x_t=f(x_{t-1},x_{t-2},\ldots,x_{t-p})$ sufficiently closed to y approaches it either for t→ ∞ ort → −∞ . If it approaches y for t → ∞ , then y is a stable fixed point, and if it approaches y for t→ −∞ , then y is unstable singular point. [12], [13]

Definition 2.3

A limit cycle of $x_t=f(x_{t-1},x_{t-2},\ldots,x_{t-p})$ is defined as a closed isolated trajectory

$x_t, x_{t+1}, x_{t+2}, \ldots, x_{t+q}=x_t$, $q>1$ … (2.4)

Where q is a smallest positive integer satisfying (2.4) known as the period of a limit cycle and $(x_{1+mq}, x_{2+mq}, \ldots, x_{p+mq})=(x_1,x_2,x_3,\ldots,x_p)$ for any integer m where the points $x_1,x_2,x_3,\ldots,x_p$ belongs to the limit cycle . A limit cycle (2.4) is stable if every trajectory sufficiently closed to the limit cycle approaches it for t → ∞ and the limit cycle is unstable if every trajectory sufficiently closed to the limit cycle approaches it for t → −∞ . [12], [13]

The stochastic process $\{X_t\}$ is called p- order autoregressive process and denoted by AR(p) if it satisfies the following stochastic difference equation

$x_t+a_1x_{t-1}+a_2x_{t-2}+\cdots+a_px_{t-p}=z_t$ … (2.5)

Where $z_t\sim$ *iid* N $(0,\sigma_t^2)$ and $a_1, a_2, \ldots, a_p$ are real constants .

By using a Backward shift operator (2.5) written as:

$$\alpha(B)x_t=z_t$$

Where the polynomial $\alpha(B)=1+a_1B+a_2B^2+\cdots+a_pzB^p$

The general solution of the (2.5) is the sum of complementary function $f(t)$ and particular solution $\alpha^{-1}(B)z_t$, i.e

$$x_t=f(t)+\alpha^{-1}(B)z_t$$

$f(t)$ is the solution of homogeneous part of (2.5) ; $\alpha(B)x_t=0$ and has the form

$$f(t)=A_1v_1^t+A_2v_2^t+\cdots+A_pv_p^t$$

Where $A_1,A_2,\ldots,A_p$ are an arbitrary constants and $v_1,v_2,\ldots,v_p$ are the roots of the characteristic equation

$v^p-\sum_{i=1}^{p}a_i\,v^{p-i}=0$ …(2.6)

and the particular solution is the solution of nonhomogeneous difference equation (2.5) has the form $x_t=\alpha^{-1}(B)z_t$ is called the stationary solution . The AR (p) defend by (2.5) is asymptotically stationary if $\lim_{t\to\infty}f(t)=0$. This limit equal to zero if $|v_i|<1$ for $i=1,2,\ldots,p$, in another word all roots of (2.6) lies inside the unit circle.[14]

3- Stability condition of Burr X autoregressive model (Burr X AR(P))

In this paragraph , the stability conditions of proposed time series model (2.2), the first step deals with the existence of a nonzero singular point of the model and the second step consist of finding the stability conditions of the nonzero singular point. In addition we find the stability condition of a limit cycle when the model possesses a limit cycle.

Burr X AR (p) model can easily applied for any kind of nonlinear damping system. When x_{t-1} changes in approaches between 0 and ∞ , the coefficients of the characteristic equation changes between $\sum_{i=1}^{p}\varphi_i$ and $\sum_{i=1}^{p}(\varphi_i+\pi_i)$.

To find the non-zero fixed point of the of the (2.2) according to the definition of fixed point we put $x_s = y$ for $s = t, t-1, \dots, t-p$, then model (2.2) became

$$y = \sum_{i=1}^{p}\left[\varphi_i + \pi_i\left(1-e^{-(\lambda y)^2}\right)^{\alpha}\right]y \quad \dots (3.1)$$

Note that the white noise $\{z_t\}$ is suppresses clearly y=0 is the fixed point of the model and to find a non-zero fixed point

$$1 = \sum_{i=1}^{p}\left[\varphi_i + \pi_i\left(1-e^{-\lambda^2y^2}\right)^{\alpha}\right]$$

$$\frac{1-\sum_{i=1}^{p}\varphi_i}{\sum_{i=1}^{p}\pi_i} = \left(1-e^{-\lambda^2y^2}\right)^{\alpha}$$

Let $k = \frac{1-\sum_{i=1}^{p}\varphi_i}{\sum_{i=1}^{p}\pi_i}$ then $\ln\left(1-e^{-\lambda^2y^2}\right)^{\alpha} = \ln k$

$$\ln\left(1-e^{-\lambda^2y^2}\right) = \frac{\ln k}{\alpha} = \ln k^{\frac{1}{\alpha}}$$

and we get

$$y = \pm\frac{1}{\lambda}\sqrt{\ln\left(\frac{1}{1-k^{\frac{1}{\alpha}}}\right)}$$

Note that in the case of the random variable of the time series $\{x_t\}$ is positive the non-zero singular point

$$y = \frac{1}{\lambda}\sqrt{\ln\left(\frac{1}{1-k^{\frac{1}{\alpha}}}\right)} \quad \dots (3.2)$$

The non-zero fixed point y is exists if $\frac{1}{1-k^{\frac{1}{\alpha}}} > 0$ that's mean $k^{\frac{1}{\alpha}} < 1$ and since $\alpha > 0$ which implies that $0 < k < 1$

Then the non-zero fixed point y for Burr x AR (P) exists if

$$0 < \frac{1-\sum_{i=1}^{p}\varphi_i}{\sum_{i=1}^{p}\pi_i} < 1 \quad \dots (3.3)$$

PROPOSITION 3.1

The non-zero fixed point of Burr X AR (P) modelis stable if the roots of characteristic equation $z^p - \sum_{j=1}^{p} h_j z^{p-j} = 0$ lies inside the unit circle where

$$h_1 = \varphi_1 + k\,\pi_1 \quad -2\alpha\ln(1-k^{\frac{1}{\alpha}})\left(\frac{1-k^{\frac{1}{\alpha}}}{k^{\frac{1}{\alpha}}}\right)\left(1-\sum_{i=1}^{p}\varphi_i\right) \quad \dots (3.4)$$

$$h_j = \varphi_j + k\,\pi_j \quad , j=2, 3\dots p \quad \text{and} \quad k=\frac{1-\sum_{i=1}^{p}\varphi_i}{\sum_{i=1}^{p}\pi_i} \quad \dots (3.5)$$

Proof:

Closed to the non-zero fixed point of the model say it y we consider the vibrational difference equation in the neighborhood of y by setting $x_s = y + y_s$ where the radius of the nieghbourhood y_s sufficiently small such that $|y_s|^n \to 0$ for $n \geq 2$ for s=t, t-1, … ,t-p by replacing x_{t-i} by $y + y_{t-i}$ for , $i = 0,1,2,3,\dots,p$ in Burr X AR(P) model (2.2) after the white noise be suppressed we get

$$y + y_t = \sum_{i=1}^{p}[\varphi_i + \pi_i(1-e^{-\lambda^{2(y+y_{t-1})^2}})^{\alpha}]\,(y+y_{t-i}) \quad \dots (3.6)$$

And by using Taylor expansion of $e^{-\lambda^{2(y+y_{t-1})^2}}$ we get

$$(1-e^{-\lambda^{2(y+y_{t-1})^2}})^{\alpha} = \left(1-e^{-\lambda^2y^2}e^{-2\lambda^2yy_{t-1}}e^{-\lambda^2y_{t-1}^2}\right)^{\alpha}$$

$$= \left(1-\left(1-k^{\frac{1}{\alpha}}\right)(1-2\lambda^2yy_{t-1}).1\right)^{\alpha}$$

Since $e^{-\lambda^2y_{t-1}^2} = e^0 = 1$ and $e^{-\lambda^2y^2} = \left(1-k^{\frac{1}{\alpha}}\right)$

where $k=\frac{1-\sum_{i=1}^{p}\varphi_i}{\sum_{i=1}^{p}\pi_i}$

After some algebraic treatment we get

$$(1-e^{-\lambda^{2(y+y_{t-1})^2}})^{\alpha} = \left(1+2\lambda^2yy_{t-1}\left(\frac{1-k^{\frac{1}{\alpha}}}{k^{\frac{1}{\alpha}}}\right)\right)^{\alpha}.k$$

$$= k + k\sum_{i=1}^{\infty} C_i^{\alpha}\left(2\lambda^2yy_{t-1}\left(\frac{1-k^{\frac{1}{\alpha}}}{k^{\frac{1}{\alpha}}}\right)\right)^{i}$$

$$= k + k.\left[C_1^{\alpha}\left(2\lambda^2yy_{t-1}\left(\frac{1-k^{\frac{1}{\alpha}}}{k^{\frac{1}{\alpha}}}\right)\right)\right] + k\left[C_2^{\alpha}\left(4\lambda^4y^2y_{t-1}^2.\left(\frac{1-k^{\frac{1}{\alpha}}}{k^{\frac{1}{\alpha}}}\right)^2\right)\right]+\dots$$

But $y_{t-1}^2 \to 0$ then

$$(1-e^{-\lambda^{2(y+y_{t-1})^2}})^{\alpha} = k + 2k\alpha\lambda^2yy_{t-1}\left(\frac{1-k^{\frac{1}{\alpha}}}{k^{\frac{1}{\alpha}}}\right) \quad \dots (3.7)$$

And by substituting (3.7) in equation (3.6) we get

$$y + y_t = \sum_{i=1}^{p}[\varphi_i + \pi_i\left(k + 2k\alpha\lambda^2yy_{t-1}\left(\frac{1-k^{\frac{1}{\alpha}}}{k^{\frac{1}{\alpha}}}\right)\right)]\,(y+y_{t-i})$$

$$y + y_t = y\sum_{i=1}^{p}\varphi_i + y\sum_{i=1}^{p}\pi_i\, k + \sum_{i=1}^{p}\varphi_i\, y_{t-i} + \sum_{i=1}^{p}\pi_i\, k\, y_{t-i} + 2k\alpha\lambda^2y^2y_{t-1}\left(\frac{1-k^{\frac{1}{\alpha}}}{k^{\frac{1}{\alpha}}}\right)\sum_{i=1}^{p}\pi_i + 2k\alpha\lambda^2yy_{t-1}\left(\frac{1-k^{\frac{1}{\alpha}}}{k^{\frac{1}{\alpha}}}\right)\sum_{i=1}^{p}\pi_i\, y_{t-i}$$

But $|y_{t-1}y_{t-i}| \to 0$

$$y + y_t = y\sum_{i=1}^{p}\varphi_i + y\sum_{i=1}^{p}\pi_i\frac{1-\sum_{i=1}^{p}\varphi_i}{\sum_{i=1}^{p}\pi_i} + \sum_{i=1}^{p}\varphi_i\, y_{t-i} + \sum_{i=1}^{p}\pi_i\frac{1-\sum_{i=1}^{p}\varphi_i}{\sum_{i=1}^{p}\pi_i}\, y_{t-i} + 2\alpha\frac{1-\sum_{i=1}^{p}\varphi_i}{\sum_{i=1}^{p}\pi_i}\lambda^2\left(-\frac{1}{\lambda^2}\ln(1-k^{\frac{1}{\alpha}})\right)\left(\frac{1-k^{\frac{1}{\alpha}}}{k^{\frac{1}{\alpha}}}\right)\sum_{i=1}^{p}\pi_i\, y_{t-1}$$

$$y_t = \sum_{i=1}^{p}(\varphi_i + k\,\pi_i)\, y_{t-i} - 2\alpha\ln(1-k^{\frac{1}{\alpha}})\left(\frac{1-k^{\frac{1}{\alpha}}}{k^{\frac{1}{\alpha}}}\right)\left(1-\sum_{i=1}^{p}\varphi_i\right) y_{t-1} \quad \dots (3.8)$$

Or $y_t = \sum_{i=1}^{p} h_i\, y_{t-i}$

Which is a linear difference equation of order p and

$$h_1 = \varphi_1 + k\,\pi_1 \quad -2\alpha\left(1-\sum_{i=1}^{p}\varphi_i\right)\ln(1-k^{\frac{1}{\alpha}})\left(\frac{1-k^{\frac{1}{\alpha}}}{k^{\frac{1}{\alpha}}}\right)$$

$$h_j = \varphi_j + k\,\pi_j \quad , j=2, 3,\dots, p$$

And the non-zero fixed point of the Burr x AR (P) model is stable if all the roots of the characteristic equation $z^p - \sum_{j=1}^{p} \mathrm{h}_j z^{p-j} = 0$ has absolute values less than one, *i.e* $|z_i| < 1 \ for \ \ i = 1,2,3,\dots,p.$ □

In the following proposition , the stability condition of a limit cycle when Burr X AR (1) has a limit cycle of period $q > 1$

Proposition 4.1

If Burr X AR (1) Model has a limit cycle of period q , then the model is orbitally stable if

$$\left|\prod_{j=1}^{q} \varphi_1 + \pi_1 \left[\left(1 - e^{-\lambda^2 x_{t+j-1}^2}\right)^{\alpha} + 2\alpha\lambda^2 x_{t+j-1}^2 e^{-\lambda^2 x_{t+j-1}^2} \left(1 - e^{-\lambda^2 x_{t+j-1}^2}\right)^{\alpha-1}\right]\right| < 1$$

Proof

Consider the following Burr X AR (1) model

$$x_t = \varphi_1 x_{t-1} + \pi_1\left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha} x_{t-1} + z_t \quad \dots (3.10)$$

Where $z_t \sim iidN(0, \sigma_z^2)$

Let $x_t, x_{t+1}, x_{t+2}, \dots, x_{t+q} = x_t$ be a limit cycle of period $q > 1$, closed to each point of a limit cycle $\mathrm{x_s}$, $\mathrm{s = t, t+1}, \dots, \mathrm{t+q}$. let $\mathrm{y_s}$ be the radius of a neighborhood of $\mathrm{x_s}$ sufficiently small such that $\mathrm{y_s^n} \to 0$ for $\mathrm{n} \geq 2$ and for $\mathrm{s = t, t+1}, \dots, \mathrm{t+q}$, replacing $\mathrm{x_s}$ in (3.10) by $x_s + y_s$ for $\mathrm{s = t, t-1}$ after suppressing a white noise we get

$$x_t + y_t = \varphi_1(x_{t-1} + y_{t-1}) + \pi_1\left(1 - e^{-\lambda^2 (x_{t-1}+y_{t-1})^{\,2}}\right)^{\alpha} (x_{t-1} + y_{t-1}) \quad \dots (3.11)$$

But $\left(1 - e^{-\lambda^2 (x_{t-1}+y_{t-1})^2}\right)^{\alpha} = \left(1 - e^{-\lambda^2 x_{t-1}^2} e^{-2\lambda^2 x_{t-1} y_{t-1}} e^{-\lambda^2 y_{t-1}^2}\right)^{\alpha}$

By using Taylor expansion and since $y_{t-1}^2 \to 0$

$$\left(1 - e^{-\lambda^2 (x_{t-1}+y_{t-1})^2}\right)^{\alpha} = \left(1 - e^{-\lambda^2 x_{t-1}^2}(1 - 2\lambda^2 x_{t-1} y_{t-1})\right)^{\alpha}$$

$$= 1 + \sum_{i=1}^{\infty} (-1)^i C_i^{\alpha} \left(e^{-\lambda^2 x_{t-1}^2}(1 - 2\lambda^2 x_{t-1} y_{t-1})\right)^i$$

$$= 1 - C_1^{\alpha} e^{-\lambda^2 x_{t-1}^2}(1 - 2\lambda^2 x_{t-1} y_{t-1}) + C_2^{\alpha} e^{-2\lambda^2 x_{t-1}^2}(1 - 2\lambda^2 x_{t-1} y_{t-1})^2 - C_3^{\alpha} e^{-3\lambda^2 x_{t-1}^2}(1 - 2\lambda^2 x_{t-1} y_{t-1})^3 + \cdots$$

$$= 1 - C_1^{\alpha} e^{-\lambda^2 x_{t-1}^2} + 2C_1^{\alpha} e^{-\lambda^2 x_{t-1}^2} \lambda^2 x_{t-1} y_{t-1} + C_2^{\alpha} e^{-2\lambda^2 x_{t-1}^2} - 4\lambda^2 C_2^{\alpha} e^{-2\lambda^2 x_{t-1}^2} x_{t-1} y_{t-1} + 4\lambda^4 C_2^{\alpha} e^{-2\lambda^2 x_{t-1}^2} x_{t-1}^2 y_{t-1}^2 - C_3^{\alpha} e^{-3\lambda^2 x_{t-1}^2} + 4\lambda^2 C_3^{\alpha} e^{-3\lambda^2 x_{t-1}^2} x_{t-1} y_{t-1} - 4\lambda^4 C_3^{\alpha} e^{-3\lambda^2 x_{t-1}^2} x_{t-1}^2 y_{t-1}^2 + 2\lambda^2 C_3^{\alpha} e^{-3\lambda^2 x_{t-1}^2} x_{t-1} y_{t-1} - 8\lambda^4 C_3^{\alpha} e^{-3\lambda^2 x_{t-1}^2} x_{t-1}^2 y_{t-1}^2 + 8\lambda^6 C_3^{\alpha} e^{-3\lambda^2 x_{t-1}^2} x_{t-1}^3 y_{t-1}^3 + \cdots$$

Since $y_{t-1}^n \to 0$ for $n \geq 2$ then

$$\left(1 - e^{-\lambda^2 x_{t-1}^2}(1 - 2\lambda^2 x_{t-1} y_{t-1})\right)^{\alpha} = 1 - C_1^{\alpha} e^{-\lambda^2 x_{t-1}^2} + C_2^{\alpha} e^{-\lambda^2 x_{t-1}^2} - C_3^{\alpha} e^{-\lambda^2 x_{t-1}^2} + \cdots + C_1^{\alpha} 2 e^{-\lambda^2 x_{t-1}^2} \lambda^2 x_{t-1} y_{t-1} - C_2^{\alpha} 4\lambda^2 e^{-2\lambda^2 x_{t-1}^2} x_{t-1} y_{t-1} + C_3^{\alpha} 6\lambda^2 e^{-3\lambda^2 x_{t-1}^2} x_{t-1} y_{t-1} + \cdots$$

$$= \left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha} + \sum_{i=1}^{\infty} C_i^{\alpha} (-1)^{i+1} 2i\lambda^2 x_{t-1} y_{t-1} \left(e^{-\lambda^2 x_{t-1}^2}\right)^i$$

$$= \left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha} - 2\lambda^2 x_{t-1} y_{t-1} \sum_{i=1}^{\infty} C_i^{\alpha} i(-1)^i \left(e^{-\lambda^2 x_{t-1}^2}\right)^i$$

But for the sum $\sum_{i=1}^{\infty} C_i^{\alpha} i(-1)^i \left(e^{-\lambda^2 x_{t-1}^2}\right)^i$

Let $i = s + 1$ at $i = 1$, $s = 0$ and then

$$\sum_{i=1}^{\infty} C_i^{\alpha} i(-1)^i \left(e^{-\lambda^2 x_{t-1}^2}\right)^i = \quad \dots (3.9)$$

$$\sum_{s=0}^{\infty} \frac{\alpha!}{(\alpha-(s+1))!\,(s+1)!} (s+1)(-1)^{s+1} \left(e^{-\lambda^2 x_{t-1}^2}\right)^{s+1}$$

$$= -\alpha e^{-\lambda^2 x_{t-1}^2} \sum_{s=0}^{\infty} \frac{(\alpha-1)!}{s!\,((\alpha-1)-s)!} (-1)^s \left(e^{-\lambda^2 x_{t-1}^2}\right)^s$$

$$= -\alpha e^{-\lambda^2 x_{t-1}^2} \left(1 + \sum_{s=1}^{\infty} C_S^{\alpha-1} (-1)^s \left(e^{-\lambda^2 x_{t-1}^2}\right)^s\right)$$

$$= -\alpha e^{-\lambda^2 x_{t-1}^2} \left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha-1} \quad \dots (3.12)$$

By substitute (3.12) in (3.11) we get

$$\left(1 - e^{-\lambda^2 x_{t-1}^2}(1 - 2\lambda^2 x_{t-1} y_{t-1})\right)^{\alpha} = \left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha} + 2\,\alpha\,\lambda^2 e^{-\lambda^2 x_{t-1}^2} \left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha-1} . x_{t-1} y_{t-1} \quad \dots (3.13)$$

Now return to (3.11) and substitute (3.13) in it we get

$$x_t + y_t = \varphi_1 x_{t-1} + \varphi_1 y_{t-1} + \pi_1 \left[\left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha} + 2\,\alpha\,\lambda^2 e^{-\lambda^2 x_{t-1}^2} \left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha-1} . x_{t-1} y_{t-1}\right] (x_{t-1} + y_{t-1})$$

$$x_t + y_t = \left[\varphi_1 + \pi_1\left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha}\right] x_{t-1} + 2\pi_1\,\alpha\,\lambda^2 e^{-\lambda^2 x_{t-1}^2} \left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha-1} x_{t-1}^2 y_{t-1} + \varphi_1 y_{t-1} + \pi_1\left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha} y_{t-1} + 2\pi_1\,\alpha\,\lambda^2 e^{-\lambda^2 x_{t-1}^2} \left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha-1} y_{t-1}^2 x_{t-1}$$

But $y_{t-1}^2 \to 0$ and $x_t = \left[\varphi_1 + \pi_1\left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha}\right] x_{t-1}$, then

$$y_t = \left[\varphi_1 + \pi_1\left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha} + 2\pi_1\,\alpha\,\lambda^2 e^{-\lambda^2 x_{t-1}^2} \left(1 - e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha-1} x_{t-1}^2\right] y_{t-1} \quad \dots (3.14)$$

But this equation is a first order difference equation with no constant coefficient which is difficult to solve analytically, then we discuss the convergence of the ratio $\left|\frac{y_t}{y_{t+q}}\right|$ to zero and this ratio converges to zero if $\left|\frac{y_t}{y_{t+q}}\right| < 1$

Let $T(X_{t-1}) = \left[\varphi_1 + \pi_1\left(1-e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha} + 2\pi_1\, \alpha\, \lambda^2\, e^{-\lambda^2 x_{t-1}^2}\left(1-e^{-\lambda^2 x_{t-1}^2}\right)^{\alpha-1} x_{t-1}^2\right] y_{t-1}$

Then $y_t = T(X_{t-1})\, y_{t-1}$

$y_{t+1} = T(X_t).\, y_t$

Now $y_{t+q} = T(X_{t+q-1}).\, y_{t+q-1}$

$y_{t+q} = T(X_{t+q-1})).\, T(X_{t+q-2}).\, y_{t+q-2}$

And after q iteration we get

$y_{t+q} = \prod_{j=1}^{q} T(X_{t+j-1}).\, y_t$ or $\left|\frac{y_{t+q}}{y_t}\right| = \left|\prod_{j=1}^{q} T(X_{t+j-1})\right|$

Finally the limit cycle (if it exists) of Burr x AR (1) model is orbit ally stable if

$$\left|\prod_{j=1}^{q} \varphi_1 + \pi_1\left[\left(1-e^{-\lambda^2 x_{t+j-1}^2}\right)^{\alpha} + 2\alpha\lambda^2 x_{t+j-1}^2\, e^{-\lambda^2 x_{t+j-1}^2}.\left(1-e^{-\lambda^2 x_{t+j-1}^2}\right)^{\alpha-1}\right]\right| < 1. \square$$

Example 3.1

we modeled the time series of monthly mean temperature of Karkuk city for years(1980- 2017), and estimate the model parameter by using statistca Software and we obtain the following Burr X AR (1) model

$$x_t = 1.440596 x_{t-1} - 0.459352\left(1-e^{-(0.807709.x_{t-1})^2}\right)^{5.817303} x_{t-1} + z_t \quad \ldots (3.15)$$

By using (3.3) we calculate $k = 0.9592 < 1$, then the nonzero singular point exists and equal to 2.7523 by using (3.2), in other hand we calculate the root of characteristic equation by using (3.4) and we see that $h_1 = 0.9816209 < 1$ and the model (3.15) have a stable nonzero singular point . Fig (3.1) shows the convergence of trajectories starting from different initial values to a nonzero fixed point of the model.

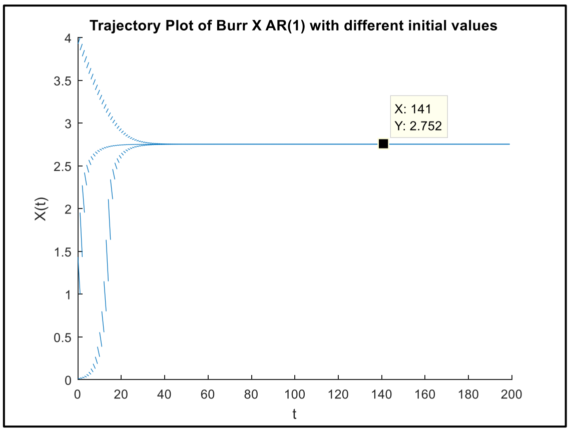

Fig (3.1): Trajectory plot of Model (3.15)

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نموذج أ استقرارية شروط حول Burr X للانحدار الذاتي

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الملخص

في هذه الورقة تم اقتراح نموذج انحدار ذاتي جديد للمتسلسلات الزمنية غير الخطية مبني اساسا على دالة التوزيع التراكمية لتوزيع Burr X وتم ايجاد شروط استقرارية النموذج المقترح بدلالة معلماته. استخدمت منهجية حركية والمعروفة بطريقة التقريب الخطية المحلية لإيجاد شروط استقرارية النقطة المنفردة غير الصفرية للنموذج المقترح, بالإضافة الى ايجاد شروط استقرارية دورة النهاية عند امتلاك النموذج المقرح لدورة نهاية.