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# A study on SS- $\pi$- regular fuzzy ideals of semi groups

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## ABSTRACT

In this paper, the notion of SS -$\pi$- regular fuzzy ideals of semi groups as a generalization of regular fuzzy ideal has been introduced and some of their important related properties have been investigated. Characterizations of fuzzy interior ideal , anti fuzzy ideal , anti fuzzy bi-ideal and anti fuzzy generalized –bi -ideal in terms of SS - $\pi$- regular fuzzy ideal have also been obtained .

## 1. Introduction

The fundamental concept of a fuzzy for short (F) set was first presented by [1], The concept F sets in the structure of groups, was first studied [2]. The concept of fuzzy ideals for short (FI) in semi group was developed by [3] On the other hand, the concept of anti fuzzy subgroups of groups introduced by [4]. The concept of anti fuzzy ideals in semi groups and characterized different classes of semi groups by the properties of their anti fuzzy ideals" explained by [5] and [6], gave some properties of anti fuzzy ideals for short (AN-F-I) in terms of regular and left (right) quasi regular semi group), where A F sub-set $p$ of a semi group Q is called anti F sub-semi group of Q if $p(\alpha\beta) \leq p(\alpha) \vee p(\beta) \,\forall\, \alpha , \beta \in$ Q" .

"A F sub-set $p$ of a semi group Q is called anti fuzzy left (right) for short(AN-F-L(R)-I ) of Q if $p(\alpha\beta) \leq p(\beta)$, $(p(\alpha\beta) \leq p(\alpha)) \,\forall\, \alpha , \beta \in$ Q ".

"A F sub-set $p$ of a semi group Q is called a anti F ideal of Q if it is both (AN-F- L-I) and (AN-F- R-I)".

"A F sub-set $p$ of a semi group Q is called anti fuzzy interior ideal for short (AN-F-IN-I) of Q if $p(\alpha\gamma\beta) \leq p(\gamma)$ ; $\forall\, \alpha, \gamma , \beta \in$ Q ".

"A F sub-set $p$ of a semi group Q is called anti fuzzy generalized bi-ideal of Q for short (AN-F-G-BI-I) if $p(\alpha\, \beta\, \gamma) \leq p(\alpha) \vee p(\gamma)$ ; $\forall\, \alpha , \beta , \gamma \in$ Q",

"A F sub - semi group $p$ is called anti Fuzzy bi-ideal of Q if $p(\alpha\, \beta\, \gamma) \leq p(\alpha) \vee p(\gamma)$ , $\forall\, \alpha , \beta , \gamma \in$ Q ". The concept of" Intra-regular left almost semi group characterized by their AN-F-I " was studies by(Khan, Asif and Faisal,2010) where "A F sub-set $p$ of a LA-semi group Q is called a F LA- sub-semi group if $p(\alpha\, \beta) \geq p(\alpha) \vee p(\beta)$ ; $\forall\, \alpha , \beta \in$ Q".

"A F sub-set $p$ of LA- semi group Q is called a F left (right) ideal of Q if $p(\alpha\, \beta) \geq p(\beta)$, $(p(\alpha\, \beta) \geq p(\alpha)) \,\forall\, \alpha , \beta \in$ Q" .

"A F LA- sub semi group $p$ of a LA- semi group Q is called a F bi-ideal if

$p((\alpha\, \beta)\gamma) \geq p(\alpha) \wedge p(\gamma) \,\forall\, \alpha , \beta , \gamma \in$ Q" .

"A F LA- sub semi group $p$ of a LA- semi group Q is AN-F-IN-I if

$p((\alpha\, \beta)\gamma) \geq p(\,\beta) \,\forall\, , \beta , \gamma \in$ Q ".

Now, we shall give the concepts of fuzzy sub-set and basic definitions with some related properties which will be used in this paper .

Let Q be a semi group , By a sub-semi group of Q we mean a nonempty C of Q s.t $C^2 \sqsubseteq$C , and by a left (right) ideal of Q we mean a nonempty sub-set C of Q s.t QC⊑C (CQ⊑C) . by two sided ideal or simply ideal , we mean a non-empty sub-set of Q which is a both "a left and right ideal of" Q, a sub-semi group C of a semi group Q is called bi- ideal of Q if CQC⊑C , by a F set $p$ in anon empty Q . “we mean a function $p$: Q ⟶[0 , 1] and the complement of $p$ denoted by $p'$ , is the fuzzy set in Q given by $p'(\alpha) = 1 - p(\alpha) \,\forall\, \alpha \in$Q.” , The concept of "fuzzy interior ideals in semi groups" was studied by (Hong and Jun , 1995) where a fuzzy sub-set $p$ in a semi group Q is called a fuzzy sub-semi group of Q- if $p(\alpha\, \beta) \geq \min\{p(\alpha) , p(\beta)\}$ ; $\forall\, \alpha , \beta \in$ Q.

A fuzzy sub-set $p$ of a semi group Q is called a fuzzy interior ideal for short (F- IN - I) of Q if : $p(\alpha\beta\gamma) \geq p(\beta)$ ; $\forall\ \alpha, \beta, \gamma \in$ Q ."

The goal of this article is to characterize an SS-$\pi$-regular Semi group by the properties of their (AN-F-IN-I) , (AN-F-L(R)-I), anti fuzzy two sided ideal for short (AN-F-T-S-I), (AN-F-G-BI-I) . We also give some properties a SS -$\pi$- regular LA-semi group and their fuzzy (left (right), two sided) ideals.

## 2. The main results

In this section, we study the concept of SS -$\pi$-regular semi groups, and we give basic properties of this concept.

Now, we give the following definitions

**Definition 2.1**

A semi group Q is called SS –$\pi$- regular if for every a ∈ Q, ∃ b, c ∈ Q, s.t $a^n = a^n$b$a^{2n}$c, for some n ∈ $Z^+$, equivalently $a^n \in a^n$Q$a^{2n}$ Q , for every a ∈ Q , for some $n \in Z^+$ .

**Example 2 . 2 :**

Let Q={1, 2, 3, 4, 5, 6} be a semi group with then

| . | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 4 | 1 | 5 | 2 | 6 | 3 |
| 3 | 5 | 3 | 1 | 6 | 4 | 2 |
| 4 | 2 | 4 | 6 | 1 | 3 | 5 |
| 5 | 3 | 6 | 2 | 5 | 1 | 4 |
| 6 | 6 | 5 | 4 | 3 | 2 | 1 |

Then it is clear that , Q is a SS- $\pi$ - regular because if n = 2

$1^2 = 1^2 3\ 1^4\ 5$ , $2^2 = 2^2 4\ 2^4\ 2$ , $3^2 = 3^2\ 6\ 3^4\ 6$ , $4^2 = 4^2 6\ \ 4^4\ 6$,

$5^2 = 5^2 1\ 5^4 1$ , $6^2 = 6^2 3\ 6^4 5$

**Definition 2.3:**

Let $p$ and $\varphi$ be any F sub-set of a semi group Q, then the product $p \circ \varphi$ is $defined\ by$

$(p \circ \varphi)_{(a^n)}$

$=$

$$\begin{cases} \bigvee_{a^n = b^n c^n} \{p(b^n) \wedge \varphi(c^n)\}; \text{ if } \exists\ \text{b}, \text{c} \in \text{Q s.t } a^n = b^n c^n \text{ for some } n \in Z^+ \\ 0\ ; otherewise. \end{cases}$$

**Definition 2.4:**

Let $p$ and $\varphi$ be any F sub-set of a semi group Q. Then the anti product $p * \varphi$ is defined by

$(p * \varphi)_{(a^n)} =$

$$\begin{cases} \bigwedge_{a^n = b^n c^n} \{p(b^n) \vee \varphi(c^n)\}; \text{ if } \exists\ \text{b} , \text{c} \in \text{Q s.t } a^n = b^n c^n \\ 1\ ; otherwise\ . \end{cases}$$

for some $n \in Z^+$.

Now we characterize SS- $\pi$ - regular semi group by the properties of their FI

**Theorem 2.5 :**

In SS -$\pi$- regular semi group Q, every F-IN-I is idempotent .

**Proof :**

Suppose that $p$ is a F-IN- I of a semi group Q , then it is clear that $p \circ p \subseteq p$

Let a ∈ Q , since Q is a SS -$\pi$- regular semi group, then ∃ x , y ∈ Q s.t $a^n = a^n$x$a^{2n}$y for some n∈ $Z^+$, we have

$a^n = a^n$x$a^{2n}$y$= a^n$x$a^n a^n$y $= a^n x(a^n x a^{2n} y) a^n y$ $= (a^n x)\, a^n($x$a^{2n})$ y$a^n$y

$(p \circ p)_{(a^n)} =$

$\bigvee_{a^n = (a^n x) a^n (\text{x}a^{2n}) \text{y} a^n y} \{p((a^n x) a^n (\text{x}a^{2n})) \wedge p(\text{y}a^n y)\}$

$\geq \{p((a^n x)\, a^n (\text{x}a^{2n})\,) \wedge p(\text{y}a^n y)\}$

$\geq p(a^n) \wedge p(a^n) = p(a^n)$ This implies that $p \subseteq p \circ p$,

Hence $p \circ p = p$ .

**Example 2 . 6 :**

Let Q $= \{r^n, s^n, t^n, v^n\}$ be a set with operation as follows :

| . | $r^n$ | $s^n$ | $t^n$ | $v^n$ |
|---|---|---|---|---|
| $r^n$ | $r^n$ | $r^n$ | $r^n$ | $r^n$ |
| $s^n$ | $r^n$ | $r^n$ | $r^n$ | $r^n$ |
| $t^n$ | $r^n$ | $r^n$ | $s^n$ | $r^n$ |
| $v^n$ | $r^n$ | $r^n$ | $s^n$ | $s^n$ |

Then we can easily see that (Q, .) is not SS -$\pi$-regular semi group

Define the F sub-set $p$ of Q as :

$p(r^n) = 0.3$ , $p(s^n) = 0.9$ , $p(t^n) = 0.5$ , $p(v^n) = 0.7$

Then it is clear that, $p$ is AN-F-IN-I of Q but not an AN-F-T-S-I of Q, because $\{r^n, t^n\}$ is not a 'two sided ideal of Q .

**Theorem 2 . 7:**

A F sub-set $p$ of SS -$\pi$- regular semi group Q is an AN-F-T-S-I of Q if and only if is any AN-F-IN-I of Q .

**Proof :**

suppose that $p$ be an AN-F-T-S-I of Q , obviously , $p$ is AN-F-IN-I of Q.

**Conversely** :

suppose that $p$ is any AN-F-IN-I of Q, let a , b ∈ Q, since Q is SS -$\pi$- regular semi group so ∃ x , y , w , z ∈ Q S.t $a^n = a^n$ x $a^{2n}$ y; $b^n = b^n$ w $b^{2n}$ z we have $p(a^n b^n) = p((a^n$ x $a^{2n}$ y $) b^n) = p((a^n$x$)\, a^n (a^n$ y $b^n)) = p((s^n\ a^n t^n) \leq p(a^n)$

where $s^n = a^n$ x and $t^n = a^n$ y $b^n$

Also $p(a^n b^n) = p((a^n b^n$ w $b^{2n})$ z$) = p((a^n b^n$ w$)\, b^n (b^n$ z$))$ $= p((d^n\ a^n u^n) \leq p(a^n)$

where $d^n = a^n\ b^n$w and $u^n = b^n$z . Hence, $p$ is an AN-F-T-S-I of Q .

**Proposition 2 . 8 :**

Suppose that Q is SS -$\pi$- regular semi group, then :

1- Every AN-F-R-I is Idempotent.

2- Every AN-F-IN-I is Idempotent .

**Proof :**

1- Let $p$ is any AN-F-R-I of semi group Q , then it is clear that $p \subseteq p * p$ . since Q is a SS –$\pi$- regular so ; ∀ a ∈ Q , ∃ x , y ∈ Q , s.t $a^n = a^n$x $a^{2n}$y , for some n∈ $Z^+$, so we have

$(p * p)_{(a^n)} = \bigwedge_{a^n = a^n \text{x}\, a^{2n} \text{y}} \{p(a^n x) \vee p(a^n a^n y)\}$

$= \bigwedge_{a^n = a^n \text{x}\, a^{2n} \text{y}} \{p(a^n x) \vee p(a^n z)\}$ where z $= a^n$y

$\leq p(a^n$x$) \vee p(a^n z) \leq p(a^n) \vee p(a^n) = p(a^n)$ this implies that $p * p \subseteq p$ and

hence $p * p = p$ .

2- Suppose that $p$ is any AN-F-IN-I of semi group Q, then it is obvious that

$p \subseteq p * p$ . since Q is a SS -$\pi$- regular so, ∀ a ∈ Q, ∃ x , y ∈ $Q$, s.t $a^n = a^n x\, a^{2n} y$, for some n ∈ $Z^+$, so we have:

$a^n = a^n x\, a^{2n} y = a^n x a^n a^n y$

$= (a^n x) a^n (x a^{2n})(y a^n y)$

$= \wedge_{a^n = a^n x\, a^{2n} y.} \{p((a^n x) a^n (x a^{2n})) \vee p(y a^n y)\}$

$\leq p((a^n x) a^n (x a^{2n})) \vee p(y a^n y)$

$\leq p(a^n) \vee p(a^n) = p(a^n)$,

this implies that $p * p \subseteq p$. hence $p * p = p$.

**Proposition 2 . 9 : [8]**

Let $p$ is any AN-F-R-I and $\varphi$ AN-F- L-I of a semi group Q,then $p * \varphi \supseteq p \cup \varphi$

It is clear that from Proposition 2.9 $p * \varphi \supseteq p \cup \varphi$ , but the converse needs not at all be true . Consider the following example

**Example 2.10 :**

Consider the semi group Q={$r^n, s^n, t^n, v^n$} with the operation as follows :

| . | $r^n$ | $s^n$ | $t^n$ | $v^n$ |
|---|---|---|---|---|
| $r^n$ | $r^n$ | $r^n$ | $r^n$ | $r^n$ |
| $s^n$ | $r^n$ | $r^n$ | $r^n$ | $r^n$ |
| $t^n$ | $r^n$ | $r^n$ | $s^n$ | $r^n$ |
| $v^n$ | $r^n$ | $r^n$ | $s^n$ | $s^n$ |

The ideals of Q are $\{r^n\}$, $\{r^n, s^n\}$, $\{r^n, s^n, t^n\}$ and $\{r^n, s^n, t^n, v^n\}$

Let us define two F sub set $p$ and $\varphi$ of Q as follows

$p(r^n) = 0.5$ , $p(s^n) = 0.6$ , $p(t^n) = 0.7$ , $p(v^n) = 0.8$ ,

$\varphi(r^n) = 0.6$ , $\varphi(s^n) = 0.7$ , $\varphi(t^n) = 0.8$ , $\varphi(v^n) = 0.9$ .

⟹ $p$ and $\varphi$ are an anti F ideal of Q , and we note that:

$(p * \varphi)_{(s^n)} = \wedge_{s^n = x^n y^n} \{p((r^n x) r^n (x r^{2n})) \vee \varphi(y r^n y)\}$

$= \wedge\{0.8, 0.8, 0.9\} = 0.8 \geq (p \cup \varphi)_{(s^n)} = 0.7$

To consider the converse of proposition 2.9 , we need to streng then the condition of semi group Q.

**Theorem 2.11 :**

If $p$ , $\varphi$ are an AN-F-T-S-I of SS -$\pi$- regular semi group Q, Then $p * \varphi = p \cup \varphi$ .

**proof :**suppose that $p$ and $\varphi$ be an AN-F-T-S-I of Q, ⟹ obviously $p * \varphi \supseteq p \cup \varphi$ . since Q is a SS –$\pi$- regular so for each element a ∈ Q, ∃ x , y ∈ Q , s.t $a^n = a^n x a^{2n} y$, for some n ∈ $Z^+$,

so we have $(p * \varphi)_{(a^n)} = \wedge_{a^n = a^n x\, a^n a^n y.} \{p(a^n x) \vee \varphi(a^n a^n y)\}$

$\leq p(a^n x) \vee \varphi(a^n a^n y) \leq p(a^n) \vee \varphi(a^n) = (p \cup \varphi)(a^n)$ ⟹ $(p * p) \subseteq p \cup \varphi$.

Hence, $p * p = p \cup \varphi$ .

**Example 2.12 :**

Let Q= $\{r^n, s^n, t^n\}$be a semi group with the following table :

| . | $r^n$ | $s^n$ | $t^n$ |
|---|---|---|---|
| $r^n$ | $r^n$ | $s^n$ | $t^n$ |
| $s^n$ | $s^n$ | $s^n$ | $t^n$ |
| $t^n$ | $t^n$ | $t^n$ | $t^n$ |

Define a F $p$ sub - set of Q by $p(r^n) = 0.6$ , $p(s^n) = 0.5$, $p(t^n) = 0.4$ .

By routine calculation , we can check that $p$ is an AN-F-I , AN- F-IN-I and anti F bi- ideal , because Q is SS -$\pi$- S regular semi group .

Now , we give other F characterizations of SS -$\pi$ - regular semi group .

**Proposition (2 . 13) :**

For a F sub-set $p$ of an SS -$\pi$- regular semi group Q , the following conditions are equivalent :

**1-** $p$ is anti F bi- ideal of Q .

**2-** $p$ is an AN-F-G-BI-I of Q.

**Proof :**

**1⟶2** suppose that $p$ is any anti F bi-ideal of Q the obviously $p$ is an AN-F-G-BI-I of Q.

**2⟶1** suppose $p$ be any AN-F-G-BI-I of Q, and x, y ∈ Q, Then since Q is an SS -$\pi$- regular of a semi group, So; ∀ x ∈ S, ∃ a, b ∈ Q s.t $x^n = x^n$ a $x^{2n}$b . thus we have :

$p(x^n y^n) = p((x^n \text{ a } x^{2n} \text{b}) y^n) = p(x^n(\text{ a } x^{2n} \text{b}) y^n) = p((x^n w^n y^n)$

$\leq p(x^n) \vee p(y^n)$ where $w^n$ = a $x^{2n}$b .

Therefore, $p$ is an anti F sub-semi group of Q . Hence , $p$ is an anti F bi- ideals of Q.

**Theorem 2. 14 :**

In SS -$\pi$- regular semi group Q, $p * \varphi \leq p \vee \varphi$, for anti fuzzy bi-ideal $p$ and AN-F-R-I $\varphi$ .

**Proof :**

Let $p$ and $\varphi$ be any AN-F-BI-I and AN-F-R-I of Q , respectively and let a ∈ Q . Then since Q is an SS -$\pi$- regular of a semi group, ∃ x , y ∈ S s.t

$a^n = a^n$ x $a^{2n}$y then we have :

$(p * \varphi)_{(a^n)} = \wedge_{a^n = b^n c^n.} \{p(b^n) \vee \varphi(c^n)\} \leq p(a^n x\, a^n) \vee \varphi(a^n y)$

$\leq p(a^n) \vee \varphi(a^n) = (p \vee \varphi)(a^n)$

And so we have $p * \varphi \leq p \cup \varphi$ .

**Theorem 2.15 :**

Let $p$ and $\varphi$ be any AN-F-IN-I of SS- $\pi$- regular of a semi group Q, then $(p * \varphi) \vee (\varphi * p) \leq p \vee \varphi$

**Proof :**

suppose that $p$ , $\varphi$ be any AN-F-IN-I of Q, and a ∈ Q, ⟹ since Q is SS -$\pi$- regular,

∃ x , y ∈ Q s.t $a^n = a^n$ x $a^{2n}$y = $((a^n x) a^n$ $(x a^{2n}))(y a^n y)$ . Hence

$(p * \varphi)_{(a^n)} = \wedge_{a^n = b^n c^n.} \{p(b^n) \vee \varphi(c^n)\} \leq p((a^n x) a^n (x a^{2n})) \vee \varphi(y a^n y)$

$\leq p(a^n) \vee \varphi(a^n) = (p \vee \varphi)(a^n)$

And so we have $p * \varphi \leq p \vee \varphi$ .similarly, we have $(\varphi * p) \leq p \vee \varphi$

Therefore $(p * \varphi) \vee (\varphi * p) \leq p \vee \varphi$.

**Theorem 2. 16 :**

Let Q be an SS -$\pi$- regular semi group. Then $\varphi * \alpha * p \subseteq \varphi \cup \alpha \cup p$. for every AN-F-L–I $\alpha$ , every AN-F-G-BI-I $\varphi$ and every AN-F-IN-I $p$ of Q .

**Proof :**

Let $\varphi$ and $p$ be any AN-F-L-I, AN-F-G-BI-I, and AN-F-IN-I $p$ of Q, respectively and Let a ∈ Q, since Q is an SS -$\pi$- regular,

∃ x, y ∈ Q s.t $a^n = a^n$ x $a^{2n}$y = $(a^n x a^n\, x a^{2n} y a^n y)$,

Then we have :$(\varphi * \alpha * p)_{(a^n)}$ $= \wedge_{a^n = a^n x a^n \, x a^{2n} y a^n y.}\{\varphi(a^n x a^n) \vee (\alpha * p)(x a^{2n} y a^n y)\}$
$\leq \varphi(a^n) \vee \{\wedge_{x a^{2n} y a^n y.}\{\alpha(x a^{2n}) \vee p(y a^n y)\}$
$\leq \varphi(a^n) \vee \alpha(a^n) \vee p(a^n) = (\varphi \cup \alpha \cup p)(a^n)$
And so we have $\varphi * \alpha * p \subseteq \varphi \cup \alpha \cup p$ .
Now , we characterized SS –$\pi$- regular LA- semi groups by the properties of their F left (right , two sided) ideals ".
Let Q be a groupoid . Then
**1-** Q is called left almost semi group if (rs)t=(ts)r , ∀ r ,s ,t ∈ Q
**2-** Medial law of a left almost semi group means (r s) (t v) = (r t) (s v) ; ∀ r ,s ,t ,v ∈ Q
**3-** In additional if Q has a left identity (necessary unique) the paramedical law mean (r s) (t v) = (v s) (t r) ; ∀ r ,s ,t ,v ∈ Q
**4-** An left almost semi group with" right identity" becomes a commutative semi group with identity . if an LA- semi group contains" left identity" , the following law holds r (s t) = s (r t) ; ∀ r ,s ,t ∈ Q.

**Definition 2.17 :**
Any element a of LA-semi group Q is called SS –$\pi$- regular if ∃ x , y ∈ Q ,
s.t $a^n = (a^n x\, a^{2n})$ y, for some n ∈ $Z^+$,
and Q is called SS -$\pi$- regular if every elements of Q is SS –$\pi$- regular .

**Example 2.18 :**
Let Q = {1, 2, 3, 4, 5} be an LA - semi group with the "left identity (5)" with Then

| . | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 5 | 1 | 2 | 3 | 4 |
| 2 | 4 | 5 | 1 | 2 | 3 |
| 3 | 3 | 4 | 5 | 1 | 2 |
| 4 | 2 | 3 | 4 | 5 | 1 |
| 5 | 1 | 2 | 3 | 4 | 5 |

Then it is clear that, Q is a SS –$\pi$- regular because if n = 2
$1^2 = 1^2 3\, 1^4\, 2$ , $2^2 = 2^2 4\, 2^4\, 1$ , $3^2 = 3^2\, 5\, 3^4\, 5$ , $4^2 = 4^2 2\, 4^4\, 3$, $5^2 = 5^2 1\, 5^4 4$ .

**Proposition 2 . 19 :**
A F sub - set $p$ of an SS -$\pi$- regular semi group Q is a F right ideal iff it is a F left ideal .

**Proof :** suppose that $p$ is a F right ideal of Q. since Q is SS – $\pi$- regular so, ∀ a ∈ Q, ∃ x , y ∈ Q , s.t $a^n = (a^n x\, a^{2n})$ y ,for some n ∈ $Z^+$ So by (1)
$p(a^n b^n) = p(((a^n x\, a^{2n})\, y) b^n)$
$= p((b^n y)(a^n x\, a^{2n})) \geq p(b^n y) \geq p(b^n)$

**Conversely :**
suppose that $p$ is a F left ideal of Q, Then by (1)
$p(a^n b^n) = p(((a^n x\, a^{2n})\, y) b^n) = p((b^n y)(a^n x\, a^{2n}))$
$\geq p(d\, a^{2n}) \geq p(a^{2n}) \geq p(a^n)$.

**Lemma 2.20 :**
Every F two sided of SS -$\pi$- regular LA - semi group Q with " left identity " is idempotent .

**Proof :**
suppose that $p$ is a F two sided ideal of Q Then it is clear that $p \circ p \subseteq p \circ Q \subseteq p$, since Q is SS – $\pi$ - regular so ; ∀ a ∈ Q , ∃ x , y ∈ Q , s.t $a^n = (a^n x\, a^{2n})$ y,
for some n ∈ $Z^+$ So by (1) $a^n = (a^n x\, a^{2n})$ y = (y x $a^n a^n) a^n$, thus we have
$(p \circ p)_{(a^n)} = \vee_{a^n = (y\, x\, a^n a^n) a^n}\, p(y\, x\, a^n a^n) \wedge p(a^n)$
$\geq p(y\, x\, a^n a^n) \wedge p(a^n)$
$\geq p(a^n) \wedge p(a^n) = p(a^n)$.
And this implies that $p \circ p \supseteq p$, hence $p \circ p = p$ .

**Theorem 2 . 21 :**
For a F sub-set $p$ of SS -$\pi$- regular LA-semi group Q ,with "left identity" the following conditions are equivalent :
**1-** $p$ is a F two sided ideal of Q .
**2-** $p$ is a F-AN-I of Q .

**Proof :**
**1⟶2** suppose $p$ be a F two sided ideal of Q, ⟹ obviously $p$ is a F-IN-I of Q.
**2⟶1** Let $p$ is a F-IN-I of Q , and a , b ∈ Q ⟹ , since Q is an SS -$\pi$- regular of LA - semi group , So ∃ x , y , u , v ∈ Q s.t $a^n = a^n$ x $a^{2n}$y, $b^n = b^n$ u $b^{2n}$v , we have :
$p(a^n b^n) = p((a^n\, x\, a^{2n})y) b^n)$ by (1)
$= p((b^n y)(a^n x a^n a^n))$ by (2)
$= p((b^n a^n)(y x a^n a^n)) = p((b^n a^n) z^n) \geq p(a^n)$ , where $z^n = y x a^n a^n$
Also $p(a^n b^n) = p(a^n (b^n u\, b^n)\, v))$ by (4)
$= p((b^n u b^{2n})(a^n v))$
$= p((w^n b^n t^n) \geq p(b^n)$ ,
where $w^n = b^n u b^n$ and $t^n = a^n$v
Hence, $p$ is a F two sided ideal

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## دراسة حول المثاليات المنتظمة المضببة من النمط -$\pi$ -SS على شبه الزمرة

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**الملخص**

في هذا البحث تم تقديم تعريف المثاليات المضببة المنتظمة من النمط -$\pi$ -SS على اشباه الزمر كتعميم للمثاليات المضببة المنتظمة على اشباه الزمر. وتم دراسة بعض الخصائص الاساسية لها والحصول على بعض النتائج الجديدة .